JG HK KL AT

Name:	____________________
Class:	12MTX________
Teacher:	____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2012 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed - 2 HOURS
(Plus 5 minutes reading time)

Directions to candidates

- Attempt all questions
- Approved calculators may be used.
- Standard Integral Tables are provided at the back of this paper.
- Write your name and class in the space provided at the top of this question paper.

Section I - TOTAL MARKS 10

- To be answered on the removable answer grid at the back of the exam paper.
- Allow about 15 minutes for this section.

Section II - TOTAL MARKS 60

- All answers to be completed on the writing paper provided. Each question is to be commenced on a new page clearly marked Question 11, Question 12, etc on the top of the page. Write your name and class at the top of each page.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Allow about 1 hour and 45 minutes for this section.

YOUR ANSWERS WILL BE COLLECTED IN ONE BUNDLE. THE MULTIPLE CHOICE SECTION I ON TOP AND THEN WRITTEN ANSWERS TO SECTION II AND THEN THE QUESTION PAPER.

SECTION I 10 MARKS

INSTRUCTIONS

- Attempt all questions
- Allow about 15 minutes for this section
- Section I answers are to be completed on the multiple-choice answer sheet attached to the back of this question paper.
- Select the alternative A, B, C or D that best answers the question

1. What is the acute angle between the lines $y = 2x - 1$ and $x - 3y + 6 = 0$?

(A) 18°

(B) 45°

(C) 63°

(D) 82°

2. Let α, β and γ be the roots of $x^3 - x^2 + cx + 12 = 0$. It is known that two roots are equal in magnitude but opposite in sign. What is the value of c?

(A) -12

(B) $-2\sqrt{3}$

(C) $2\sqrt{3}$

(D) 12

3. $P(x) = x^3 + 4x^2 - 5x + 4$ divided by $x - 2$, expressed in the form of $P(x) = Q(x).A(x) + R(x)$ is

(A) $P(x) = (x-2)(x^2 + 2x - 1) + 2$

(B) $P(x) = (x-2)(x^2 + 6x - 17) - 30$

(C) $P(x) = (x-2)(x^2 + 6x + 7) + 18$

(D) $P(x) = (x-2)(x^2 + 2x - 9) - 14$

4. The curve $y = \sin x$ is rotated about the x-axis from $x = 0$ to $x = \dfrac{\pi}{2}$. Find the volume of the solid formed.

(A) $\dfrac{\pi}{4}(\pi - 1)$ units^3

(B) $\dfrac{\pi^2}{4}$ units^3

(C) $\dfrac{\pi}{2}$ units^3

(D) $\dfrac{\pi}{4}$ units^3

5. If the velocity v of a particle moving on the x-axis is given by

$$v^2 = -3x^2 + 20x + 7$$

Which of the following expresses its acceleration in terms of x?

(A) $\ddot{x} = -3\left(x - 3\frac{1}{3}\right)$

(B) $\ddot{x} = -3(x-2)$

(C) $\ddot{x} = -3(x-3)$

(D) $\ddot{x} = -2(x-2)$

6. Find the sum of the coefficients of $(1 + x)^{16}$

(A) 131 072

(B) 65 536

(C) 17

(D) 32 768

7. Evaluate $\int_e^{e^2} \frac{dx}{x \ln 2}$

(A) ln2

(B) $\frac{1}{\ln 2}$

(C) $\frac{-1}{2e^2}$

(D) $\frac{1}{2e^2}$

8. It is known that $\int_0^4 f(x)\,dx = 6$. Hence, the value of $\int_3^7 f[(x-3)+2]\,dx$ is

(A) 8

(B) 18

(C) 14

(D) 16

9. If $\frac{dN}{dt} = 0.1(N - 100)$ and $N = 300$ when $t = 0$, which of the following is true ?

(A) $N = 200 + 100e^{0.1t}$

(B) $N = 300 + 100e^{0.1t}$

(C) $N = 100 + 200e^{0.1t}$

(D) $N = 100 + 300e^{0.1t}$

10. If $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 \ldots\ldots + \binom{n}{n}x^n$,

then the expression $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots\ldots + n\binom{n}{n} =$

(A) 0
(B) $n(2)^{n-1}$
(C) 1
(D) 2^n

END OF SECTION 1

SECTION II 60 MARKS
INSTRUCTIONS

- Answer all questions on the writing paper provided
- Allow about 1 hour and 45 minutes for this section
- Begin each question on a new page.
- Show all necessary working.

Question 11 (15 marks) BEGIN A NEW PAGE **Marks**

(a) A particle moves in a straight line and its position x metres at time t seconds is given by

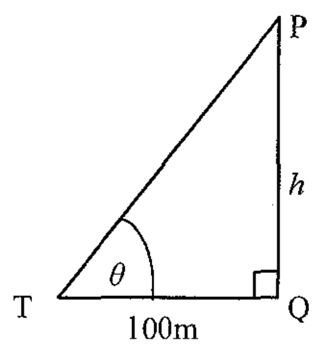
$$x = 2 + \sin 4t + \sqrt{3}\cos 4t.$$

(i) By first expressing $\sin 4t + \sqrt{3}\cos 4t$ in the form of $R\sin(4t + \alpha)$, prove that it is undergoing simple harmonic motion. **4**

(ii) Find the equilibrium position and the amplitude of the motion. **1**

(iii) Find the maximum speed of the particle. **1**

(b) A parachutist, P, jumps out of a plane at height *h(t)* metres above the ground and by the time he reaches 3000 m, he is falling at a constant rate of 5.5 m/s. Point Q is on horizontal ground directly below him. An observer at T is 100 m from Q and the angle of elevation from this point to the parachutist is *θ(t)* radians.



(i) Show that $\dfrac{dh}{d\theta} = \dfrac{100}{\cos^2\theta}$. **1**

(ii) Show that the rate of decrease of the angle of elevation when $h = 2000$ m is 0.000137 rad/s. **3**

(c) Use the substitution $u = \tan\theta$ to find the exact value of this integral **2**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{\tan\theta}\,d\theta.$$

(d) Find all values of x for which $\dfrac{6}{x} \ge x - 1$ **3**

Question 12 (15 marks) BEGIN A NEW PAGE **Marks**

(a) Differentiate $[1 + \cos^{-1}(3x)]^3$ with respect to x. **2**

(b) Write down the general solution of $\sqrt{3}\tan\theta - 1 = 0$.
Leave your answer in exact radian form. **1**

(c) (i) Rewrite $-3 - x^2 - 4x$ in the form $b^2 - (x + a)^2$ where a and b are integers. **1**

(ii) Hence, or otherwise, evaluate $\int \dfrac{dx}{\sqrt{-3 - x^2 - 4x}}$. **1**

(d) Consider the function $f(x) = \sin^{-1}(x - 1)$.

(i) State the domain and range of $y = f(x)$. **2**

(ii) Draw the graph of $y = f(x)$. **1**

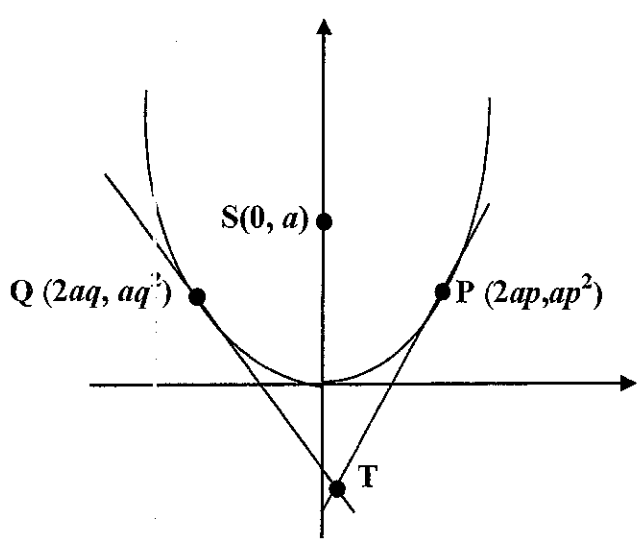
(iii) The area bounded by the curve $y = f(x)$, the y-axis and the line $y = \frac{\pi}{2}$ is rotated about the y- axis. Find the volume of the solid formed. **2**

(e) Consider the parabola $4ay = x^2$ where the focal length is a, $(a > 0)$, and the tangents at P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ intersect at the point T.
Let S $(0,a)$ be the focus of the parabola.

(i) Find the coordinates of T.
(You may assume that the equation of the tangent at P is $y = px - ap^2$) **1**

(ii) Show that the length $SP = a(p^2 + 1)$ **1**

(iii) Suppose P and Q move on the parabola in such a way that $SP + SQ = 4a$. Show that T is constrained to move on a parabola. **3**



Question 13 (15 marks) BEGIN A NEW PAGE **Marks**

(a) A particle is projected from the top of a cliff 200m high. The horizontal and vertical components of the velocity when t = 0 are $20\sqrt{3}\,m/s$ and $30m/s$ respectively.

(i) Determine the parametric equations of the path of the stone after t seconds. (take $g = 10m/s^2$) **2**

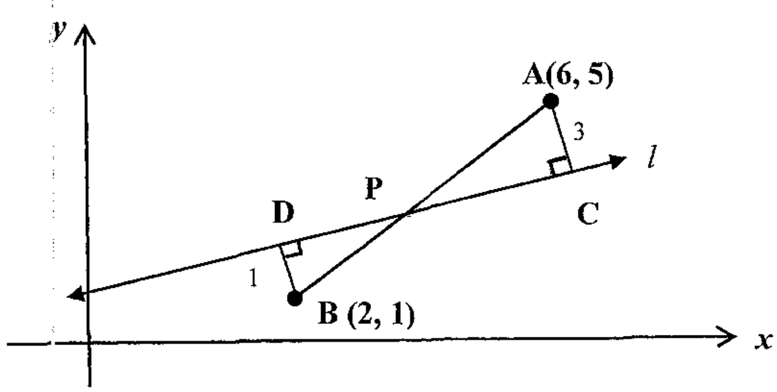
(ii) Find when the particle hits the ground. **2**

(iii) Find the velocity and the angle of impact of the particle when it hits the ground. **3**

(b) Evaluate $\lim\limits_{t \to 0} \left[\dfrac{4\sin^2 t}{t^2} \right]$ **1**

(c) The equation $2\cos^3\theta - \cos^2\theta + \cos\theta - 1 = 0$ has solutions $\cos a$, $\cos b$ and $\cos c$. Prove that $\sec a + \sec b + \sec c = 1$. **2**

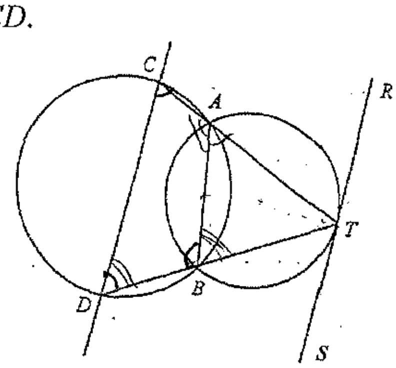
(d)



The points $A(6, 5)$ and $B(2, 1)$ are 3 units and 1 unit respectively from the line l and are on opposite sides of l, as shown in the diagram.

Find the coordinates of the point P, where the interval AB crosses the line l. **3**

(e) Two unequal circles intersect at A and B. The line RS is a tangent to the smaller circle at T. The lines TA and TB meet the larger circle at C and D respectively. Prove that $RS \parallel CD$. **2**



Question 14 (15 marks) BEGIN A NEW PAGE **Marks**

(a) (i) Prove that $\tan(x + h) - \tan x = \dfrac{\sin h}{\cos(x + h)\cos x}$. **2**

(ii) Hence, find the derivative of $\tan x$ from first principles. **1**

(b) (i) Show that $(1 + x)^{2n}\left(1 - \dfrac{1}{x}\right)^{2n} = \left(x - \dfrac{1}{x}\right)^{2n}$. **1**

(ii) Hence, by equating the constant terms, deduce that

$({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 \dots + ({}^{2n}C_{2n})^2 = (-1)^n\ {}^{2n}C_n$. **2**

(c) (i) Show that the equation $2x^3 - 3x^2 + 0.999 = 0$ has a root near $x = 1$. **1**

(ii) Explain why Newton's method fails if the first approximation taken for $2x^3 - 3x^2 + 0.999 = 0$ is $x = 1$. **2**

(iii) Using $x = 1.5$ find, by one application of Newton's method, a better approximation of the root of the equation $2x^3 - 3x^2 + 0.999 = 0$. **2**

(d) Prove by mathematical induction, $\sum_{j=1}^{n} \sin(2j - 1)x = \dfrac{1 - \cos 2nx}{2\sin x}$ **4**

Hint : You may find $\sin(2k + 1)x = \sin(2kx + x)$ useful.

END OF THE PAPER

Ext 1 AP4

MCQ ANSWERS

1) B

2) A

3) C

4) B

5) A

6) B

7) B

8) C

9) C

10) B

Q11)

a) (i) $x = 2 + \sin 4t + \sqrt{3}\cos 4t$

$$\sin 4t + \sqrt{3}\cos 4t = R\sin(4t+\alpha)$$
$$= R[\sin 4t\cos\alpha + \cos 4t\sin\alpha]$$
$$= R\cos\alpha\sin 4t + R\sin\alpha\cos 4t$$

$\therefore \left.\begin{array}{l} R\cos\alpha = 1 \\ R\sin\alpha = \sqrt{3}\end{array}\right\}$ $\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

(Both R and α) ✓

$$R^2\sin^2\alpha + R^2\cos^2\alpha = 3 + 1 = 4$$
$$R^2 = 4 \Rightarrow R = 2$$

$$x = 2 + \sin 4t + \sqrt{3}\cos 4t$$
$$\left.\begin{array}{l} x = 2 + 2\sin(4t + \frac{\pi}{3}) \\ \dot{x} = 4 \times 2\cos(4t + \frac{\pi}{3}) \\ \ddot{x} = -16 \times 2\sin(4t + \frac{\pi}{3}) \end{array}\right\} ✓$$
$$\ddot{x} = -16(x-2) \quad ✓ \quad \text{as } x - 2 = 2\sin(4t + \tfrac{\pi}{3})$$

$\therefore$ It is undergoing SHM.

(ii) $\left.\begin{array}{l}\text{Equilibrium position} = 2 \\ \text{amplitude} = 2\end{array}\right\}$ ✓

(iii) Max. speed = 8 ms^{-1} ✓

Q11) (b)

(i) $\tan\theta = \frac{h}{100}$

$\therefore h = 100\tan\theta$ ✓

$\frac{dh}{d\theta} = 100\sec^2\theta = \frac{100}{\cos^2\theta}$

(ii) $\frac{d\theta}{dt} = \frac{d\theta}{dh} \times \frac{dh}{dt}$ given $\frac{dh}{dt} = -5.5$

when $h = 2000$, $\tan\theta = \frac{h}{100}$

$\therefore \theta = \tan^{-1}(20) = 1.52084$ ✓

$\therefore \frac{d\theta}{dt} = \frac{\cos^2(1.52084)}{100} \times -5.5$ ✓

$= -0.000137$ rad s^{-1}. ✓ (-1 for not showing negative)

$\therefore$ rate of decrease $= 0.000137$

(c) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{\tan\theta}\, d\theta$

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$\theta = \frac{\pi}{3},\ u = \sqrt{3}$

$\theta = \frac{\pi}{6},\ u = \frac{1}{\sqrt{3}}$

$= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{du}{u}$ ✓

$= \left[\ln u\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$= \ln\sqrt{3} - \ln\frac{1}{\sqrt{3}}$

$= \ln\left(\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}\right) = \ln 3$ ✓

Q11) (d)

$\frac{6}{x} \geq x - 1 \quad (x \neq 0)$

$(\times x^2)$ $\quad 6x \geq x^2(x-1)$ ✓

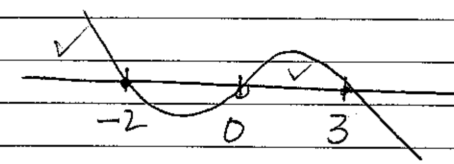
$6x - x^2(x-1) \geq 0$

$x[6 - x(x-1)] \geq 0$

$x[6 - x^2 + x] \geq 0$

$x(6 + x - x^2) \geq 0$

$x(2 + x)(3 - x) \geq 0$ ✓



$x \leq -2$, $0 < x \leq 3$ ✓

(-1 for $0 \leq x \leq 3$)

Q12) (a)

$$\frac{d}{dx}\left[1+\cos^{-1}(3x)\right]^3 = 3\left[1+\cos^{-1}(3x)\right]^2\left[\frac{-3}{\sqrt{1-9x^2}}\right]$$

(b) $\tan\theta = \frac{1}{\sqrt{3}}$

$\therefore\ \theta = n\pi + \frac{\pi}{6}$

(c) (i) $-\left[x^2+4x+3\right] = -\left[x^2+4x+2^2-2^2+3\right]$

$= -\left[(x+2)^2-1\right]$

$= 1-(x+2)^2$

(ii) $\displaystyle\int \frac{dx}{\sqrt{-3-x^2-4x}} = \int \frac{dx}{\sqrt{1-(x+2)^2}}$

$= \sin^{-1}(x+2) + C$

Q12) d) $f(x) = \sin^{-1}(x-1)$

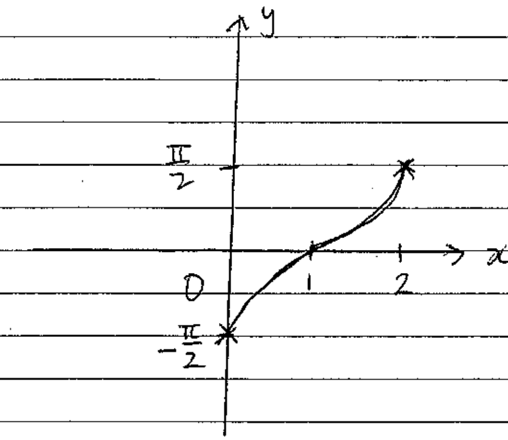
(i) Domain

$-1 \le x-1 \le 1$

$0 \le x \le 2$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(ii)



(iii)

Volume $= \pi\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\, dy$

$y = \sin^{-1}(x$

$\sin y = x-1$

$\sin y + 1 = x$

$= \pi\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin y+1)^2\, dy$

$= \pi\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 y + 2\sin y + 1\, dy$

$= \pi\left[\frac{y}{2} - \frac{\sin 2y}{4} + y - 2\cos y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= \pi\left[\frac{3y}{2} - \frac{\sin 2y}{4} - 2\cos y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$\frac{3\pi^2}{2}$

Q12) (e)

(i) $y = px - ap^2$ (tangent at P) — ①

$y = qx - aq^2$ (tangent at Q).

$\therefore \quad px - ap^2 = qx - aq^2$

$(p-q)x = a(p^2-q^2)$

$\cancel{(p-q)}x = a(p+q)\cancel{(p-q)}$

$x = a(p+q)$

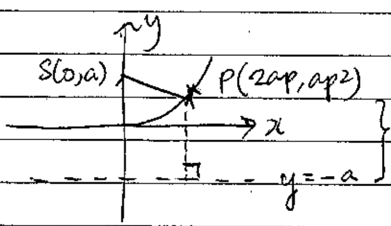
Sub. into ①

$y = pa(p+q) - ap^2$

$y = ap^2 + apq - ap^2 = apq.$

$\therefore$ T has coordinates $T(a(p+q), apq)$ ✓

(ii) Using definition,



$ap^2 - (-a) = ap^2 + a = a(p^2+1)$ ✓

(or use distance formula to find SP.)

12 (e) (iii)

$SP = a(p^2+1)$

Similarly, $SQ = a(q^2+1)$

$SP + SQ = 4a \quad \Rightarrow \quad \cancel{a}(p^2+1) + \cancel{a}(q^2+1) = 4\cancel{a}$

$p^2 + q^2 = 2$ ✓

For $T(a(p+q), apq)$

$x = a(p+q) \qquad y = apq$

$p+q = \frac{x}{a} \quad , \quad pq = \frac{y}{a}$

$(p+q)^2 = \frac{x^2}{a^2}$

$p^2 + 2pq + q^2 = \frac{x^2}{a^2}$

Working ✓

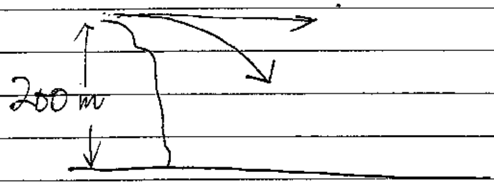
But $p^2+q^2 = 2$, $\quad 2 + 2pq = \frac{x^2}{a^2}$

and $pq = \frac{y}{a}$, $\quad 2 + \left(\frac{2y}{a}\right) = \frac{x^2}{a^2}$ ✓

$2a^2 + 2ay = x^2$ which is a parabola.

Q13)

(a) (i)



$\ddot{y} = -10$	$\ddot{x} = 0$
$\dot{y} = -10t + C$	$\dot{x} = C_2$
$t=0,\ \dot{y} = 30 = -10(0) + C$	$t=0,\ \dot{x} = 20\sqrt{3} = C_2$
$\therefore \dot{y} = 30 - 10t$	$\therefore \dot{x} = 20\sqrt{3}$
$y = 30t - 5t^2 + C_1$	$x = 20\sqrt{3}t + C_3$
$t=0,\ y = 200 = 0 + C_1$	$t=0,\ x = 0 + C_3 = 0$
$\therefore y = 30t - 5t^2 + 200$ ✓	$\therefore x = 20\sqrt{3}t.$ ✓

(~~to~~ gain 1 m all workings must be shown correctly.)

(ii) $y = 0$

$30t - 5t^2 + 200 = 0$

$6t - t^2 + 40 = 0$

$t^2 - 6t - 40 = 0$ ✓ working

$(t - 10)(t + 4) = 0$

$t = 10s$ ✓

Q13 (a) (iii)

When $t = 10$,

$\dot{x} = 20\sqrt{3}$,

$\dot{y} = 30 - 10(10) = -70.$

$V^2 = \dot{x}^2 + \dot{y}^2 = 70^2 + 1200$

$\therefore V = -10\sqrt{61}$ ✓ (0 for not showing negat[ive] sign.)

$\tan\theta = \dfrac{\dot{y}}{\dot{x}} = \dfrac{-70}{20\sqrt{3}}$ ✓

$\therefore \theta = \tan^{-1}(-2.0207\ldots) = 116.3^\circ$

$= 116^\circ$ ✓

(must be obtuse.)

(−1 for showing acute.).

Q13) b)

$$\lim_{t \to 0} \left[\frac{4 \sin t}{t} \cdot \frac{\sin t}{t} \right] = 4 \quad \checkmark$$

(c) $2\cos^3\theta - \cos^2\theta + \cos\theta - 1 = 0$

$A = 2$
$B = -1$
$C = 1$
$D = -1$

$\cos a$, $\cos b$, $\cos c$ are roots.

$\therefore \sec a + \sec b + \sec c$

$$= \frac{1}{\cos a} + \frac{1}{\cos b} + \frac{1}{\cos c}$$

$$= \frac{\cos b \cos c + \cos a \cos c + \cos a \cos b}{\cos a \cos b \cos c} \quad \checkmark$$

1 – showing sum of roots two at a time

$$= \frac{\text{Sum of roots 2 at a time}}{\text{Product of roots}}$$

1 – showing product of root.

$$= \frac{C/A}{-D/A} \quad \checkmark$$

$$= \frac{C}{-D} = \frac{1}{-(-1)} = 1.$$

Q13)

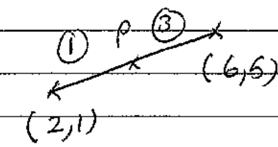
d) $\angle BDP = \angle ACP = 90^\circ$ (given)
$\angle DPB = \angle CPA$ (Vertically opp. $\angle$s)
$\therefore \Delta DPB \parallel\!\mid \Delta CPA$ (equiangular) ✓

$$\frac{DB}{CA} = \frac{DP}{CP}$$

$$\frac{1}{3} = \frac{DP}{CP} \qquad \therefore DP : CP = 1 : 3. \quad \checkmark$$

(show all work for similarity)

$$P(x,y) = \left(\frac{2(3) + 6(1)}{4}, \frac{1(3) + 5(1)}{4} \right)$$



$= (3, 2)$. ✓

e) $\angle RTA = \angle ABT$ (alt seg. theorem)
$\angle ABT = \angle ACD$ (ext. $\angle$ of cyclic quad.) ✓

As $\angle ACD = \angle RTA$,
$CD \parallel RS$ (equal alternate angles imply parallel lines) ✓

Q14 a) (i) $\tan(x+h) - \tan x = \dfrac{\sin h}{\cos(x+h)\cos x}$

$$LHS = \frac{\tan x + \tan h}{1 - \tan x \tan h} - \tan x$$

$$= \frac{\cancel{\tan x} + \tan h - \cancel{\tan x} + \tan^2 x \tan h}{1 - \tan x \tan h}$$

$$= \frac{\tan h + \tan^2 x \tan h}{1 - \tan x \tan h}$$ ✓

$$= \frac{\frac{\sin h}{\cos h}(1 + \tan^2 x)}{1 - \tan x\left(\frac{\sin h}{\cos h}\right)}$$

$$= \frac{\sin h\,(1 + \tan^2 x)}{\cos h - \tan x \sin h}$$

$$= \frac{\sin h\left(1 + \frac{\sin^2 x}{\cos^2 x}\right)}{\cos h - \frac{\sin x}{\cos x}\sin h}$$

$$= \frac{\sin h\,(\cos^2 x + \sin^2 x)}{\cos h \cos^2 x - \sin x \sin h \cos x}$$

$$= \frac{\sin h}{\cos x(\cos h \cos x - \sin x \sin h)}$$ ✓

$$= \sin h$$

Q14

a) (ii) $$\frac{d}{dx}[\tan x] = \lim_{h\to 0}\frac{\tan(x+h) - \tan x}{h}$$

$$= \lim_{h\to 0}\frac{\sin h}{\cos x\,\cos(h+x)\cdot h}$$

$$= \lim_{h\to 0}\frac{\sin h}{h}\cdot\frac{1}{\cos x\cos(x+h)}$$ ✓

$$= 1 \times \frac{1}{\cos x \cos x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

as $\lim_{h\to 0}\frac{\sin h}{h} = 1$.

(Need to show all necessary steps to get 1 m)

alt solution to (a)(i)

$$\tan(x+h) - \tan x = \frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}$$

$$= \frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos x\cos(x+h)}$$

$$= \frac{\sin[(x+h) - x]}{\cos x\cos(x+h)}$$

$$= \frac{\sin h}{\cos x\cos(x+h)}$$

Q14)

b)(i) $(1+x)^{2n}\left(1-\frac{1}{x}\right)^{2n}$

$= \left[(1+x)\left(1-\frac{1}{x}\right)\right]^{2n}$ ✓

$= \left[1-\frac{1}{x}+x-1\right]^{2n}$

$= \left(x-\frac{1}{x}\right)^{2n}$

(ii) RHS, $T_{r+1} = \binom{2n}{r}x^{2n-r}\left(-\frac{1}{x}\right)^r$

$= \binom{2n}{r}x^{2n-r}(-1)^r(x)^{-r}$

$= \binom{2n}{r}x^{2n-2r}(-1)^r$

Let $r=n$, then $\binom{2n}{n}(-1)^n$ ✓

is the coef. of the constant term

LHS, $\left[\binom{2n}{0}+\binom{2n}{1}x+\binom{2n}{2}x^2+\cdots\cdots+\binom{2n}{2n}x^{2n}\right]\times$

$\left[\binom{2n}{0}-\binom{2n}{1}\left(\frac{1}{x}\right)+\binom{2n}{2}\left(\frac{1}{x^2}\right)-\cdots\cdots\binom{2n}{2n}\left(-\frac{1}{x}\right)^{2n}\right]$

$\therefore$ Constant term $=\binom{2n}{0}\binom{2n}{0}-\binom{2n}{1}\binom{2n}{1}+\binom{2n}{2}\binom{2n}{n}+\cdots\binom{2n}{2n}\binom{2n}{2n}$ ✓

Equating constants from both sides,

$\binom{2n}{n}(-1)^n = \binom{2n}{0}^2-\binom{2n}{1}^2+\binom{2n}{2}^2+\cdots\cdots+\binom{2n}{2n}^2$

Q14)

c) $f(x) = 2x^3-3x^2+0.999$

(i) $f(1) = 2(1)-3(1)+0.999 = -0.001$

$\therefore$ There is a root close to the root. ✓

(ii) $f'(x) = 6x^2-6x$

~~at $x=1$, $f'(1)=6(1)^2-6(1)=0$~~. ✓

It is a stationary point, $\therefore$ there is no ✓ tangent which will intersect the x-axis.

(iii) $f'(x) = 6x^2-6x$

~~$f'(1.5) = 4.5$~~

~~$f(1.5) = 0.999$~~

~~$x_2 = 1.5 - \frac{f(1.5)}{f'(1.5)}$~~ $= 1.278$ ✓ (calculator readout)

✓

Q 14)

d) $\sum_{j=1}^{n} \sin(2j-1)x = \frac{1-\cos 2nx}{2\sin x}$

Step 1 : Test $n=1$

$LHS = \sin(2(1)-1)x = \sin x$

$RHS = \frac{1-\cos 2x}{2\sin x} = \frac{1-(1-2\sin^2 x)}{2\sin x} = \sin x$

It is true for $n=1$ ✓

Step 2: Assume that it is true for $n=k$.

$\sum_{j=1}^{k} \sin(2j-1)x = \frac{1-\cos 2kx}{2\sin x}$

Step 3: Need to prove that it is true for $n=k+1$

i.e: $\sum_{j=1}^{k+1} \sin(2j-1)x = \frac{1-\cos 2(k+1)x}{2\sin x}$

$LHS = \sum_{j=1}^{k} \sin(2j-1)x + \sin(2(k+1)-1)x$

$= \frac{1-\cos 2kx}{2\sin x} + \sin(2k+1)x$ (from step 2) ✓

$= \frac{1-\cos 2kx + 2\sin x \sin(2k+1)x}{2\sin x}$

$= \frac{1-\cos 2kx + 2\sin x \sin(2kx+x)}{2\sin x}$

$= \frac{1-\cos 2kx + 2\sin x(\sin 2kx\cos x + \cos 2kx \sin x)}{2\sin x}$

$= \frac{1-\cos 2kx + 2\sin x \sin 2kx \cos x + 2\cos 2kx \sin^2 x}{2\sin x}$ ✓

$= \frac{1-\cos 2kx + \sin 2kx \sin 2x + \cos 2kx(1-\cos 2x)}{2\sin x}$

$= \frac{1-\cancel{\cos 2kx} + \sin 2kx \sin 2x + \cancel{\cos 2kx} - \cos 2kx \cos 2x}{2\sin x}$

$= \frac{1-(\cos 2kx \cos 2x - \sin 2kx \sin 2x)}{2\sin x}$ ✓

$= \frac{1-\cos(2kx+2x)}{2\sin x}$

$= \frac{1-\cos 2(k+1)x}{2\sin x} = RHS$.

∴ It is true for $n=k+1$.

As it is true for $n=1$, it must be true for $n=2,3\ldots$ and so on as it is true for $n=k+1$. ∴ By induction, it is true for all $n \geq 1$.