

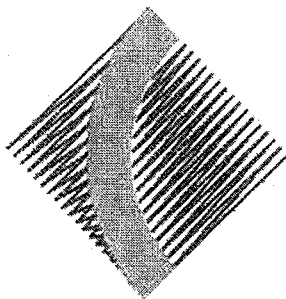
HK
JG
GS
AT
MB

Name: _____

Class: 12MTX__

Teacher: _____

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2015 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS EXTENSION 1

*Time allowed - 2 HOURS
(Plus 5 minutes reading time)*

DIRECTIONS TO CANDIDATES:

- Attempt all questions
- Write your name and class in the space provided at the top of this question paper and on each booklet
- Multiple choice questions are to be answered on the multiple choice answer sheet
- Each question in Section 2 is to be commenced in a new booklet clearly marked Question 11, Question 12, etc .
- All necessary working should be shown in every question in Section 2. Full marks may not be awarded for careless or badly arranged work.
- Board of Studies approved calculators may be used.
- A Standard Integrals sheet is provided.

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10.

- 1 What is the solution to the inequality $\frac{2}{3-x} < 1$?
- (A) $x < 1$
- (B) $x > 1$
- (C) $1 < x < 3$
- (D) $x < 1$ or $x > 3$
- 2 Let $x = 0.8$ be a first approximation to the root of the equation $3 \ln x + x = \cos 2x$. What is the second approximation to the root using Newton's method (to 3 significant figures)?
- (A) 0.742
- (B) 0.776
- (C) 0.981
- (D) 0.985
- 3 The displacement of a particle moving in simple harmonic motion is given by $x = 3 \cos \pi t$ where t is the time in seconds. The period of oscillation is:
- (A) π
- (B) $\frac{2\pi}{3}$
- (C) 2
- (D) 3

4 When the polynomial $P(x) = x^3 - 5x^2 + kx + 2$ is divided by $(x + 1)$, the remainder is 3. What is the value of k ?

(A) -7

(B) -5

(C) 5

(D) 7

5 What is the acute angle to the nearest degree between the lines $y = 1 - 3x$ and $4x - 6y - 5 = 0$?

(A) 15°

(B) 38°

(C) 52°

(D) 75°

6 Which of the following is equivalent to the expression $\sqrt{\frac{4+4\cos 2x}{1-\cos 2x}}$?

(A) $2\cot^2 x$

(B) $2|\cot x|$

(C) $2|\tan x|$

(D) $2\tan^2 x$

7 Which of the following is the derivative of $\tan^{-1}(e^{-x})$?

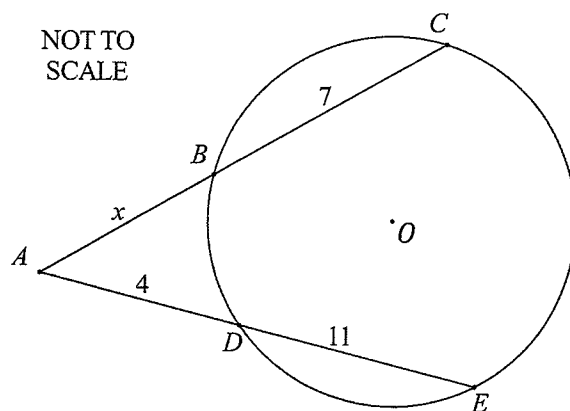
(A) $\frac{e^x}{1+e^{2x}}$

(B) $\frac{-e^{-x}}{1+e^{2x}}$

(C) $\frac{-e^{-x}}{1+e^{-2x}}$

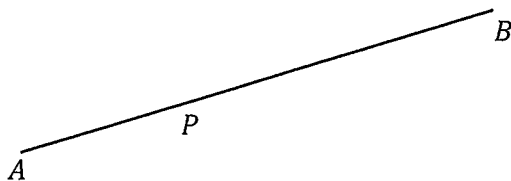
(D) $\frac{e^x}{1+e^{-2x}}$

- 8 In the diagram below, BC and DE are chords of a circle. CB and ED are produced to meet at A.



What is the value of x ?

- (A) $\frac{11}{28}$
- (B) $\frac{28}{11}$
- (C) 5
- (D) 12
- 9 The point P divides the interval AB in the ratio 3:7.



In what external ratio does A divide the interval PB?

- (A) 3:10
- (B) 3:4
- (C) 7:3
- (D) 10:3
- 10 The statement $7^n - 3^n$ is always divisible by 10 is true for:
- (A) All integers $n \geq 1$
- (B) All integers $n \geq 2$
- (C) All odd integers $n \geq 1$
- (D) All even integers $n \geq 2$

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hours and 45 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 11 (15 marks)	Marks
(a) Differentiate $x^2 \cos^{-1} x$.	2
(b) Find the exact value of $\sin \frac{\pi}{12}$ with a rational denominator.	2
(c) Find $\int \frac{3x+6}{(x+2)^2+25} dx$.	2
(d) In a particular racing car, the probability that the brake pads will need to be replaced during a race is 0.2. The car competes in a 12-race series. (i) What is the probability that the pads will need to be replaced on exactly 3 occasions? Give your answer to 2 decimal places.	1
(ii) What is the probability that the pads will need to be replaced on 3 occasions at most? Give your answer to 2 decimal places.	2
(e) The point $A(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$ with focus S . The tangent at A intersects the x -axis at B . (i) Draw a neat diagram showing this information.	1
(ii) Find the coordinates of the point B .	2
(iii) Show that the lengths of SO , SB and SA are three terms of a geometric sequences.	3

Question 12 (15 marks) – Start a new writing booklet

Marks

- (a) Use Mathematical Induction to show that for all positive integers $n \geq 1$: **3**

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + n \times 2^{n-1} = 1 + (n-1)2^n.$$

- (b) A particle is moving on a horizontal line relative to a fixed point O on the line. At time t seconds, the particle has displacement x metres from O , velocity $v \text{ ms}^{-1}$ and acceleration $a \text{ ms}^{-2}$ given by $a = x - 2$. Initially, the particle is 1 metre to the right of O and is moving away from O at a speed of 1 ms^{-1} .

- (i) Show that $v = 2 - x$. **2**

- (ii) Find the displacement in terms of time. **2**

- (iii) Explain why the particle can never be more than 2 metres to the right of O . **1**

- (c) A soft drink taken from a cool room has a temperature of 3°C . It is placed in a room of constant temperature 25°C . **2**

The soft drink's temperature T after t minutes is given by

$$T = 25 - Ce^{-0.04t}, \text{ where } C \text{ is a positive constant.}$$

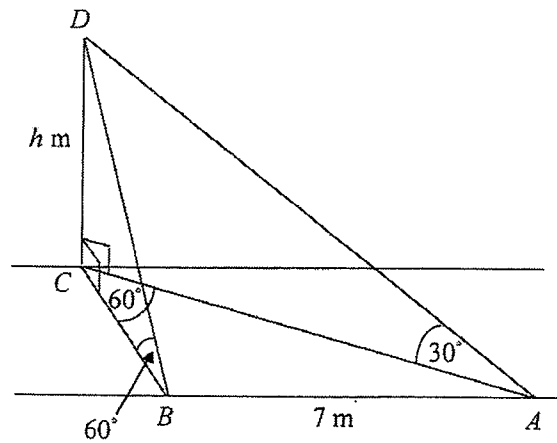
How long will it take for the soft drink to reach a temperature of 15°C ? Give your answer in minutes and seconds.

- (d) In a class of 27 students, there are 14 boys and 13 girls. The class needs to elect two boys and two girls for the student council. How many different combinations are possible? **2**

Question 12 continues on the next page

- (e) A road on horizontal ground has two parallel edges.

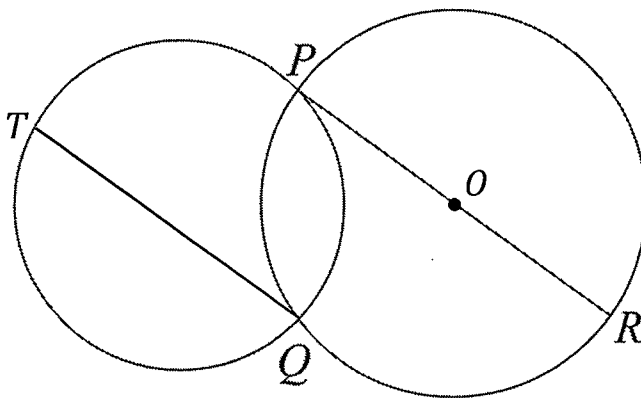
3



CD is a vertical flagpole of height h metres which stands with its base C on one edge of the road. A and B are two points on the other edge of the road such that $AB = 7$ m and $\angle ACB = 60^\circ$. From A and B , the angles of elevation of the top D of the flagpole are 30° and 60° respectively.

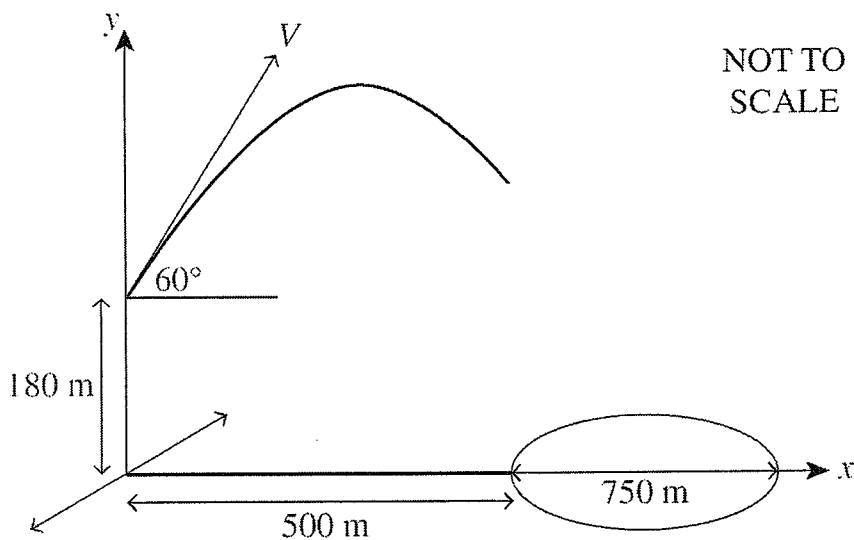
Find the exact height of the flagpole.

- (a) Two circles intersect at P and Q . PR is the diameter of circle PRQ .



Copy this diagram into your writing booklet.

- (i) Draw a straight line through P , parallel to QR , meeting circle PRQ at S . **2**
 Prove that QS is a diameter of circle PRQ .
- (ii) Prove that the circles have equal radii if TQ is parallel to PR . **2**
- (b) A shell is fired from a cannon at an angle of 60° to the horizontal, with a velocity of $40\sqrt{3}$ m/s. (Assume $g = 10$ m/s².)



- (i) The cannon is on top of a hill 180 metres above a lake of diameter 750 metres. The base of the hill is 500 metres from the lake. **3**
- If the shell is projected towards the centre of the lake, how long will it take to land?
- (ii) Does the shell land in the lake? Justify your answer. **2**

Question 13 continues on the next page

Question 13 (continued)**Marks**(c) (i) Expand $(1-x)^{2n}$ using the Binomial Theorem. **1**(ii) Hence, prove the following identity: **2**

$${}^{2n}C_1 + 3{}^{2n}C_3 + \dots + (2n-1){}^{2n}C_{2n-1} = 2{}^{2n}C_2 + 4{}^{2n}C_4 + \dots + 2n{}^{2n}C_{2n}$$

(d) The area between $x=2$, $y=\ln x$ and the x -axis is rotated about the y -axis. Find the volume of the solid that is formed. **3****Question 14** (15 marks) – Start a new writing booklet**Marks**(a) A function is defined as $f(x) = \frac{2x+1}{x-1}$.(i) Write down the equations of any vertical and horizontal asymptotes. **1**(ii) State the domain of the inverse function $f^{-1}(x)$. **1**(iii) Sketch the graph of $y = f^{-1}(x)$, clearly labelling all important features of the graph. **2**(b) (i) Show that $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$. **3**(ii) Show that the smallest positive solution of the equation $\tan 3x = \cot 2x$ is $x = \frac{\pi}{10}$. **2**(iii) If x satisfies $\tan 3x = \cot 2x$, show that x also satisfies the equation $5 \tan^4 x - 10 \tan^2 x + 1 = 0$. **2**(iv) Hence, deduce that $\tan \frac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}$. **4****End of Examination**

HSC Mathematics Extension 1

AP4 2015

NAME.....

CLASS 12 MTX.....

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

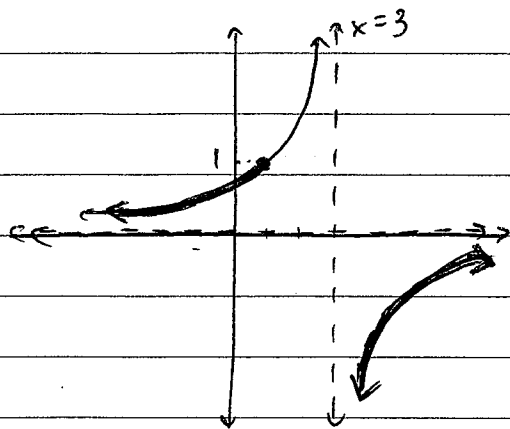
A B C D
 correct
 ↙

1. A B C D
2. A B C D
3. A B C D
4. A B C D
5. A B C D
6. A B C D
7. A B C D
8. A B C D
9. A B C D
10. A B C D

Question	Marks
1-10	/ 10
11	/ 15
12	/ 15
13	/ 15
14	/ 15
Total	/ 70

2015 Ext 1 AP4 Solutions (Section I)

$$1. \frac{2}{3-x} < 1$$



$$\therefore x < 1 \text{ or } x > 3,$$

ie. (D)

1. D

2. B

3. C

4. A

5. D

6. B

7. C

8. C

9. A

10. D

$$2. \text{ Let } f(x) = 3\ln x + x - \cos 2x$$

$$\therefore f'(x) = \frac{3}{x} + 1 + 2\sin 2x$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.8 - \frac{f(0.8)}{f'(0.8)}$$

$$= 0.8 - \frac{3\ln(0.8) + 0.8 - \cos(1.6)}{\frac{3}{0.8} + 1 + 2\sin(1.6)}$$

$$x_2 = 0.7763275 \dots$$

ie. (B)

$$3. \text{ Period} = \frac{2\pi}{\pi}$$

$$= \frac{2\pi}{\pi}$$

$$= 2$$

ie. (C)

$$4. P(-1) = 3$$

$$\text{ie. } (-1)^3 - 5(-1)^2 + k(-1) + 2 = 3$$

$$-1 - 5(1) - k + 2 = 3$$

$$-k - 4 = 3$$

$$-k = 7$$

$$k = -7$$

ie. (A)

$$5. y = 1 - 3x \rightarrow y = -3x + 1$$

$$4x - 6y - 5 = 0 \rightarrow y = \frac{2}{3}x - \frac{5}{6}$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{-3 - \frac{2}{3}}{1 + (-3 \times \frac{2}{3})} \right|$$

$$= \left| \frac{-\frac{9}{3} - \frac{2}{3}}{1 + (-2)} \right|$$

$$= \left| \frac{\left(\frac{-11}{3}\right)}{-1} \right|$$

$$\tan \theta = \frac{11}{3}$$

$$\theta \approx 74.74488\dots$$

ie. (D)

$$\begin{aligned}
 6. \sqrt{\frac{4+4\cos 2x}{1-\cos 2x}} &= 2\sqrt{\frac{1+\cos 2x}{1-\cos 2x}} \\
 &= 2\sqrt{\frac{1+\cos^2 x - \sin^2 x}{1-\cos^2 x + \sin^2 x}} \\
 &= 2\sqrt{\frac{2\cos^2 x}{2\sin^2 x}} \\
 &= 2\sqrt{\cot^2 x} \\
 &= 2|\cot x|
 \end{aligned}$$

ie. (B)

$$\begin{aligned}
 7. \frac{d}{dx} \tan^{-1}(e^{-x}) &= -e^{-x} \times \frac{1}{1+(e^{-x})^2} \\
 &= \frac{-e^{-x}}{1+e^{-2x}}
 \end{aligned}$$

ie. (C)

8. $AB \cdot AC = AD \cdot AE$ (products of intercepts on intersecting secants are equal)

$$x(x+7) = 4 \times 15$$

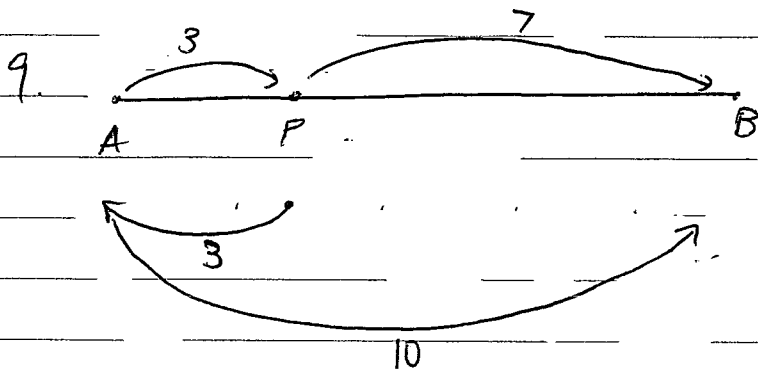
$$x^2 + 7x - 60 = 0$$

$$(x+12)(x-5) = 0$$

$$x = -12 \text{ or } 5 \text{ but } x > 0$$

$$\therefore x = 5 \text{ only}$$

ie. (C)



ie. (A)

10. Let $S(n) = 7^n - 3^n$

$$S(1) = 4 \quad \times$$

$$S(2) = 40 \quad \checkmark$$

$$S(3) = 316 \quad \times$$

$$S(4) = 2320 \quad \checkmark$$

⋮

only true for even values
ie. (D)

This result can be proven by Mathematical Induction.

Section II

11. (a) $\frac{d}{dx} x^2 \cos^{-1} x = (\cos^{-1} x)(2x) + (x^2) \left(\frac{-1}{\sqrt{1-x^2}} \right)$ \checkmark product rule
 \checkmark $\frac{d}{dx} \cos^{-1} x$
 $= 2x \cos^{-1} x - \frac{x^2}{\sqrt{1-x^2}}$

(b) $\sin \frac{\pi}{12} = \sin \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$

$$= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

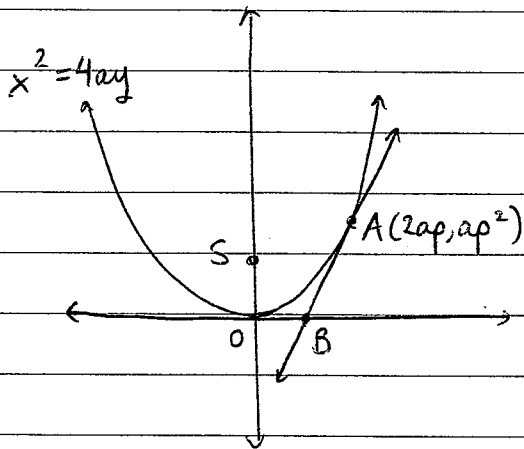
$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

(c) $\int \frac{3x+6}{(x+2)^2+25} dx = \frac{3}{2} \int \frac{2x+4}{x^2+4x+29} dx$
 $= \frac{3}{2} \ln(x^2+4x+29) + c$

(d) Omitted

(e) (i)



$$(ii) y = \frac{x^2}{4a}$$

$$\therefore \frac{dy}{dx} = \frac{2x}{4a}$$

$$= \frac{x}{2a}$$

$$\text{At } A, \frac{dy}{dx} = \frac{2ap}{2a}$$

$$= p.$$

Equation of tangent at A:

$$y - ap^2 = p(x - 2ap)$$

$$y - ap^2 = px - 2ap^2$$

$$y = px - ap^2.$$

When $y = 0$,

$$px - ap^2 = 0$$

$$px = ap^2$$

$$x = ap$$

\therefore Coordinates of B are $(ap, 0)$.

(iii) Focus S is (0, a), ∴ SO = a units.

$$\begin{aligned}
 SB &= \sqrt{(ap)^2 + a^2} \\
 &= \sqrt{a^2 p^2 + a^2} \\
 &= a\sqrt{p^2 + 1} \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 SA &= \sqrt{(2ap)^2 + (ap^2 - a)^2} \\
 &= \sqrt{4a^2 p^2 + a^2 p^4 - 2a^2 p^2 + a^2} \\
 &= \sqrt{2a^2 p^2 + a^2 p^4 + a^2} \\
 &= a^{\frac{1}{2}} \sqrt{p^4 + 2p^2 + 1} \\
 &= a\sqrt{(p^2 + 1)^2} \\
 &= a(p^2 + 1) \text{ units.}
 \end{aligned}$$

∴ SO, SB, SA = a, a√(p²+1), a(p²+1).

This is a GP with first term a & common ratio √(p²+1).

$$12. (a) \text{ RTP: } 1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + n \times 2^{n-1} = 1 + (n-1)2^n$$

Test $n=1$:

$$\text{LHS} = 1 \times 2^0$$

$$= 1$$

$$\text{RHS} = 1 + (1-1)2^1$$

$$= 1$$

\therefore True for $n=1$.

Assume true for $n=k$:

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + k \times 2^{k-1} = 1 + (k-1)2^k \quad \{k \in \mathbb{Z}^+\}$$

Prove true for $n=k+1$:

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + k \times 2^{k-1} + (k+1)2^k = 1 + k \times 2^{k+1}$$

$$\text{LHS} = (1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + k \times 2^{k-1}) + (k+1)2^k$$

$$= 1 + (k-1)2^k + (k+1)2^k \quad \text{by assumption}$$

$$= 1 + (k-1+k+1)2^k$$

$$= 1 + 2k \times 2^k$$

$$= 1 + k \times 2 \times 2^k$$

$$= 1 + k \times 2^{k+1}$$

$$= \text{RHS}$$

\therefore True by principle of Mathematical Induction
for all $n \in \mathbb{Z}^+$.

(b) (i) Let initial direction of movement be the positive direction.

Since $a = x - 2$,

$$\frac{d(\frac{1}{2}v^2)}{dx} = x - 2$$

$$\frac{1}{2}v^2 = \frac{1}{2}(x-2)^2 + c'$$

$$v^2 = (x-2)^2 + c$$

Initially, $x = 1$ & $v = 1$.

$$\therefore 1 = (1-2)^2 + c$$

$$1 = (-1)^2 + c$$

$$1 = 1 + c$$

$$c = 0$$

$$\therefore v^2 = (x-2)^2$$

$$v = \pm(x-2)$$

When $t = 0$, $x = 1$ and $v = 1$.

$$\therefore v = -(x-2) \text{ only}$$

$$v = 2 - x \text{ as required.}$$

(ii) $v = 2 - x$

$$\frac{dx}{dt} = 2 - x$$

$$\frac{dt}{dx} = \frac{1}{2-x}$$

$$t = -\ln(2-x) + c$$

When $t = 0$, $x = 1$

$$\therefore 0 = -\ln 1 + c$$

$$c = 0$$

$$t = -\ln(2-x)$$

$$-t = \ln(2-x)$$

$$2-x = e^{-t}$$

$$x = 2 - e^{-t}$$

(iii) Initially, particle is moving to the right from $x=1$ and slowing down since acceleration is to the left ($a=-1$). If the particle could reach $x=2$, $v=a=0$ and motion would not continue. $\therefore 1 \leq x \leq 2$ and $v \geq 0$.

(c) When $t=0$, $T=3$.

$$\therefore 3 = 25 - Ce^0$$

$$C = 22$$

$$\therefore T = 25 - 22e^{-0.04t} \quad \checkmark$$

Soft drink will reach 15°C when:

$$25 - 22e^{-0.4t} = 15$$

$$-22e^{-0.4t} = -10$$

$$e^{-0.4t} = \frac{5}{11}$$

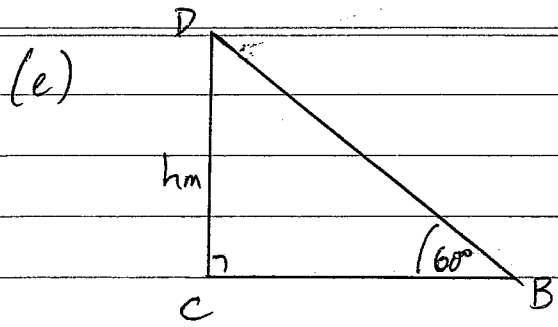
$$-0.4t = \ln \frac{5}{11}$$

$$t = \frac{\ln \left(\frac{5}{11} \right)}{-0.4}$$

$$\approx 19.71 \text{ minutes} \quad \checkmark$$

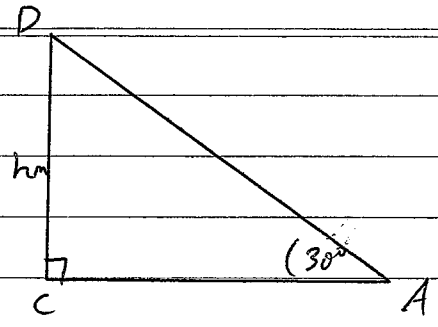
(or 19 mins 43 sec)

(d) Omitted



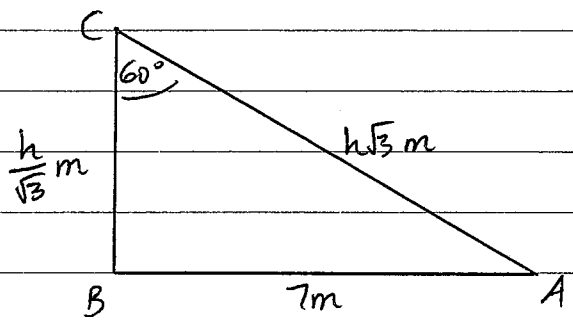
$$\tan 60^\circ = \frac{h}{BC}$$

$$BC = \frac{h}{\sqrt{3}}$$



$$\tan 30^\circ = \frac{h}{AC}$$

$$AC = h\sqrt{3}$$



$$\cos 60^\circ = \frac{\left(\frac{h}{\sqrt{3}}\right)^2 + (h\sqrt{3})^2 - 7^2}{2\left(\frac{h}{\sqrt{3}} \times h\sqrt{3}\right)}$$

$$\frac{1}{2} = \frac{\frac{h^2}{3} + 3h^2 - 49}{2h^2}$$

$$3h^2 = h^2 + 9h^2 - 147$$

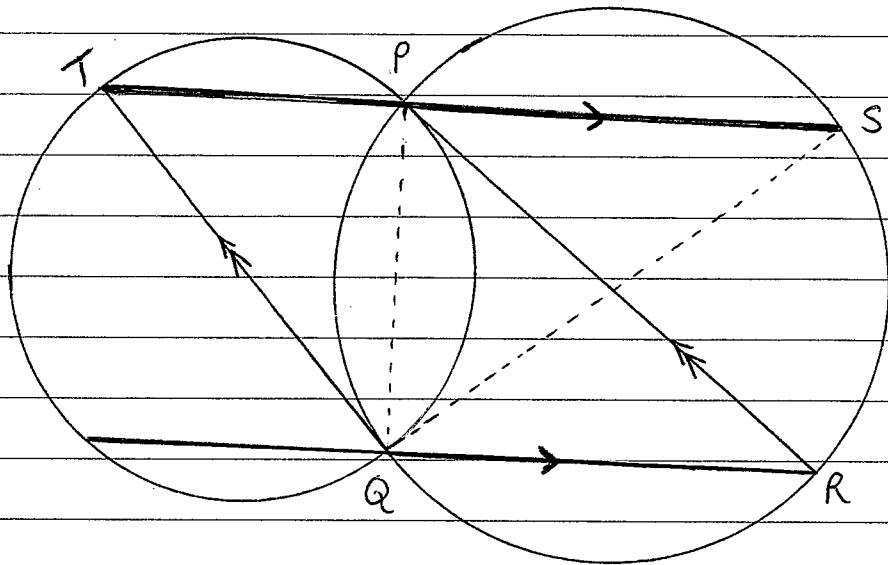
$$-7h^2 = -147$$

$$h^2 = 21$$

$$h = \sqrt{21} \text{ only } \{h > 0\}$$

∴ Flagpole is $\sqrt{21}$ metres. ✓

13. (a)



(i) $\angle PQR = 90^\circ$ (angle in a semicircle is 90°)

$\angle SPQ + \angle PQR = 180^\circ$ (co-interior angles on $PS \parallel QR$ are supplementary)

$$\angle SPQ = 90^\circ$$

$\therefore QS$ is a diameter ($\angle SPQ$ is an angle in a semicircle)

(ii) $\angle TPQ = \angle PQR$ (alternate angles on $PS \parallel QR$ are equal)
 $= 90^\circ$

$\therefore QT$ is a diameter ($\angle TPQ$ is an angle in a semicircle)

If $TP \parallel QR$ & $TQ \parallel PR$ (given),
 then $PRQT$ is a parallelogram (two pairs of parallel sides).

$\therefore PR = QT$ (opposite sides of a parallelogram are equal)

$\therefore PR$ & QT are equal diameters,
 hence the circles have equal radii.

$$(b) (i) \cos 60^\circ = \frac{\dot{x}_0}{40\sqrt{3}}$$

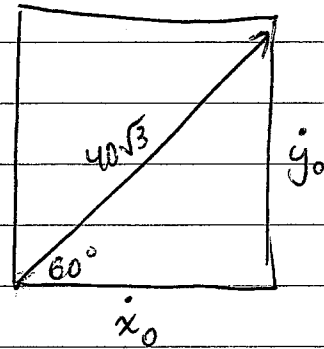
$$\dot{x}_0 = 40\sqrt{3} \times \frac{1}{2}$$

$$\dot{x}_0 = 20\sqrt{3}$$

Since \dot{x} is constant,

$$\dot{x} = 20\sqrt{3}$$

$$x = 20\sqrt{3}t$$



$$\sin 60^\circ = \frac{\dot{y}_0}{40\sqrt{3}}$$

$$\dot{y}_0 = 40\sqrt{3} \times \frac{\sqrt{3}}{2} \quad \checkmark$$

$$\dot{y}_0 = 60$$

Since \dot{y} is subject to gravity & $g = 10 \text{ m/s}^2$,

$$\ddot{y} = 60 - 10t$$

$$y = -5t^2 + 60t + C \quad \{C \in \mathbb{R}\}$$

Since shell begins at $y = 180$,

$$y = -5t^2 + 60t + 180 \quad \checkmark$$

For $y = 0$:

$$5t^2 - 60t - 180 = 0$$

$$t^2 - 12t - 36 = 0$$

$$t = \frac{12 \pm \sqrt{144 - 4(-36)(1)}}{2}$$

$$= \frac{12 \pm 12\sqrt{2}}{2}$$

$$= 6 \pm 6\sqrt{2} \quad \text{but } t > 0$$

$$\therefore t = 6 + 6\sqrt{2} \text{ only.}$$

\therefore Shell takes $6 + 6\sqrt{2}$ seconds to land. \checkmark

$$(ii) x = 20\sqrt{3}(6 + 6\sqrt{2}) \quad \checkmark$$

$$\approx 501.78$$

\therefore Yes, the shell lands in the lake. \checkmark

$$(c)(i) (1-x)^{2n} = 1 - {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots + {}^{2n}C_{2n} x^{2n}$$

(ii) Differentiating both sides:

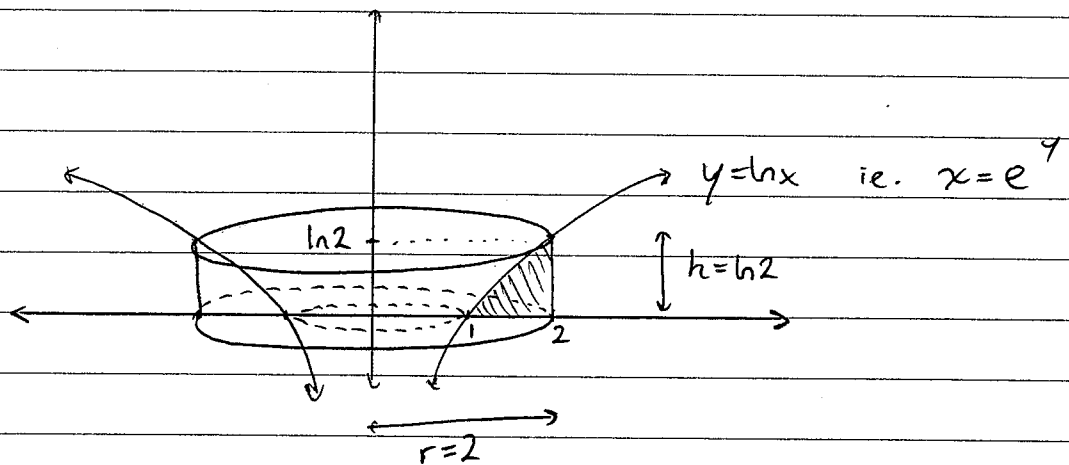
$$-2n(1-x)^{2n-1} = -{}^{2n}C_1 + 2{}^{2n}C_2 x + \dots + 2n{}^{2n}C_{2n} x^{2n-1} \checkmark$$

let $x=1$:

$$\checkmark 0 = -{}^{2n}C_1 + 2{}^{2n}C_2 - 3{}^{2n}C_3 + 4{}^{2n}C_4 - \dots + 2n{}^{2n}C_{2n}$$

$$\therefore {}^{2n}C_1 + 3{}^{2n}C_3 + \dots + (2n-1){}^{2n}C_{2n-1} = 2{}^{2n}C_2 + 4{}^{2n}C_4 + \dots + 2n{}^{2n}C_{2n}$$

(d)



$$\begin{aligned} V &= \pi r^2 h - \pi \int_0^{\ln 2} x^2 dy \\ &= \pi \times 2^2 \times \ln 2 - \pi \int_0^{\ln 2} e^{2y} dy \checkmark \\ &= 4\pi \ln 2 - \pi \left[\frac{1}{2} e^{2y} \right]_0^{\ln 2} \\ &= \frac{\pi}{2} (8 \ln 2 - e^{2 \ln 2} - e^0) \checkmark \\ &= \frac{\pi}{2} (8 \ln 2 - 4 + 1) \\ &= \frac{\pi}{2} (8 \ln 2 - \frac{3}{1}) \checkmark \end{aligned}$$

14. (a) $f(x) = \frac{2x+1}{x-1}$

(i) Vertical asymptote @ $x=1$
Horizontal asymptote @ $y=2$ $\left[\begin{array}{l} x \neq 1 \\ xy \neq 2 \\ \text{No mark} \end{array} \right] \left[\begin{array}{l} x=1 \text{ or } x \neq 1 \\ y \neq 2 \text{ or } y=2 \\ \text{1 mark} \end{array} \right]$

(ii) Since $f(x)$ has a horizontal asymptote @ $y=2$,
 $f^{-1}(x)$ has a vertical asymptote of $x=2$
 \therefore Domain is $x < 2$ or $x > 2$.

(iii) $f^{-1}(x)$ is defined by

$$x = \frac{2y+1}{y-1}$$

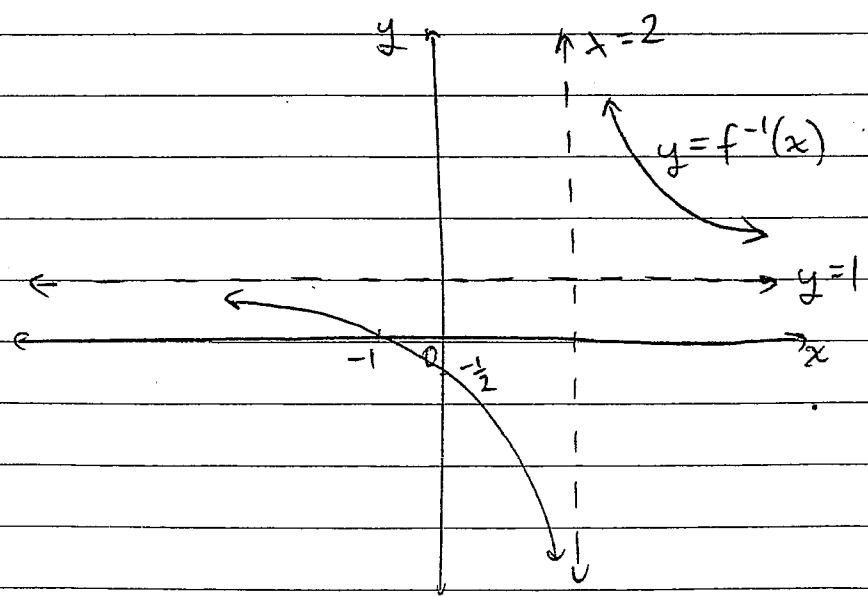
$$xy - x = 2y + 1$$

$$-x - 1 = 2y - xy$$

$$-x - 1 = y(2 - x)$$

$$y = \frac{-x-1}{2-x}$$

$$y = \frac{x+1}{x-2}$$



✓ Shape & intercepts

✓ Reflects part (i) & (ii)

$$(b)(i) \text{ RTP } \tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

$$\text{LHS} = \tan 3x$$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \quad \checkmark$$

$$= \frac{\left(\frac{2\tan x}{1 - \tan^2 x} \right) + \tan x}{1 - \left(\frac{2\tan x}{1 - \tan^2 x} \right) \tan x} \quad \checkmark$$

$$= \frac{2\tan x + \tan x(1 - \tan^2 x)}{(1 - \tan^2 x) - 2\tan^2 x} \quad \checkmark$$

$$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} \quad \text{as required.}$$

$$(ii) \tan 3x = \cot 2x$$

$$= \frac{\cos 2x}{\sin 2x} \quad \checkmark$$

$$= \frac{\sin\left(\frac{\pi}{2} - 2x\right)}{\cos\left(\frac{\pi}{2} - 2x\right)}$$

$$\tan 3x = \tan\left(\frac{\pi}{2} - 2x\right) \quad \checkmark$$

$$3x = \frac{\pi}{2} - 2x$$

$$5x = \frac{\pi}{2}$$

$$x = \frac{\pi}{10}$$

$$(iii). \quad \tan 3x = \cot 2x$$

$$\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = \frac{1}{\tan 2x} \quad \text{from part (i)}$$

$$\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = \frac{1 - \tan^2 x}{2 \tan x} \quad \checkmark$$

$$6 \tan^2 x - \tan^4 x = 1 - \tan^2 x - 3 \tan^2 x + 3 \tan^4 x \quad \checkmark$$

$$-5 \tan^4 x + 10 \tan^2 x - 1 = 0$$

$$5 \tan^4 x - 10 \tan^2 x + 1 = 0 \quad \text{as required.}$$

$$(iv):- \quad 5 \tan^4 x - 10 \tan^2 x + 1 = 0$$

$$\text{let } u = \tan^2 x.$$

$$5u^2 - 10u + 1 = 0$$

$$u = \frac{10 \pm \sqrt{100 - 4(5)(1)}}{10}$$

$$\tan^2 x = \frac{10 \pm \sqrt{80}}{10} \quad \checkmark$$

Since $\frac{\pi}{10}$ is the solution and $0 < \frac{\pi}{10} < \frac{\pi}{2}$
 $\therefore \tan x > 0$

\checkmark

Also, Since $\frac{\pi}{10} < \frac{\pi}{4}$ and $\tan x$ is

an increasing function $\therefore \tan \frac{\pi}{10} < \tan \frac{\pi}{4}$

$$\tan \frac{\pi}{10} < 1 \quad \checkmark$$

$$\therefore \tan \frac{\pi}{10} = \sqrt{\frac{5 - 2\sqrt{5}}{5}}$$

$$\tan \frac{\pi}{5} = \frac{2 \tan \frac{\pi}{10}}{1 - \tan^2 \frac{\pi}{10}} \quad \checkmark$$

$$= \frac{2 \left(\sqrt{\frac{5 - 2\sqrt{5}}{5}} \right)}{1 - \frac{5 - 2\sqrt{5}}{5}}$$

$$\tan \frac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}, \text{ as required.}$$