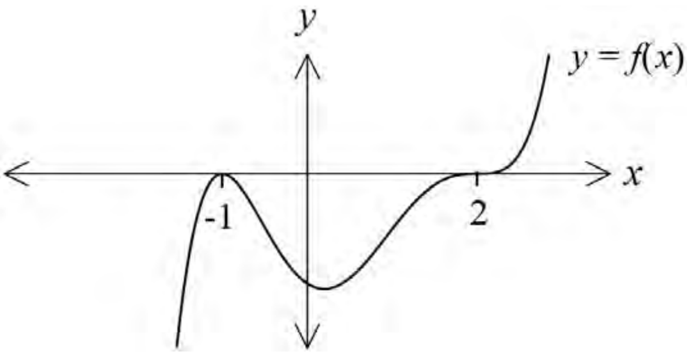
Section I

10 marks
Attempt Questions 1 – 10
Use the multiple choice answer sheet located at the back of the paper.
Allow about 15 minutes for this section

---

1. Which of the following could be the equation of the polynomial *P(x)*?



(A) $P(x)=(x-1)(x+2)$

(B) $P(x)=(x+1)^2(x-2)^3$

(C) $P(x)=(x-1)^2(x+2)^3$

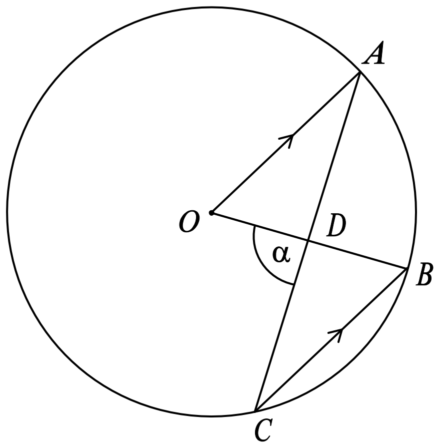
(D) $P(x)=(x+1)(x-2)^3$

2. $\sin 2\alpha \cos 2\alpha =$

(A) $\frac{1}{2}\sin 4\alpha$ (B) $4\sin\alpha\cos\alpha$ (C) $\frac{1}{2}\sin 2\alpha$ (D) $2\sin 2\alpha$

3. The points *A*, *B* and *C* lie on the circle with centre *O*. *OA* is parallel to *CB*. *AC* intersects *OB* at *D* and $\angle ODC = \alpha$.

What is the size of $\angle OAD$ in terms of α?



(A) $\frac{\alpha}{2}$ (B) $\frac{\alpha}{3}$ (C) $\frac{2\alpha}{3}$ (D) 3α

4. If $x = t^2$ and $y = \sqrt{t}$ which of the following is an expression for $\dfrac{dy}{dx}$?

(A) $t^{\frac{1}{2}}$

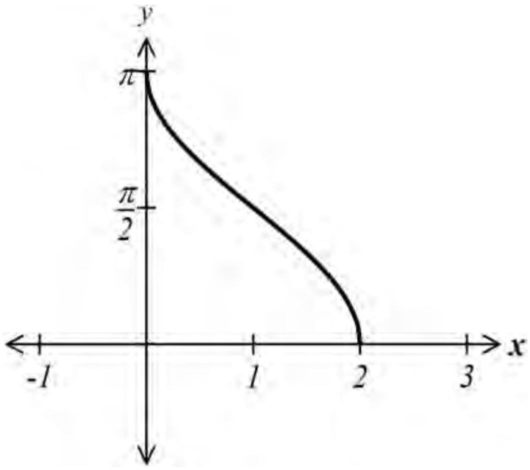
(B) $x^{-\frac{1}{4}}$

(C) $\dfrac{3}{2}x^{-\frac{1}{2}}$

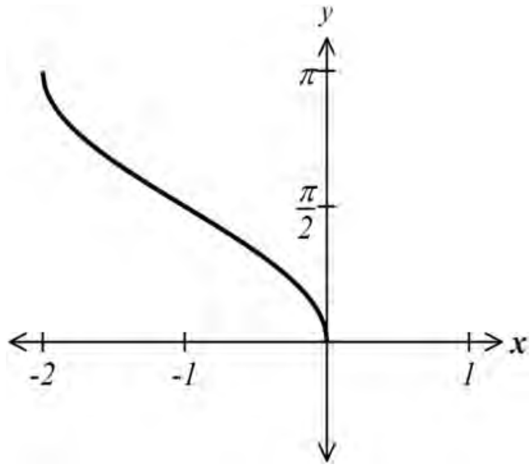
(D) $\dfrac{1}{4}t^{-\frac{3}{2}}$

5. Which of the following represents the graph of $y = \cos^{-1}(x+1)$?

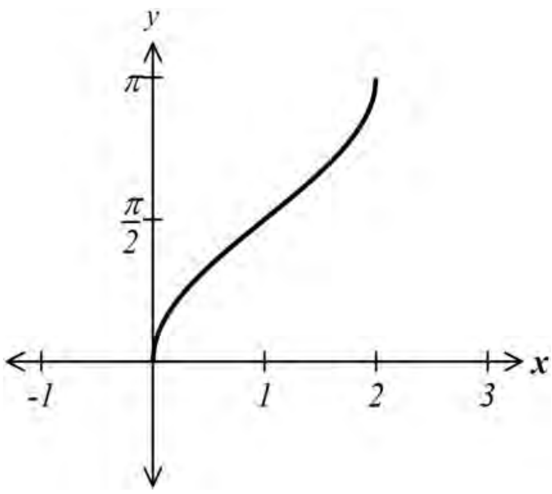
(A)



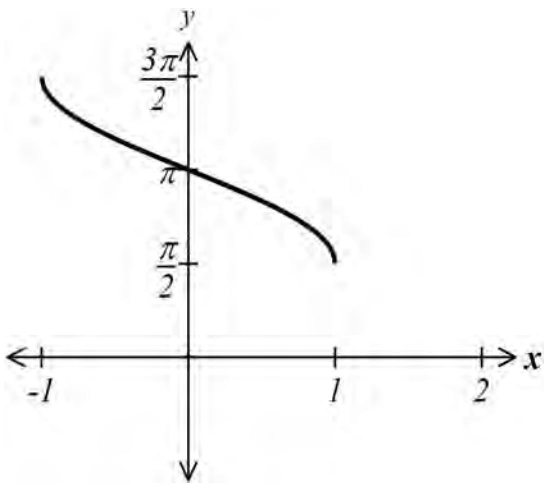
(B)



(C)



(D)



6. What is the indefinite integral for $\int\left(2\sin^2 x - x^2\right)dx$?

(A) $2x - \sin 2x - \frac{x^3}{3} + C$

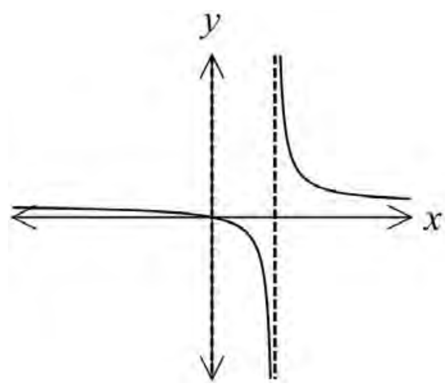
(B) $x - \frac{1}{2}\sin 2x - \frac{x^3}{3} + C$

(C) $2x - \cos 2x - \frac{x^3}{3} + C$

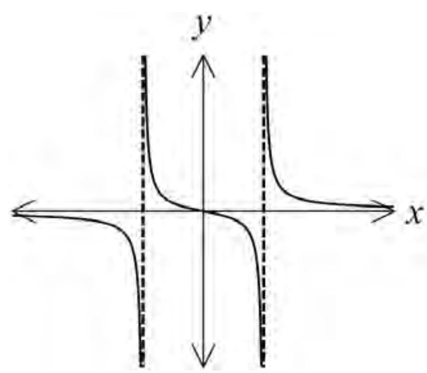
(D) $x - \frac{1}{2}\cos 2x - \frac{x^3}{3} + C$

7. Which of the following is a graph of $y = \frac{x^2}{x^2 - 4}$?

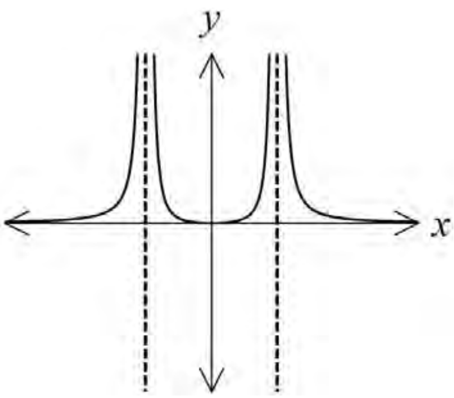
(A)



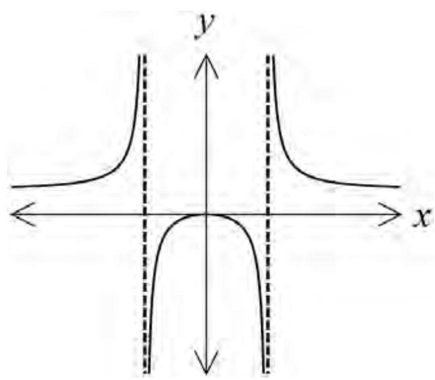
(B)



(C)



(D)



8. If $u = x^3 + 1$ then $\int_0^1 x^2 (x^3+1)^3 \, dx$ is equivalent to,

(A) $\frac{1}{3}\int_0^1 u^3 du$

(B) $3\int_0^1 u^3 du$

(C) $\frac{1}{3}\int_1^2 u^3 du$

(D) $3\int_1^2 u^3 du$

9. The coefficient of x^5 in the expansion of $\left(2x - \frac{3}{x}\right)^9$ is given by,

(A) $-{}^9C_5 (2)^4 (3)^5$

(B) ${}^9C_2 (2)^7 (3)^2$

(C) ${}^9C_5 (2)^5 (3)^4$

(D) $-{}^9C_2 (2)^2 (3)^7$

10. An object moves in a straight line so that at time *t* its displacement from a fixed origin is *x* and its velocity is *v*. The acceleration is $5-6x$. Which of the following is the correct equation for velocity given that $v = 2$ when $x = 1$?

(A) $v = 5x - 3x^2$

(B) $v = \sqrt{5x - 3x^2}$

(C) $v = \sqrt{10x - 6x^2}$

(D) $v = \sqrt{20x - 12x^2}$

Section II
60 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.
In questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of k if $x-4$ is a factor of $P(x) = x^3 - 3kx^2 + 32$. **1**

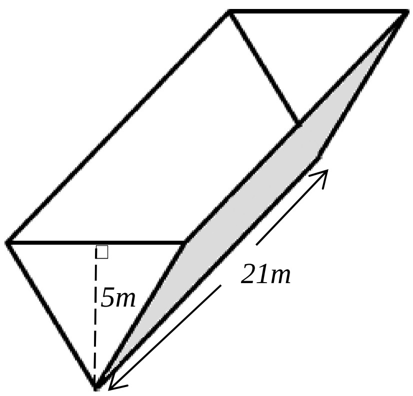
(b) Evaluate $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2}$, giving your answer as an exact value. **3**

(c) Find the acute angle between the lines $x-y+2=0$ and $2x-y-1=0$, to the nearest minute. **2**

(d) Solve $\dfrac{1}{3x} \geq \dfrac{1}{x+2}$ **3**

(e) Solve $\cos\theta(2\sin\theta - 1) = 0$ and express your answers as general solutions. **2**

(f) A water trough has a vertical cross section in the shape of an equilateral triangle as shown in the diagram. It is initially empty and is being filled with water at the rate of 4 cubic metres per hour.



(i) Given that the trough is 5 metres high and 21 metres long, show that the volume of water in the trough at depth D metres is given by, **2**

$$V = 7\sqrt{3}D^2$$

(ii) Find the exact rate at which the water level is rising when the water has a depth of $1\cdot5$ metres? **2**

Question 12 (15 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Use mathematical induction to prove that **3**

$$\sum_{r=1}^{n} \ln\left(\frac{r}{r+2}\right) = \ln\left(\frac{2}{(n+1)(n+2)}\right) \text{ for } n \geq 1.$$

(b) The height of a giraffe has been modeled using the equation

$$H = 5.40 - 4.80e^{-kt}$$

where H is the height in metres, t is the age in years and k is a positive constant.

(i) If a 6 year old giraffe has a height of 5.16 metres, find the value of k. **1**

(ii) Find the limiting height of the giraffe. **1**

(c) Use the substitution $t = \tan\frac{x}{2}$ to show that $\dfrac{1+\cos x+\sin x}{1-\cos x+\sin x} = \cot\dfrac{x}{2}$. **2**

(d) (i) Expand $\left(x+\dfrac{1}{x}\right)^5$ in descending powers of x. **1**

(ii) If $x+\dfrac{1}{x}=n$, express $x^5+\dfrac{1}{x^5}$ in terms of n. **2**

(e) Consider the function $f(x) = 2-\log_e x$.

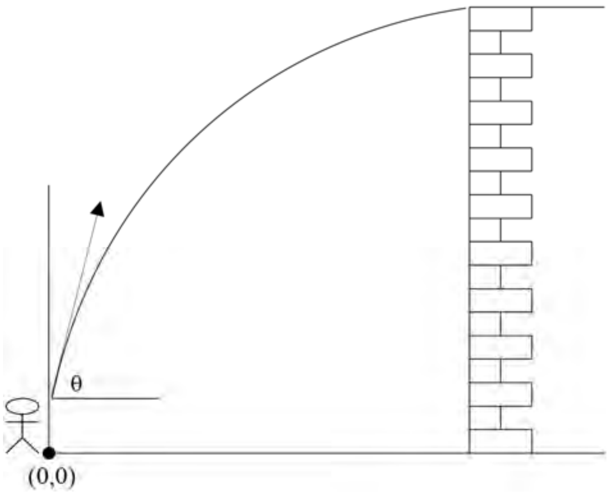
(i) Find the equation of the inverse function $f^{-1}(x)$. **1**

(ii) Explain why the x coordinate X of the point of intersection P of the graphs $y = f(x)$ and $y = f^{-1}(x)$ satisfies the equation $e^{2-X} - X = 0$. **2**

(iii) Use one applications of Newton's Method with an initial value of $X = 1\cdot 4$ to find the value of X correct to two decimal places. **2**

Question 13 (15 Marks) Use a SEPARATE writing booklet. **Marks**

(a) A basketball player of height 2 metres throws a ball on to the top of a flat roofed building. The height of the roof is 8 metres above the ground. The basketball player throws the ball at an initial velocity of $16\,ms^{-1}$ when he is 4 metres from the base of the building.



NOT TO SCALE

(i) Derive expressions for the vertical and horizontal components of the displacement of the ball from the point of projection. Assume $\ddot{x}=0$ and $\ddot{y}=-10$ and use the origin as shown on the diagram. **2**

(ii) Show that the cartesian equation of the ball's path is **2**

$$y=-\frac{5}{256}x^2(1+\tan^2\theta)+x\tan\theta+2$$

(iii) What are the two angles of projection he must throw the ball between to ensure that the ball lands at the top of the building? Answer correct to the nearest degree. **2**

(b) (i) Show that $(p-q)^2=2(p^2+q^2)-(p+q)^2$ **1**

(ii) $P(2p,p^2)$ and $Q(2q,q^2)$ are two points on the parabola $x^2=4y$ and M is the midpoint of PQ. If P and Q move on the parabola so that $p-q=1$, show that the locus of M is the parabola $x^2=4y-1$. **2**

Question 13 Continues on the next page

(c) Use the substitution $x = \sin\theta$ where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ to evaluate **3**

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx\,.$$

(d) A particle is travelling in a straight line. Its displacement $(x\,cm)$ from O at a given time $(t\ seconds)$ after the start of motion is given by

$$x = 2 + \sin^2 t$$

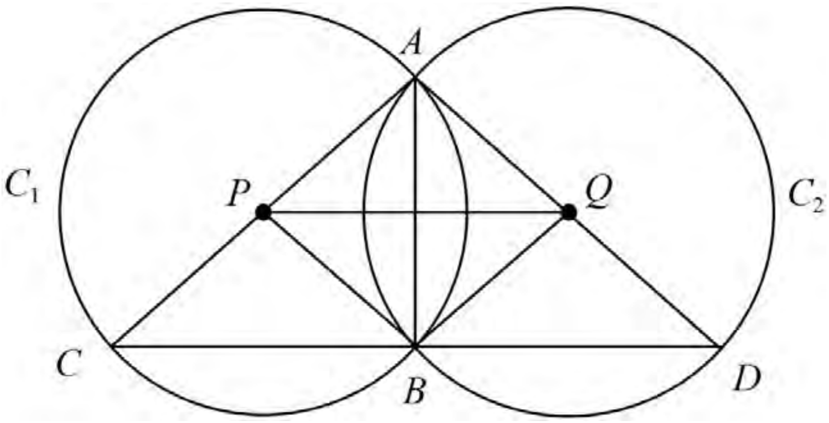
It is travelling in simple harmonic motion.

(i) Find the centre of motion. **1**

(ii) Find the total distance travelled by the particle in the first $\frac{3\pi}{2}$ seconds. **2**

Question 14 (15 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Two circles C_1 and C_2 centred at P and Q with equal radii r intersect at A and B respectively. AC is a diameter in circle C_1 and AD is a diameter in C_2.



Redraw the diagram in your answer booklet.

(i) Show that ΔABC is congruent to ΔABD. **2**

(ii) Show that $PB // AD$. **2**

(iii) Show that $PQDB$ is a parallelogram. **1**

(b) Use the binomial expansion of $(1+x)^n$ and integration to show that **3**

$$ {}^nC_0 + \frac{1}{3}\,{}^nC_2 + \frac{1}{5}\,{}^nC_4 + \ldots = \frac{2^n}{n+1} \quad \text{for } n = 1, 2, 3, \ldots $$

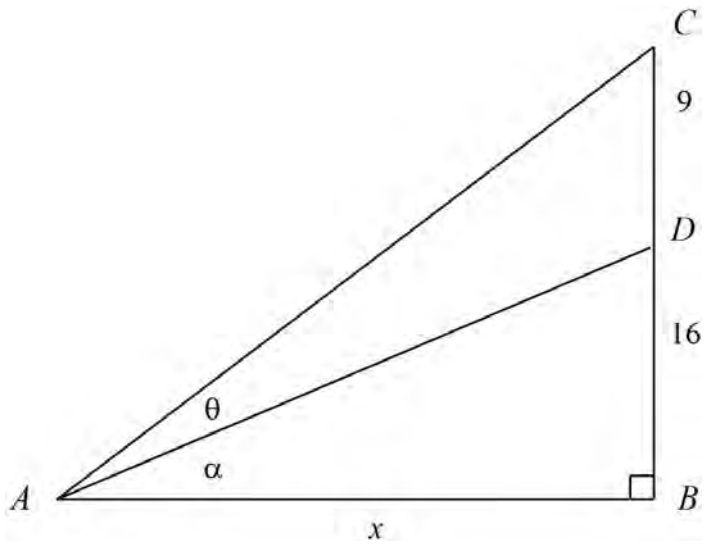
Question 14 Continues on the next page

Question 14 Continued **Marks**

(c) (i) Show that: **2**

$$\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right) \quad \text{, where } AB \neq -1$$

(ii) **3**



In ΔABC above, $\angle CAD = \theta$, $\angle DAB = \alpha$, $AB = x, CD = 9$ and $BD = 16$.

Show that $\theta = \tan^{-1}\left(\frac{9x}{x^2 + 400}\right)$

(iii) Hence, or otherwise, find the value of x such that θ is a maximum. **2**

End of Paper

2013 CTHS Mathematics Extension 1 Section I - Answer Sheet

Student Name ________________________________

Class_________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

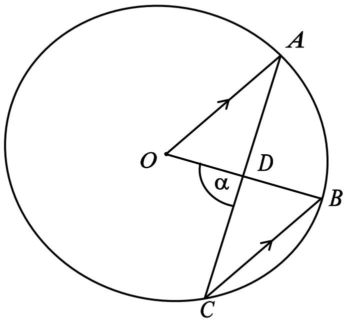
- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

Correct

A ● B ~~●~~ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

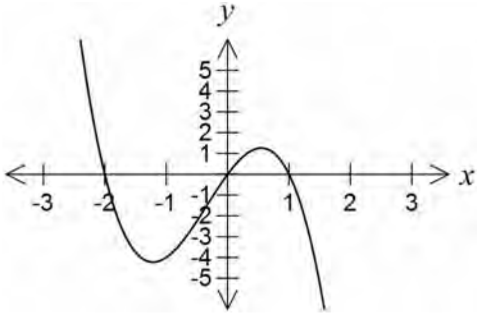
2013 AP4 Mathematics Extension 1 Solutions

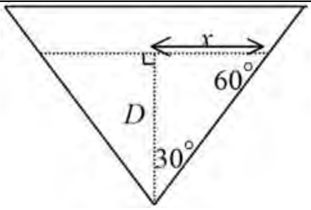
Q	Solution	Marks
Section 1 – Multiple Choice		
1	(B)	1
2	$\sin 4\alpha = 2\sin 2\alpha \cos 2\alpha$ $\therefore\ \sin 2\alpha \cos 2\alpha = \frac{1}{2}\sin 4\alpha$ (A)	1
3	 Let $\angle OAC = \beta$ $OA \parallel CB$ (given) $\angle OAC = \angle ACB$ (alternate angles of parallel lines are equal, $OA \parallel CB$) $\therefore \angle ACB = \beta$ $\angle AOB = 2\angle ACB$ (angle at the centre is twice the angle at the circumference, standing on the same arc) $\angle AOB = 2\beta$ $\angle ODA = 180° - \alpha$ (angle sum of a straight angle) In ΔOAD $2\beta + \beta + 180° - \alpha = 180°$ (anlge sum of a triangle) $3\beta = \alpha$ $\alpha = \frac{\beta}{3}$ (B)	1

Q	Solution	Marks
4	$x = t^2 \quad y = \sqrt{t}$ $y^2 = t$ $\therefore x = (y^2)^2$ $= y^4$ $y = x^{\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$ $= \frac{1}{4}(t^2)^{-\frac{3}{4}}$ $= \frac{1}{4}t^{-\frac{3}{2}}$ (D)	1
5	(B)	1
6	$\cos 2x = 1 - 2\sin^2 x$ $2\sin^2 x = 1 - \cos 2x$ $\int (2\sin^2 x - x^2)\,dx$ $= \int (1 - \cos 2x - x^2)\,dx$ $= x - \frac{\sin 2x}{2} - \frac{x^3}{3} + C$ (B)	1
7	(D)	1
8	$u = x^3 + 1 \qquad x = 1 \quad x = 0$ $\frac{du}{dx} = 3x^2 \qquad u = 2 \quad u = 1$ $\int_0^1 x^2(x^3+1)^3\,dx = \frac{1}{3}\int_0^1 (x^3+1)^3 3x^2\,dx$ $= \frac{1}{3}\int_1^2 u^3\,du$ (C)	1
9	${}^9C_k(2x)^{9-k}\left(-\frac{3}{x}\right)^k$ $= {}^9C_k 2^{9-k}(-3)^k x^{9-k}\left(\frac{1}{x}\right)^k$ $= {}^9C_k 2^{9-k}(-3)^k x^{9-2k}$ Co-efficient of x^5 is when $9 - 2k = 5$ $2k = 4$ $k = 2$ ${}^9C_2 2^{9-2}(-3)^2$ (B)	1

Q	Solution	Marks
10	$a=5-6x \qquad v=2 \qquad x=1$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=a$ $\int\left(\frac{1}{2}v^2\right)=\int(5-6x)$ $\frac{1}{2}v^2=5x-\frac{6x^2}{2}+C$ $\frac{1}{2}\times 2^2=5\times 1-\frac{6\times 1^2}{2}+C$ $C=0$ $v^2=10x-6x^2$ $v=\pm\sqrt{10x-6x^2}$ Given conditions $v=2$ when $x=1$ $v>0$ $\therefore v=\sqrt{10x-6x^2}$ (C)	1

Q	Solution	Marks
Section 2 – Written responses		
11 (a)	$P(4)=4^3-3k\times4^2+32$ $P(4)=0$ $\therefore 4^3-3k\times4^2+32=0$ $64-48k+32=0$ $48k=96$ $k=2$	1
11 (b)	$\int_0^{\frac{\sqrt{3}}{2}}\frac{dx}{9+4x^2}$ $=\int_0^{\frac{\sqrt{3}}{2}}\frac{dx}{4\left(\frac{9}{4}+\frac{4x^2}{4}\right)}$ $=\frac{1}{4}\int_0^{\frac{\sqrt{3}}{2}}\frac{dx}{\frac{9}{4}+x^2}$ $=\frac{1}{4}\left[\frac{2}{3}\tan^{-1}\left(\frac{2x}{3}\right)\right]_0^{\frac{\sqrt{3}}{2}}$ $=\frac{1}{4}\times\frac{2}{3}\left[\tan^{-1}\frac{2\times\frac{\sqrt{3}}{2}}{3}-\tan^{-1}0\right]$ $=\frac{1}{6}\tan^{-1}\frac{\sqrt{3}}{3}$ $=\frac{1}{6}\times\frac{\pi}{6}$ $=\frac{\pi}{36}$	3
11 (c)	$x-y+2=0$ $m_1=1$ $2x-y-1=0$ $m_2=2$ $\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ $=\left\lvert\frac{1-2}{1+1\times2}\right\rvert$ $=\frac{1}{3}$ $=18°26'$	2

Q	Solution	Marks
11 (d)	$\frac{1}{3x}\geq\frac{1}{x+2}$ $\frac{1}{3x}-\frac{1}{x+2}\geq0$ $\frac{x+2-3x}{3x(x+2)}\geq0$ $\frac{-2x+2}{3x^2+6x}\geq0$ $(3x^2+6x)^2\times\frac{-2x+2}{3x^2+6x}\geq0\times(3x^2+6x)^2$ $(3x^2+6x)\times(-2x+2)\geq0$ $6x(x+2)(1-x)\geq0$  $x<-2,\qquad 0<x\leq1,$	3
11 (e)	$\cos\theta(2\sin\theta-1)=0$ $\cos\theta=0\qquad\sin\theta=\frac{1}{2}$ $\therefore\theta=2\pi n\pm\frac{\pi}{2}\qquad\theta=\pi n+(-1)^n\times\frac{\pi}{6}$	2

Q	Solution	Marks
11 (f) (i)	 $\tan 30° = \frac{x}{D}$ $\frac{1}{\sqrt{3}} = \frac{x}{D}$ $x = \frac{D}{\sqrt{3}}$ Area of cross-section at height D metres $A = \frac{1}{2} \times \left(2 \times \frac{D}{\sqrt{3}}\right) \times D$ $= \frac{D^2}{\sqrt{3}}$ Volume of water in trough at height D metres $V = \frac{D^2}{\sqrt{3}} \times 21$ $= \frac{D^2 \times 21\sqrt{3}}{3}$ $= 7\sqrt{3}D^2$	2

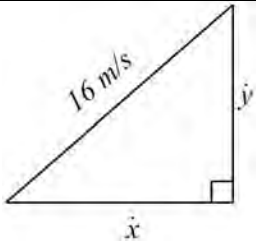
Q	Solution	Marks
11 (f) (ii)	$\frac{dV}{dt} = 4$ $V = 7\sqrt{3}D^2$ $\frac{dV}{dD} = 14\sqrt{3}D$ $\frac{dD}{dt} = \frac{dD}{dV} \times \frac{dV}{dt}$ $\frac{dD}{dt} = \frac{1}{14\sqrt{3}D} \times 4$ When $D = 1 \cdot 5$ $\frac{dD}{dt} = \frac{1}{14\sqrt{3} \times 1 \cdot 5} \times 4$ $= \frac{4}{21\sqrt{3}}$ $\therefore$ the water level is rising at a rate of $\frac{4}{21\sqrt{3}} mh^{-1}$ when the depth of the water is $1 \cdot 5\, m$.	2

Q	Solution	Marks
12 (a)	Prove true for $n=1$ $LHS=\ln\left(\frac{1}{1+2}\right)$ $=\ln\left(\frac{1}{3}\right)$ $RHS=\ln\left(\frac{2}{(1+1)(1+2)}\right)$ $=\ln\left(\frac{2}{2\times 3}\right)$ $=\ln\left(\frac{1}{3}\right)$ $\therefore$ true for $n=1$ Assume true for $n=k$ $\sum_{r=1}^{n}\ln\left(\frac{r}{r+2}\right)=\ln\left(\frac{2}{(k+1)(k+2)}\right)$ i.e. $\ln\left(\frac{1}{3}\right)+\ln\left(\frac{2}{4}\right)+\ln\left(\frac{3}{5}\right)+...+\ln\left(\frac{k}{k+2}\right)=\ln\left(\frac{2}{(k+1)(k+2)}\right)$ Prove true for $n=k+1$ i.e. $\ln\left(\frac{1}{3}\right)+\ln\left(\frac{2}{4}\right)+\ln\left(\frac{3}{5}\right)+...+\ln\left(\frac{k}{k+2}\right)+\ln\left(\frac{k+1}{k+3}\right)=\ln\left(\frac{2}{(k+2)(k+3)}\right)$ $LHS=\ln\left(\frac{1}{3}\right)+\ln\left(\frac{2}{4}\right)+\ln\left(\frac{3}{5}\right)+...+\ln\left(\frac{k}{k+2}\right)+\ln\left(\frac{k+1}{k+3}\right)$ $=\ln\left(\frac{2}{(k+1)(k+2)}\right)+\ln\left(\frac{k+1}{k+3}\right)$ $=\ln\left(\frac{2}{(k+1)(k+2)}\times\frac{k+1}{k+3}\right)$ $=\ln\left(\frac{2}{(k+2)(k+3)}\right)$ $=RHS$ $\therefore$ true for $n=k+1$ $\therefore$ proved true for all $n\geq 1$ by mathematical induction	3

Q	Solution	Marks
12 (b) (i)	$H=5.40-4.80e^{-kt}$ When $t=6$, $H=5.16$ $5.16=5.40-4.80e^{-k\times 6}$ $4.80e^{-6k}=5.40-5.16$ $e^{-6k}=\dfrac{0.24}{4.80}$ $\log_e e^{-6k}=\log_e\left(\dfrac{0.24}{4.80}\right)$ $-6k=\log_e\left(\dfrac{0.24}{4.80}\right)$ $k=\dfrac{\log_e\left(\dfrac{0.24}{4.80}\right)}{-6}$ $k=0.499288712$ $k=0.50$	1
12 (b) (ii)	As $t\to\infty$ $e^{-0.5297t}\to 0$ $\therefore H\to 5.40$ as $t\to\infty$	1
12 (c)	$\dfrac{1+\cos x+\sin x}{1-\cos x+\sin x}=\cot\dfrac{x}{2}$ $LHS=(1+\cos x+\sin x)\div(1-\cos x+\sin x)$ $=\left(1+\dfrac{1-t^2}{1+t^2}+\dfrac{2t}{1+t^2}\right)\div\left(1-\dfrac{1-t^2}{1+t^2}+\dfrac{2t}{1+t^2}\right)$ $=\dfrac{(1+t^2)+(1-t)^2+2t}{1+t^2}\div\dfrac{(1+t^2)-(1-t)^2+2t}{1+t^2}$ $=\dfrac{(1+t^2)+(1-t)^2+2t}{1+t^2}\times\dfrac{1+t^2}{(1+t^2)-(1-t^2)+2t}$ $=\dfrac{2+2t}{2t^2+2t}$ $=\dfrac{2(1+t)}{2t(t+1)}$ $=\dfrac{1}{t}$ $=\dfrac{1}{\tan\left(\dfrac{x}{2}\right)}$ $=\cot\left(\dfrac{x}{2}\right)$ $=RHS$	2

Q	Solution	Marks
12 (d) (i)	$\left(x+\frac{1}{x}\right)^5 = {}^5C_5x^5\left(\frac{1}{x}\right)^0 + {}^5C_4x^4\left(\frac{1}{x}\right)^1 + {}^5C_3x^3\left(\frac{1}{x}\right)^2 + {}^5C_2x^2\left(\frac{1}{x}\right)^3 + {}^5C_1x^1\left(\frac{1}{x}\right)^4 + {}^5C_0x^0\left(\frac{1}{x}\right)^5$ $= x^5 + 5x^3 + 10x + 10\left(\frac{1}{x}\right) + 5\left(\frac{1}{x^3}\right) + \left(\frac{1}{x^5}\right)$	1
12 (d) (ii)	$\left(x+\frac{1}{x}\right)^5 = \left(\frac{1}{x^5}\right) + 5\left(\frac{1}{x^3}\right) + 10\left(\frac{1}{x}\right) + 10x + 5x^3 + x^5$ $x+\frac{1}{x}=n$ $\therefore x^5 + \left(\frac{1}{x^5}\right) = \left(x+\frac{1}{x}\right)^5 - \left[5\left(\frac{1}{x^3}\right)+10\left(\frac{1}{x}\right)+10x+5x^3\right]$ $= \left(x+\frac{1}{x}\right)^5 - \left[5\left(x^3+\frac{1}{x^3}\right)+10\left(x+\frac{1}{x}\right)\right]$ $= \left(x+\frac{1}{x}\right)^5 - \left[5\left(x+\frac{1}{x}\right)\left(x^2-x\times\frac{1}{x}+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)\right]$ $= \left(x+\frac{1}{x}\right)^5 - \left[5\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)+10\left(x+\frac{1}{x}\right)\right]$ $= \left(x+\frac{1}{x}\right)^5 - \left\{5\left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2-2-1\right]-10\left(x+\frac{1}{x}\right)\right\}$ $= \left(x+\frac{1}{x}\right)^5 - \left\{5\left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2-3\right]-10\left(x+\frac{1}{x}\right)\right\}$ $= n^5 - \left(5n(n^2-3)\right) - 10n$ $= n^5 - 5n^3 + 15n - 10n$ $= n^5 - 5n^3 + 5n$	2

Q	Solution	Marks
12 (e) (i)	$f(x) = 2 - \log_e x$ $f: \; y = 2 - \log_e x$ $f^{-1}: \; x = 2 - \log_e y$ $\log_e y = 2 - x$ $e^{2-x} = y$ $y = e^{2-x}$	1
12 (e) (ii)	$y = 2 - \log_e x$ and $y = e^{2-x}$ Since they are inverse functions, they will intersect on the line $y = x$ at $x = X$ $y = e^{2-X}$ and $y = X$ $\therefore e^{2-X} = X$ $e^{2-X} - X = 0$	2
12 (e) (iii)	Let $g(x) = e^{2-X} - X$ $g'(x) = -e^{2-X} - 1$ $x_2 \approx X - \dfrac{g(X)}{g'(X)}$ $\approx 1\cdot 4 - \dfrac{e^{2-1\cdot 4} - 1\cdot 4}{-e^{2-1\cdot 4} - 1}$ ≈ 1.549575135 ≈ 1.55	2

Q	Solution	Marks
13 (a) (i)	 $\ddot{x}=0 \qquad\qquad \ddot{y}=-10$ $\int \ddot{x}\ dt=\int dt \qquad \int \ddot{y}\ dt=\int -10\ dt$ $\therefore\quad \dot{x}=C \qquad \therefore\quad \dot{y}=-10t+C$ $at\ t=0,\ \dot{x}=16\cos\theta \qquad at\ t=0,\ \dot{y}=16\sin\theta$ $\dot{x}=16\cos\theta \qquad \dot{y}=-10t+16\sin\theta$ $\int \dot{x}\ dt=\int 16\cos\theta\, dt \qquad \int \dot{y}\ dt=\int(-10t+16\sin\theta)\ dt$ $x=16\cos\theta t+C \qquad y=-\frac{10t^2}{2}+16\sin\theta t+C$ $at\ t=0,\ x=0\ \ \therefore C=0 \qquad at\ t=0,\ y=2\ \ \therefore C=2$ $\therefore\ \ x=16\cos\theta t \qquad y=-5t^2+16\sin\theta t+2$	2
13 (a) (ii)	From (i) $t=\frac{x}{16\cos\theta}$ Sub into $y=-5t^2+16\sin\theta t+2$ $y=-5\left(\frac{x}{16\cos\theta}\right)^2+16\sin\theta\left(\frac{x}{16\cos\theta}\right)+2$ $y=-\frac{5}{256}\left(\frac{x^2}{\cos^2\theta}\right)+\tan\theta\times x+2$ $y=-\frac{5}{256}x^2\sec^2\theta+x\tan\theta+2$ $y=-\frac{5}{256}x^2(1+\tan^2\theta)+x\tan\theta+2$	2
13 (a) (iii)	When $x=4,\ y=8$ Substitute into $y=-\frac{5}{256}x^2(1+\tan^2\theta)+x\tan\theta+2$ $8=-\frac{5}{256}\times 4^2(1+\tan^2\theta)+4\times\tan\theta+2$ $8=-\frac{5}{16}(1+\tan^2\theta)+4\times\tan\theta+2$ $128=-5(1+\tan^2\theta)+64\times\tan\theta+32$ $128=-5-5\tan^2\theta+64\tan\theta+32$ $5\tan^2\theta-64\tan\theta+101=0$ $\tan\theta=\frac{64\pm\sqrt{(-64)^2-4\times5\times101}}{2\times5}$ $\tan\theta=\frac{64\pm\sqrt{2076}}{10}$ $\theta=62^\circ\ or\ 85^\circ$ $\therefore$ the ball must be thrown between 62° and 85°	2

Q	Solution	Marks
13 (b) (i)	Show that $(p-q)^2 = 2(p^2+q^2)-(p+q)^2$ $RHS = 2(p^2+q^2)-(p+q)^2$ $= 2p^2+2q^2-(p^2+2pq+q^2)$ $= 2p^2+2q^2-p^2-2pq-q^2$ $= p^2-2pq-q^2$ $= (p-q)^2$ $= LHS$	1
13 (b) (ii)	$P(2p,p^2)$, $Q(2q,q^2)$ $x = \dfrac{2p+2q}{2}$ $= p+q$ Given that $p-q=1$ $y = \dfrac{p^2+q^2}{2}$ $y = \dfrac{\left(\dfrac{(p-q)^2+(p+q)^2}{2}\right)}{2}$ from (i) $= \dfrac{(p-q)^2+(p+q)^2}{4}$ $\therefore y = \dfrac{1^2+x^2}{4}$ using $p-q=1$ and $x=p+q$ $4y = x^2+1$ $\therefore x^2 = 4y-1$	2

Q	Solution	Marks
13 (c)	$x=\sin\theta$ When $x=\frac{\sqrt{3}}{2}$ $x=\frac{1}{2}$ $\frac{dx}{d\theta}=\cos\theta$ $\theta=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\theta=\sin^{-1}\left(\frac{1}{2}\right)$ $\therefore dx=\cos\theta\, d\theta$ $=\frac{\pi}{3}$ $=\frac{\pi}{6}$ $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\frac{x^2}{\sqrt{1-x^2}}dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\times\cos\theta\, d\theta$ $=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^2\theta}{\sqrt{\cos^2\theta}}\times\cos\theta\, d\theta$ $=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^2\theta}{\cos\theta}\times\cos\theta\, d\theta$ $=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sin^2\theta\, d\theta$ $=\left[\frac{\theta}{2}-\frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\left(\frac{\frac{\pi}{3}}{2}-\frac{1}{4}\sin 2\left(\frac{\pi}{3}\right)\right)-\left(\frac{\frac{\pi}{6}}{2}-\frac{1}{4}\sin 2\left(\frac{\pi}{6}\right)\right)$ $=\left(\frac{\pi}{6}-\frac{1}{4}\sin\frac{2\pi}{3}\right)-\left(\frac{\pi}{12}-\frac{1}{4}\sin\frac{\pi}{3}\right)$ $=\frac{\pi}{12}$	3

Q	Solution	Marks
13 (d) (i)	$x = 2+\sin^2 t$ $\dot{x} = 2\sin t\cos t$ $\ddot{x} = 2\cos t\cos t - \sin t \times 2\sin t$ $= 2\cos^2 t - 2\sin^2 t$ When $\ddot{x}=0$, particle is at the centre $2\cos^2 t - 2\sin^2 t = 0$ $\cos^2 t - \sin^2 t = 0$ $\cos 2t = 0$ $2t = \frac{\pi}{2}, \frac{3\pi}{2}$ $t = \frac{\pi}{4}, \frac{3\pi}{4}$ at $t = \frac{\pi}{4}$ $x = 2+\sin^2\left(\frac{\pi}{4}\right)$ $= 2+\left(\frac{1}{\sqrt{2}}\right)^2$ $= \frac{5}{2}$	1
13 (d) (ii)	$\dot{x} = 2\sin t\cos t$ When $\dot{x}=0$ $2\sin t\cos t = 0$ $\sin t\cos t = 0$ $\sin t = 0$ $t = 0, \pi, 2\pi, \ldots$ $\cos t = 0$ $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ at $t=0$ $\quad x = 2+(\sin 0)^2$ $= 2$ $t = \frac{\pi}{2}$ $\quad x = 2+(\sin\frac{\pi}{2})^2$ $= 3$ $t = \pi$ $\quad x = 2+(\sin\pi)^2$ $= 2$ $t = \frac{3\pi}{2}$ $\quad x = 2+(\sin\frac{3\pi}{2})^2$ $1+1+1=3$ $\therefore$ total distance travelled in the first $\frac{3\pi}{2}$ seconds is 3 *m*.	2

Q	Solution	Marks
14 (a) (i)	There are many methods which can be used to answer this question. Here is one: $AC = AD$ (circles C_1 and C_2 have equal radii, given) AB is common $\angle ABC = 90°$ (angles in a semicircle are 90°) $\angle ABD = 90°$ (angles in a semicircle are 90°) $\therefore \angle ABC = \angle ABD = 90°$ $\therefore \Delta ABC \equiv \Delta ABD\,(RHS)$	2
14 (a) (ii)	$\angle CAB = \angle DAB$ (corresponding angles of congruent triangles are equal) Let $\angle CAB = \theta$ $\therefore \quad \angle DAB = \theta$ $\therefore \angle CPB = 2\angle CAB$ (angle at the centre is twice the angle at the circumference standing on the same arc) $\therefore \angle CPB = 2\theta$ $\angle PCB = 90° - \angle CAB$ (complementary angles of ΔCAB) $\therefore \angle PCB = 90° - \theta$ $\angle PBC + \angle PCB + \angle CPB = 180°$ (angle sum of a triangle is 180°) $\therefore \angle PBC = 180° - (90° - \theta) - 2\theta$ $\angle PBC = 90° - \theta$ $\angle ADB = 90° - \angle DAB$ (complementary angles of ΔDAB) $\therefore \angle ADB = 90° - \theta$ $\therefore \angle PBC = \angle ADB = 90° - \theta$ $\therefore PB \parallel AD$ (corresponding angles are equal, $\therefore$ the lines are parallel)	2
14 (a) (iii)	$PB = QD$ (radii of a circle) $QD \parallel PB$ (since $QD \parallel AP$ from (ii)) $\therefore PQDB$ is a parallelogram (one pair of opposite sides are equal and parallel)	1

Q	Solution	Marks
14 (b)	$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + {}^nC_4x^4 + \ldots + {}^nC_nx^n$ $\int (1+x)^n\,dx = \int ({}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + {}^nC_4x^4 + \ldots + {}^nC_nx^n)\,dx$ $\int_{-1}^{1} (1+x)^n\,dx = \int_{-1}^{1} ({}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + {}^nC_4x^4 + \ldots + {}^nC_nx^n)dx$ $\left[\frac{(1+x)^n}{n+1}\right]_{-1}^{1} = \left({}^nC_0 \times 1 + \frac{{}^nC_1(1)^2}{2} + \frac{{}^nC_2(1)^3}{3} + \frac{{}^nC_3(1)^4}{4} + \ldots + \frac{{}^nC_n(1)^{n+1}}{n+1}\right) -$ $\left({}^nC_0 \times (-1) + \frac{{}^nC_1(-1)^2}{2} + \frac{{}^nC_2(-1)^3}{3} + \frac{{}^nC_3(-1)^4}{4} + \ldots + \frac{{}^nC_n(-1)^{n+1}}{n+1}\right)$ $\frac{2^n}{n+1} + \frac{0^n}{n+1} = \left({}^nC_0 \times 1 + \frac{{}^nC_1(1)^2}{2} + \frac{{}^nC_2(1)^3}{3} + \frac{{}^nC_3(1)^4}{4} + \ldots + \frac{{}^nC_n(1)^{n+1}}{n+1}\right) -$ $\left({}^nC_0 \times (-1) + \frac{{}^nC_1(-1)^2}{2} + \frac{{}^nC_2(-1)^3}{3} + \frac{{}^nC_3(-1)^4}{4} + \ldots + \frac{{}^nC_n(-1)^{n+1}}{n+1}\right)$ $\frac{2^{n+1}}{n+1} - \left({}^nC_0 - \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 - \frac{1}{4}\,{}^nC_3 + \ldots + \frac{{}^nC_n(-1)^{n+1}}{n+1}\right)$ $= {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \frac{1}{4}\,{}^nC_3 + \ldots + \frac{1}{n+1}\,{}^nC_n$ $\frac{2^{n+1}}{n+1} = 2\left({}^nC_0 + \frac{1}{3}\,{}^nC_2 + \frac{1}{5}\,{}^nC_4 + \ldots + \frac{1}{n+1}\,{}^nC_n\right)$ $\frac{2 \times 2^n}{2 \times (n+1)} = \left({}^nC_0 + \frac{1}{3}\,{}^nC_2 + \frac{1}{5}\,{}^nC_4 + \ldots + \frac{1}{n+1}\,{}^nC_n\right)$ $\therefore\ \frac{2^n}{n+1} = {}^nC_0 + \frac{1}{3}\,{}^nC_2 + \frac{1}{5}\,{}^nC_4 + \ldots + \frac{1}{n+1}\,{}^nC_n$ for $n = 1, 2, 3\ldots$	3

Q	Solution	Marks
14 (c) (i)	If $\alpha = \tan^{-1} A \to \tan\alpha = A$ If $\beta = \tan^{-1} B \to \tan\beta = B$ Now $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$ $= \dfrac{A-B}{1+AB}$ $\therefore \alpha - \beta = \tan^{-1}\left(\dfrac{A-B}{1+AB}\right)$ i.e. $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\dfrac{A-B}{1+AB}\right)$	2
14 (c) (ii)	$\tan(\theta+\alpha) = \dfrac{25}{x}$ $\therefore \theta+\alpha = \tan^{-1}\left(\dfrac{25}{x}\right)$........1 $\tan\alpha = \dfrac{16}{x}$ $\therefore \alpha = \tan^{-1}\left(\dfrac{16}{x}\right)$..............2 $From 1 \quad \theta = \tan^{-1}\left(\dfrac{25}{x}\right) - \alpha$ $From\ 2 \quad \theta = \tan^{-1}\left(\dfrac{25}{x}\right) - \tan^{-1}\left(\dfrac{16}{x}\right)$ Now using the result from (i) $\theta = \tan^{-1}\left(\dfrac{25/x - 16/x}{1 + 25/x \times 16/x}\right)$..........3 $\dfrac{25/x - 16/x}{1 + 25/x \times 16/x} = \dfrac{25x - 16x}{x^2+400} = \dfrac{9x}{x^2+400}$ $\theta = \tan^{-1}\left(\dfrac{9x}{x^2+400}\right)$	3

Q	Solution	Marks
14 (c) (iii)	θ will be a maximum when $\dfrac{9x}{x^2+400}$ is a maximum.	2

Let $y=\dfrac{9x}{x^2+400}$

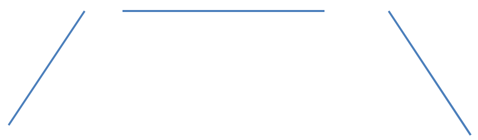
$$\frac{dy}{dx}=\frac{9\left(x^2+400\right)-9x\times 2x}{\left(x^2+400\right)^2}$$

$$=\frac{3600-9x^2}{\left(x^2+400\right)}$$

$\dfrac{dy}{dx}=0$ when $3600-9x^2=0$

$\therefore x=20,\ x>0$

x	*19*	20	21
y'	+0.46	*0*	*-0.44*



$\therefore\ \theta$ is a maximum when $x=20$