Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1. Which expression is a correct factorisation of $a^3 - 64$?

(A) $(a-4)(a^2+4a+16)$

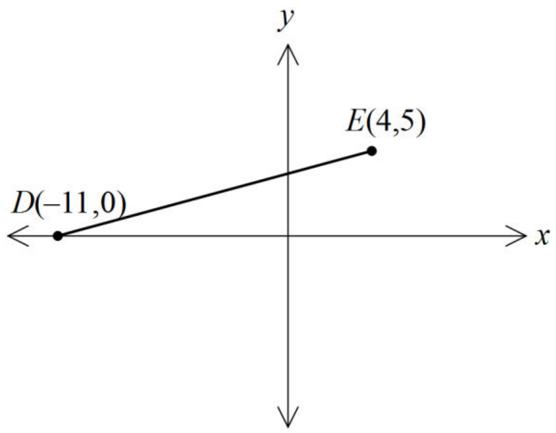
(B) $(a-4)(a^2-4a+16)$

(C) $(a+4)(a^2+4a+16)$

(D) $(a+4)(a^2-4a+16)$

2. The interval *DE* is divided internally in the ratio 3:2 by the point *F*.

Find the *x*-coordinate of *F*.



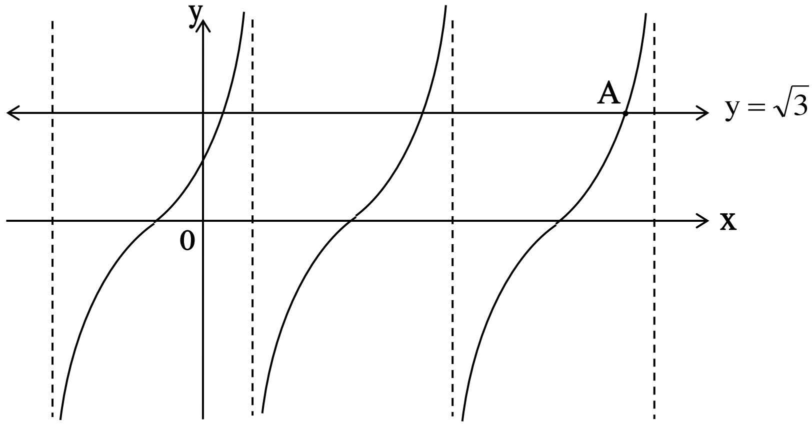
(A) -5

(B) $-\frac{7}{2}$

(C) -2

(D) $-\frac{3}{2}$

3. The graph of the curve $y = \tan\left(\frac{x}{3} + \frac{\pi}{4}\right)$ and the line $y = \sqrt{3}$ are as shown.



What are the coordinates of A?

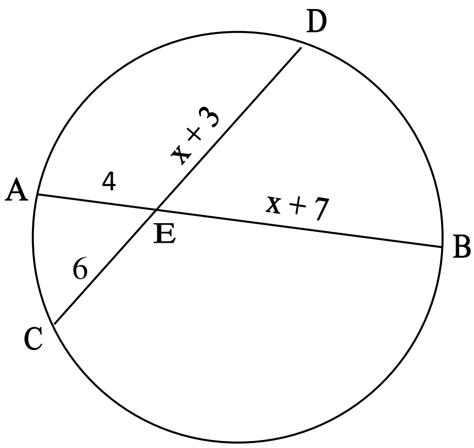
(A) $\left(\frac{\pi}{3}, \sqrt{3}\right)$

(B) $\left(\frac{13\pi}{4}, \sqrt{3}\right)$

(C) $\left(\frac{25\pi}{4}, \sqrt{3}\right)$

(D) $\left(\frac{19\pi}{3}, \sqrt{3}\right)$

4. Two chords AB and CD intersect at E as shown.



What is the value of x?

(A) 7

(B) 6

(C) 5

(D) 4

5. Evaluate $\lim_{x \to 0} \frac{\sin 4x}{3x}$.

(A) 0

(B) $\frac{3}{4}$

(C) $\frac{4}{3}$

(D) ∞

6. A particle is moving in a simple harmonic motion between x = – 1 and x = 5. It covers the distance from x = – 1 to x = 5 in one second.

Which of the following could be the equation for the motion of this particle?

(A) $x = 3 + 2 \sin \pi t$

(B) $x = 2 + 3 \sin 2\pi t$

(C) $x = 2 + 3 \cos 2\pi t$

(D) $x = 2 + 3 \sin \pi t$

7. Find $\int \frac{1}{9 + 25x^2}\, dx$.

(A) $\frac{1}{15} \tan^{-1} \frac{5x}{3} + C$

(B) $\frac{1}{25} \tan^{-1} \frac{5x}{3} + C$

(C) $\frac{1}{25} \tan^{-1} \frac{3x}{5} + C$

(D) $\frac{1}{15} \tan^{-1} \frac{3x}{5} + C$

8. Find the value of the constant term in the binomial expansion $\left(5x - \frac{3}{x^2}\right)^{12}$

(A) ${}^{12}\mathbf{C}_6\ 5^6\ 3^6$

(B) ${}^{12}\mathbf{C}_8\ 5^8\ 3^4$

(C) $-\ {}^{12}\mathbf{C}_4\ 5^8\ 3^4$

(D) $-\ {}^{12}\mathbf{C}_8\ 5^8\ 3^4$

9. A particle moves in simple harmonic motion such that $v^2 + 9x^2 = k$. What is the period of the particle's motion?

(A) $\frac{2\pi}{k}$

(B) 3π

(C) $\frac{3k}{2\pi}$

(D) $\frac{2\pi}{3}$

10. What is the domain of $y = cos^{-1}\left(\frac{x}{3}\right)$?

(A) $-\pi \leq x \leq \pi$

(B) $-3 \leq x \leq 3$

(C) $-\pi \leq x \leq \pi$

(D) $-\frac{1}{3} \leq x \leq \frac{1}{3}$

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

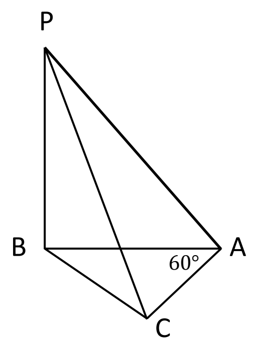
Question 11 (15 marks) Use a new writing booklet. **Marks**

(a) A curve is represented by the parametric equations $x = 7t$ and $y = 4t^2$. **1**

What is the Cartesian equation of the curve?

(b) A vertical pole PB is held by two ropes PA due east and PC due south of it. Given that $\angle BAC = 60°$, find the height of the post, to 2 decimal places, if **3**

$PA = 10m \text{ and } PC = 8m.$



(c) Use the substitution $u = 2x + 1$ to evaluate $\displaystyle\int_0^2 \frac{x}{(2x+1)^2}\,dx$. **3**

Question 11 continues on page 6.

Question 11 continued

(d) At 8:30 a.m. a sandwich which has an initial temperature $22°$ C, is placed in a refrigerator that is set to a constant temperature of $3°$ C.

The sandwich cools at a rate that is proportional to the difference between the temperature of the refrigerator and the temperature (T) of the sandwich.

The rate of temperature change can be expressed as:

$$\frac{dT}{dt} = -\mathrm{k}(T - 3),$$

where t is the number of minutes after the sandwich is placed in the refrigerator.

(i) Show that $T = 3 + Ae^{-\mathrm{k}t}$ satisfies this equation. **1**

(ii) After 10 minutes in the refrigerator, the sandwich has a temperature of $12°$ C. **3**

To the nearest minute, at what time will the sandwich's temperature drop to $5°$ C?

(e) The polynomial $P(x) = x^3 - 3x^2 + kx + 33$ has roots α, β, γ.

(i) Find the value of $\alpha + \beta + \gamma$. **1**

(ii) Find the value of $\alpha\beta\gamma$. **1**

(iii) It is known that two of the roots are equal in magnitude but opposite in sign. Find the third root and hence find the value of k. **2**

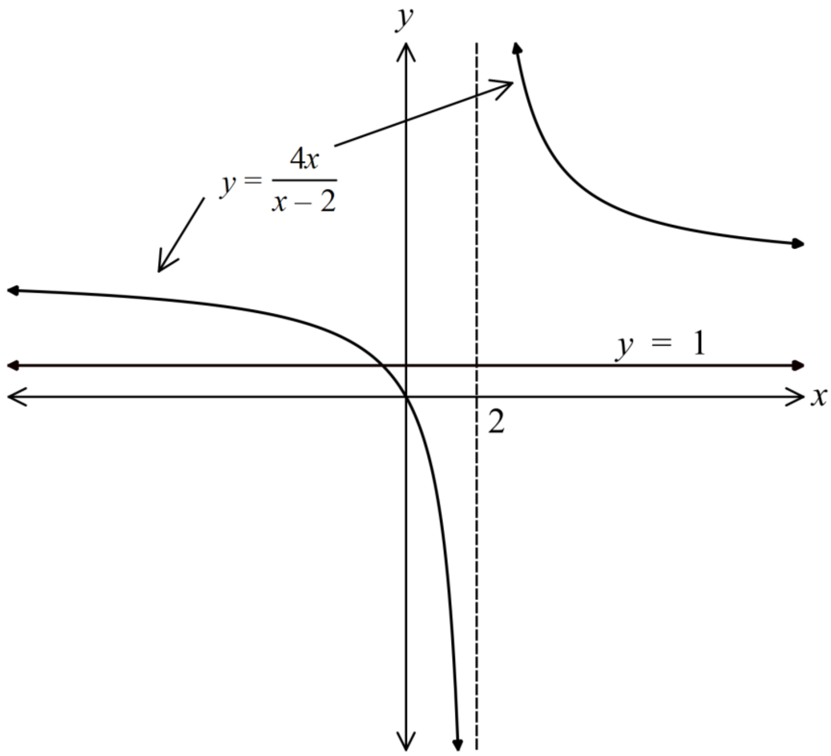
End of Question 11.

Question 12 (15 marks) Use a new writing booklet.

(a) (i) Solve the inequality $\dfrac{4x}{x-2} \leq 1$ by algebraic methods. **3**

(ii) The graph below shows the functions $y = 1$ and $y = \dfrac{4x}{x-2}$. **1**

Copy the graph into your answer booklet and explain how the graph could be used to illustrate the solution found in part (i).



(b) (i) Show that the derivative of $y = \tan^{-1}\left(\dfrac{x^3}{2}\right)$ is $\dfrac{6x^2}{4+x^6}$. **2**

(ii) Hence find $\displaystyle\int \frac{x^2}{4+x^6}\,dx$. **1**

Question 12 continues on page 8.

Question 12 continued

(c) A particle is moving in simple harmonic motion with its acceleration given by

$$\ddot{x} = -12\sin 2t.$$

Initially, the particle is at the origin and has a positive velocity of 6 m/s.

(i) Show that the particle's velocity has equation $\dot{x} = -12\sin^2 t + 6$. **2**

(ii) Show that $\ddot{x} = -4x$. **2**

(iii) State its amplitude and period **1**

(d) (i) Write down the coefficient of x^n in the binomial expansion $(1 + x)^{2n}$. **1**

(ii) Show that $(1 + x^2 + 2x)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k}(x + 2)^{n-k}$. **2**

End of Question 12

Question 13 (15 marks) Use a new writing booklet

(a) Prove by mathematical induction that for all integers $n > 1$, **3**

$$12^n > 7^n + 5^n$$

(b) A particle is moving along the x-axis. Initially the particle is 1 metre to the right of the origin, travelling at a velocity of 3 metres per second and its acceleration is given by

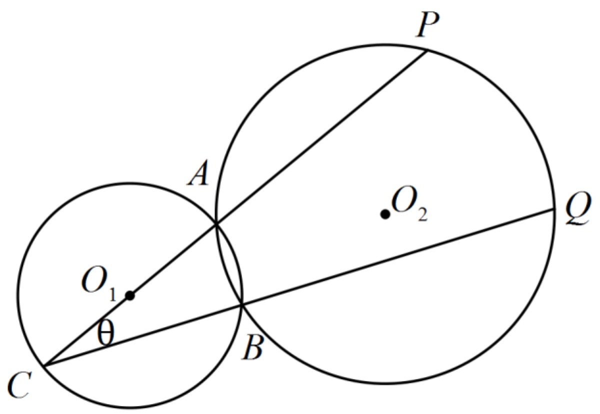
$$\ddot{x} = 2x^3 + 4x,$$

where x is the displacement of the particle at time t.

(i) Show that $\dot{x} = x^2 + 2$. **2**

(ii) Hence, or otherwise, find an expression for x in terms of t. **3**

(c) Two circles with centres O_1 and O_2 intersect at points A and B as shown in the diagram.



AC is the diameter in the circle with centre O_1 and it intersects the other circle at A and P.

When the chord CB is produced it intersects the second circle again at Q.

$\angle ACB = \theta$.

Copy or trace the diagram into your writing booklet.

(i) Prove that AQ is a diameter of the circle with centre O_2. **2**

(ii) Show that $\angle ABO_1 = 90 - \theta$. **2**

Question 13 continues on page 10.

Question 13 continued

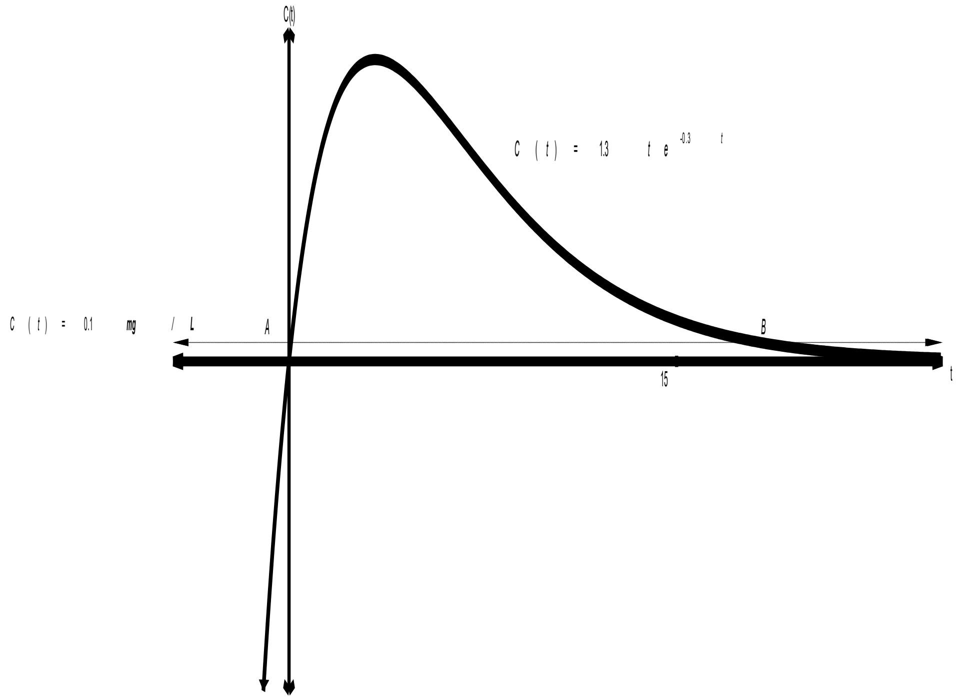
(d) A patient was administered with a drug.

The concentration of the drug in the patient's blood followed the rule:

$$C(t) = 1.3t\,e^{-0.3t}$$

where time, t, is measured in hours and $C(t)$ is measured in mg/L.

This rule is graphed below.



The doctor left instructions that the patient must not receive another dose of the medicine until the concentration of the drug had dropped to below 0.1 mg/L.

2

(i) Using $t = 15$ as a first approximation, use one application of Newton's method to find approximately when the concentration of the drug in the blood of the patient reaches 0.1 mg/L.

(ii) Would it be appropriate to use your answer in (i) as the time when the drug would next be administered? Explain your answer. **1**

End of Question 13.

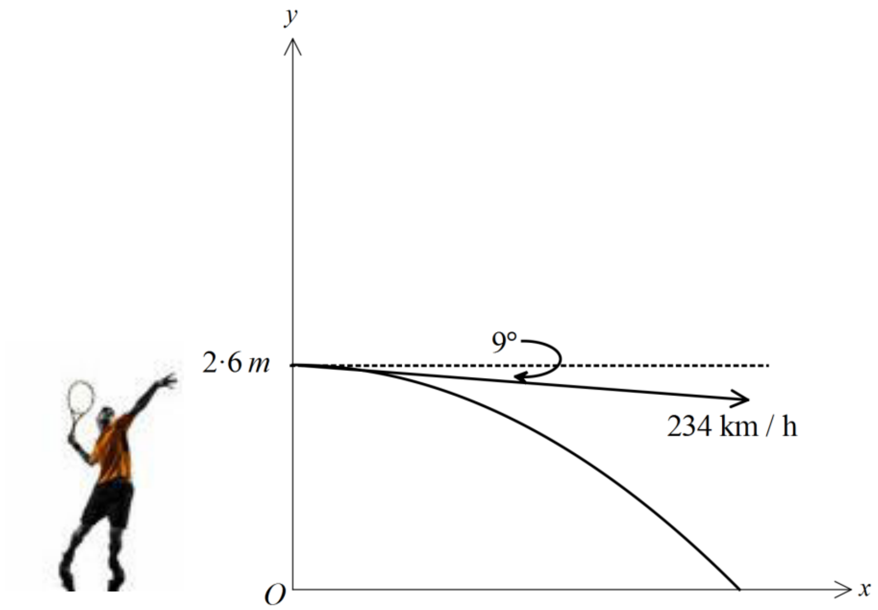
Question 14 (15 marks) Use a new writing booklet **Marks**

(a) (i) Sketch **neatly** on the same set of axes the graphs $y = x^2$ and $y = \cos(\pi x)$ for $0 \leq x \leq 1.5$. (your diagram should be at least half a page) **2**

(ii) On the same diagram, sketch the graph of $y = x^2 + \cos(\pi x)$. Label the three curves on your diagram. **1**

(iii) Using the graph, determine the number of positive real roots of the equation $x^2 + \cos(\pi x) = 0$. **1**

(b) Sam served a tennis ball to his opponent. The racquet hit the ball when the ball was 2.6 metres above the ground. The initial speed of the ball as 234 km/h at an angle of 9° below the horizontal.



Let the origin be a point on the ground, directly below where the racquet hit the ball.

Let gravity equal 10 m/s^2.

(i) Show that the motion of the ball (in metres) can be expressed by the equations **2**

$$x = 65t \cos 9°$$

and $$y = 2.6 - 5t^2 - 65t \sin 9°.$$

(ii) The net at the centre of the court is 11.9 metres from the origin. The net is 91cm tall. Show that the ball will not make it over the net. **2**

(iii) What will the velocity of the ball be when it hits the net? **2**

Question 14 continues on page 12.

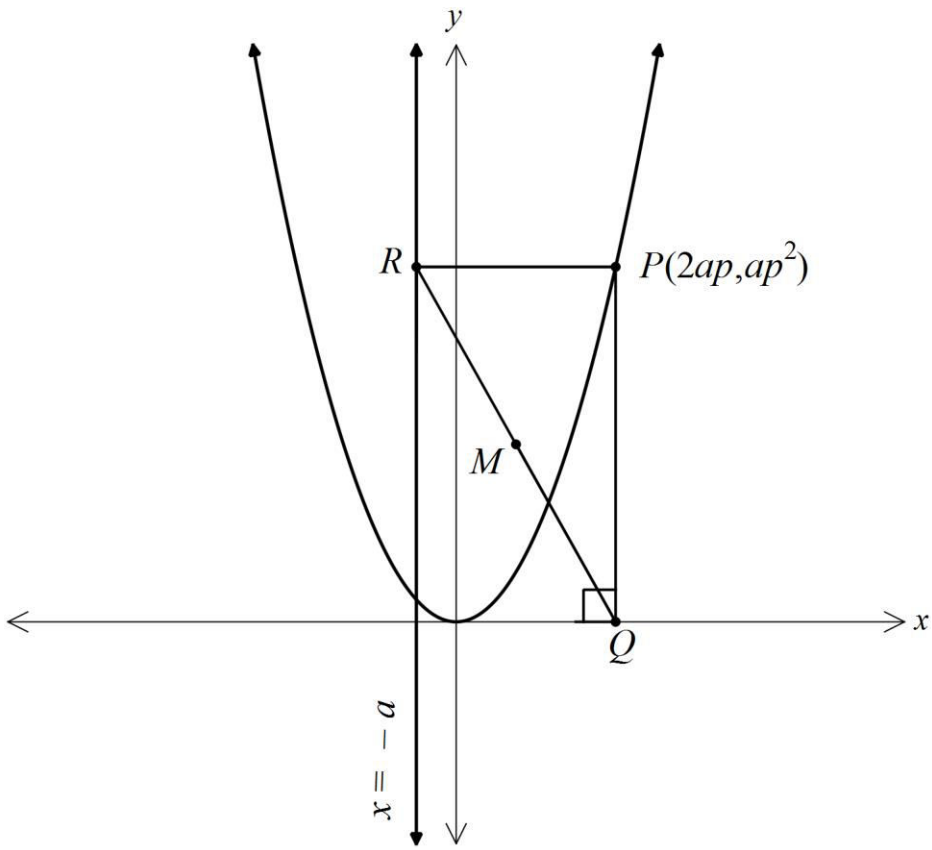
Question 14 continued

(c) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$.

The point Q is a point on the x-axis such that PQ is parallel to the y-axis.

The point R is a point on the line $x = -a$ such that RP is parallel to the x-axis.

M is the midpoint of interval RQ.



(i) Show that M has coordinates $\left(\dfrac{a(2p-1)}{2}, \dfrac{ap^2}{2}\right)$. **2**

(ii) Show that the locus of the point M is a parabola with equation $y = \dfrac{x^2}{2a} + \dfrac{x}{2} + \dfrac{a}{8}$. **2**

(iii) Find the equation for the axis of symmetry for the parabola which forms the locus of M (in part (ii)). **1**

End of Exam.

CTHS Trial AP4 2016

Mathematics Extension 1 Course

Name ______________________ Teacher ____________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

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Mathematics Extension 1 Course

Name Solutions Teacher ____________

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A ✖ B ✖ (correct) C ○ D ○

1.	A ●	B ○	C ○	D ○
2.	A ○	B ○	C ●	D ○
3.	A ○	B ○	C ●	D ○
4.	A ○	B ○	C ●	D ○
5.	A ○	B ○	C ●	D ○
6.	A ○	B ○	C ○	D ●
7.	A ●	B ○	C ○	D ○
8.	A ○	B ●	C ○	D ○
9.	A ○	B ○	C ○	D ●
10.	A ○	B ●	C ○	D ○

1 A
2 C
3 C
4 C
5 C
6 D
7 A
8 B
9 D
10 B

Q1. $a^3 - 64 = a^3 - 4^3$
$= (a-4)(a^2+4a+16)$

2. $(-11, 0)\ (4, 5)\quad 3:2$
$x = \frac{3 \times 4 + 2 \times -11}{3+2}$
$= -2$

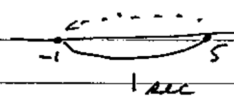
3. $y = \tan\left(\frac{x}{3} + \frac{\pi}{4}\right)$

asymptotes at
$\left(-\frac{3\pi}{2} - \frac{\pi}{4}\right), \left(\frac{3\pi}{2} - \frac{\pi}{4}\right)\left(\frac{9\pi}{2} - \frac{\pi}{4}\right)$
$\left(\frac{15\pi}{2} - \frac{\pi}{4}\right)$
ie $-\frac{7\pi}{4}\quad \frac{5\pi}{4}\quad \frac{17\pi}{4}\quad \frac{29\pi}{4}$

$\tan\left(\frac{x}{3} + \frac{\pi}{4}\right) = \sqrt{3}\qquad \frac{\pi}{3}, \frac{4\pi}{3}$
$\therefore \left(\frac{x}{3} + \frac{\pi}{4}\right) = \frac{7\pi}{3}$
$4x + 3\pi = 28\pi$
$x = 25\pi$

4. $4(x+7) = 6(x+3)$
$4x + 28 = 6x + 18$
$x = 5.$

5. $\lim\limits_{x \to 0} \frac{\sin 4x}{3x}$
$= \frac{4}{3} \lim\limits_{x \to 0} \frac{\sin 4x}{4x}$
$= \frac{4}{3}$

6. 

$\therefore T = 2\text{ sec.}$
$\frac{2\pi}{n} = 2$
$\therefore n = \pi$
centre of motion = 2
amplitude = 3.
$\therefore 2 + 3\sin \pi t$

7. $\int \frac{1}{9+25x^2}\, dx = \int \frac{1}{25\left(\frac{9}{25} + x^2\right)}\, dx$
$= \frac{1}{25} \cdot \frac{5}{3} \tan^{-1} \frac{5x}{3} +$
$= \frac{1}{15} \tan^{-1} \frac{5x}{3} + c$

8. $T_k = {}^{12}C_k (5x)^k \left(\frac{-3}{x^2}\right)^{12-k}$
$= {}^{12}C_k\, 5^k (-3)^{12-k}\, x^k\, x^{-2(12-k)}$
for constant
$x^{k-24+2k} = x^0$
$k = 8$
$\therefore$ constant $= {}^{12}C_8\, 5^8 (-3)^4$
$= {}^{12}C_8\, 5^8\, 3^4$

9 $v^2 + 9x^2 = k$
$v^2 = k - 9x^2$
~~[illegible]~~
$acc = \ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
$\frac{1}{2}v^2 = \frac{k}{2} - \frac{9x^2}{2}$
$\ddot{x} = -9x$
but $\ddot{x} = -n^2 x$
$\therefore -n^2 = -9$
$n^2 = 9$
$n = 3\quad (n > 0)$
$\therefore T = \frac{2\pi}{3}$

10 $y = \cos^{-1} \frac{x}{3}$
ie
$\cos y = \frac{x}{3}\ ;\ 3\cos y = x$
domain = 4
$-3 \leq x \leq 3.$

4

Question 11

a) $x = 7t.\qquad y = 4t^2$
$t = \frac{x}{7}\qquad y = 4 \cdot \frac{x^2}{49}$ 1

b) let $PB = h$ m
$\triangle PBA\quad AB^2 = 100 - h^2$ 1
$\triangle PCB\quad BC^2 = 64 - h^2$ 1

Since $AB \perp BC$
$\tan 60° = \sqrt{3} = \left(\frac{BC}{AB}\right)$ 1
$\left(\frac{BC}{AB}\right)^2 = 3 = \frac{64 - h^2}{100 - h^2}$
$300 - 3h^2 = 64 - h^2$
$2h^2 = 236$
$h = 10.86$ m.

c) $u = 2x + 1\qquad x = \frac{u-1}{2}$ 1
$\frac{du}{2} = dx$
$x = 0\quad u = 1$
$x = 2\quad u = 5$
$\int_1^5 \frac{(u-1)/2}{u^2} \frac{du}{2}$ 1
$= \frac{1}{2}\int_1^5 \frac{u-1}{2u^2}\, du$
$= \frac{1}{4}\int_1^5 u^{-1} - u^{-2}\, du$
$= \frac{1}{4}\left[\ln u + u^{-1}\right]_1^5$ 1
$= \frac{1}{4}\left(\ln 5 - \frac{4}{5}\right)$

11.d $T = 3 + Ae^{-kt}$

i) $T - 3 = Ae^{-kt}$ ①

$\frac{dT}{dt} = -kAe^{-kt}$

$= -k(T-3)$ from ①

ii) $T = 3 + Ae^{-kt}$

$T = 22 \quad t = 0$

$22 = 3 + A$

$A = 19$

at $t = 10 \quad T = 2$

$\therefore 12 = 3 + 19e^{-10k}$

$k = \frac{\ln 9/19}{-10}$ (1)

$= 0.07472144018$

when $T = 5°C$

$5 = 3 + 19e^{-kt}$ (1)

$t = \frac{\ln 2/19}{-0.07472144018}$

$\doteqdot 30\text{min}$

So sandwich will reach $5°C$ at 9am (1)

11(e).

$P(x) = x^3 - 3x^2 + kx + 33$

i) $\alpha + \beta + \gamma = \frac{-b}{a}$

$= 3.$ (1)

ii) $\alpha\beta\gamma = \frac{-d}{a} = -33.$ (1)

iii) let Roots be $\beta, -\beta, \gamma$

$\therefore -\beta + \beta + \gamma = 3$

$\gamma = 3$ (1)

$k = \alpha\beta + \alpha\gamma + \beta\gamma$

$= -\beta\beta + -\beta\gamma + \beta\gamma$

$= -\beta^2$ Ⓐ

$\alpha\beta\gamma = -33$

ie $-\beta^2\gamma = -33$

$\gamma = 3$

$\therefore -3\beta^2 = -33$ (1)

$-\beta^2 = -11$

$= k.$ from Ⓐ

12 a) i) $\frac{4x}{x-2} \le 1$ ($x \ne 2$)

$(x-2)(4x) \le (x-2)^2$ (1)

$4x^2 - 8x \le x^2 - 4x + 4$

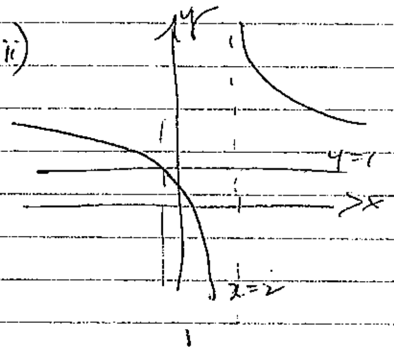
$3x^2 - 4x - 4 \le 0.$

$(3x+2)(x-2) \le 0$ (1)

$-\frac{2}{3} \le x \le 2$ But $x \ne 2$

$\therefore -\frac{2}{3} \le x < 2$ (1)

ii)



to solve $y \le 1$ & $y = \frac{4x}{x-2}$ (1)

* we want the region where * curve is below $y=1$

ie b/w pt of intersection & < 2.

12 b) i) $y = \tan^{-1}\left(\frac{x^3}{2}\right)$.

$\frac{dy}{dx} = \left[\frac{1}{1+\left(\frac{x^3}{2}\right)^2}\right] \times \frac{3x^2}{2}$ (1)

$= \frac{1}{1+\frac{x^6}{4}} \times \frac{3x^2}{2}$

$= \frac{3x^2}{2+\frac{x^6}{2}} \times \frac{2}{2}$ 1

$= \frac{6x^2}{4+x^6}$

ii) $\int \frac{x^2}{4+x^6}\,dx$

$= \frac{1}{6}\int \frac{6x^2}{4+x^6}\,dx$ 1

$= \frac{1}{6}\tan^{-1}\left(\frac{x^3}{2}\right) + c.$

Q14 c.

$\ddot{x} = -12 \sin 2t$

i) $t = 0$ $x = 0$ m $v = 6$ m/s.

$\dot{x} = -12 \int \sin 2t \, dt$

$= -12 \int 2 \sin t \cos t \, dt$ (1)

$= -12 \sin^2 t + c$

$\dot{x} = 6 = -12 \sin^2 0 + c$ (1)

$\therefore c = 6$

$\dot{x} = -12 \sin^2 t + 6$

ii) $x = \int -12 \sin^2 t + 6$

$= \int -12 \left(\frac{1 - \cos 2t}{2}\right) + 6 \, dt$

$= \int -6 + 6\cos 2t + 6 \, dt$

$= \int 6 \cos 2t$

$= 3 \sin 2t + c$ (1)

$t = 0$ $x = 0$ $\therefore c = 0$

$x = 3 \sin 2t$

$\ddot{x} = -12 \sin 2t$ (1)

$= -4 \times 3 \sin 2t$

$\ddot{x} = -4x$

(iii) (1)

14 d) i) $(1+x)^{2n}$ $\quad {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \cdots \quad {}^{2n}C_{2n} x^{2n}$

$T_{k+1} = {}^{2n}C_k x^k$ (1)

x^n term has coeff. ${}^{2n}C_n$

ii) $(1 + x^2 + 2x)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} (x+2)^{n-k}$.

$LHS = [1 + (x^2 + 2x)]^n$

$= \sum_{k=0}^{n} \binom{n}{n-k} 1^k (x^2 + 2x)^{n-k}$ (1)

$\binom{n}{n-k} = \binom{n}{k}$ (1)

$= \sum_{k=0}^{n} \binom{n}{k} [x(x+2)]^{n-k}$

$= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} (x+2)^{n-k}$. given

$LHS = [1 + (x^2 + 2x)]^n$

$= \sum_{k=0}^{n} \binom{n}{n-k} [x^2 + 2x]^{n-k}$

$= \sum_{k=0}^{n} \binom{n}{n-k} [x(x+2)]^{n-k}$

$= \sum_{k=0}^{n} \binom{n}{n-k} x^{n-k} (x+2)^{n-k}$

However $\binom{n}{n-k} = \binom{n}{k}$

$\therefore = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} (x+2)^{n-k}$

Or $[1 + x^2 + 2x]^n$

$= [x^2 + 2x + 1]^n$

$= [(x^2 + 2x) + 1]^n$

$= \sum_{k=0}^{n} \binom{n}{k} (x^2 + 2x)^{n-k}$

$= \sum_{k=0}^{n} x^{n-k} (x+2)^{n-k}$

Q.13

a) for $n=2$

$LHS = 12^2 = 144$

$RHS = 7^2+5^2 = 49+25 = 74$ (1)

$\therefore LHS > RHS \quad \therefore$ true for $n=2$

Assume true for $n=k$

ie $12^k > 7^k + 5^k$

Prove true for $n=k+1$

ie $12^{k+1} > 7^{k+1} + 5^{k+1}$

$LHS = 12^{k+1}$

$= 12 \cdot 12^k$

$> 12(7^k+5^k)$

$> 12\times 7^k + 12\times 5^k$ (1)

$= 7.7^k + 5.5^k + 5.7^k + 7.5^k$

~~RHS = 7^{k+1} + 5^{k+1} = 7×7^k + 5×5^k~~

since $k \geq 2$ $\therefore 5.7^k + 7.5^k > 0$

~~since k is positive~~ and

$12\times 7^k + 12\times 5^k > 7(7^k) + 5(5^k)$ (1)

$\therefore$ true for $n=k+1$

Since true for $n=2$ & for $n=k+1$ then true for $n>1$.

13 b)

$\ddot{x} = 2x^3+4x$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

ie $\frac{1}{2}v^2 = \int 2x^3+4x \; dx$

$v^2 = 2\left[\frac{x^4}{2} + 2x^2\right] + c$

$= x^4+4x^2+c$ (1)

when $t=0 \quad x=1 \quad \dot{x}=3$

$\therefore 9 = 1+4+c$

$c = 4$

$\therefore v^2 = x^4+4x^2+4$

$= (x^2+2)^2$ (1)

$v = x^2+2$ since when $x=1$ moving to right $v>0$

or $\dot{x} = x^2+2$.

$\frac{dx}{dt} = \dot{x} = x^2+2$

$\frac{dt}{dx} = \frac{1}{x^2+2}$ (1)

$t = \int \frac{1}{2+x^2}\,dx$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + c$ (1)

at $t=0 \quad x=1$

$0 = \frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}} + c$

$c = -\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$

13bii cont.) $t = \frac{1}{\sqrt{2}} \tan^{-1}\frac{x}{\sqrt{2}} - \frac{1}{\sqrt{2}} \tan^{-1}\frac{1}{\sqrt{2}}$

$\tan^{-1}\frac{x}{\sqrt{2}} = \sqrt{2}t + \tan^{-1}\frac{1}{\sqrt{2}}$

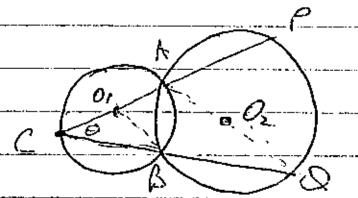
$x = \sqrt{2}\tan\left(\sqrt{2}t + \tan^{-1}\frac{1}{\sqrt{2}}\right)$ (1)

or

$= \sqrt{2}\dfrac{\left(\tan(\sqrt{2}t) + \frac{1}{\sqrt{2}}\right)}{1 - \tan(\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)}$

$= \dfrac{2\tan(\sqrt{2}t) + \sqrt{2}}{\sqrt{2} - \tan(\sqrt{2}t)}$

13c)



Join A to B, A to Q, O_1 to B.

$\angle ABC = 90°$ (CA diameter, angle in semicircle = 90°) (1)

$\angle ABC + \angle ABQ = 180°$ (supplementary ∠s on a straight line)

$\therefore 90 + \angle ABQ = 180$

$\angle ABQ = 90°$ (1)

Since $\angle ABQ = 90$ then ΔABQ is triangle in semicircle

$\therefore$ AQ must pass through O_2

ii) In ΔABC $B\hat{C}A = \theta$

$A\hat{B}C = 90$ (1)

$\therefore B\hat{A}C = 90 - \theta$

$CO_1 = O_1A = O_1B$ – all radii (1)

$\therefore O_1A = O_1B$; $\angle O_1\hat{A}B = \angle O_1\hat{B}A$

13d. $C(t) = 1.3te^{-0.3t}$

$C(t) = 0.1$

for next dose $f(t) = 1.3te^{-0.3t} - 0.1$

$f(15) = 1.3 \times 15e^{-0.3\times15} - 0.1$

$\doteqdot 0.1166254325$

(1)

$f'(t) = 1.3t \times -0.3e^{-0.3t} + e^{-0.3t} \times 1.3$

$u = 1.3t$ $v = e^{-0.3t}$

$u' = 1.3$ $v' = -0.3e^{-0.3t}$

$= 1.3e^{-0.3t}(1 - 0.3t)$

$f'(15) = 1.3 \times 15 \times 0.3e^{-0.3\times15} + e^{-0.3\times15} \times 1.3$

$\doteqdot -0.050545973425$

$t_2 = t_1 - \frac{f(t)}{f'(t)}$

$= 15 - \frac{0.116625\ldots}{-0.050545\ldots}$

$= 17.307$

$\doteqdot$ 17 hrs & 18 mins. [exact (1)

~~15.43340405~~

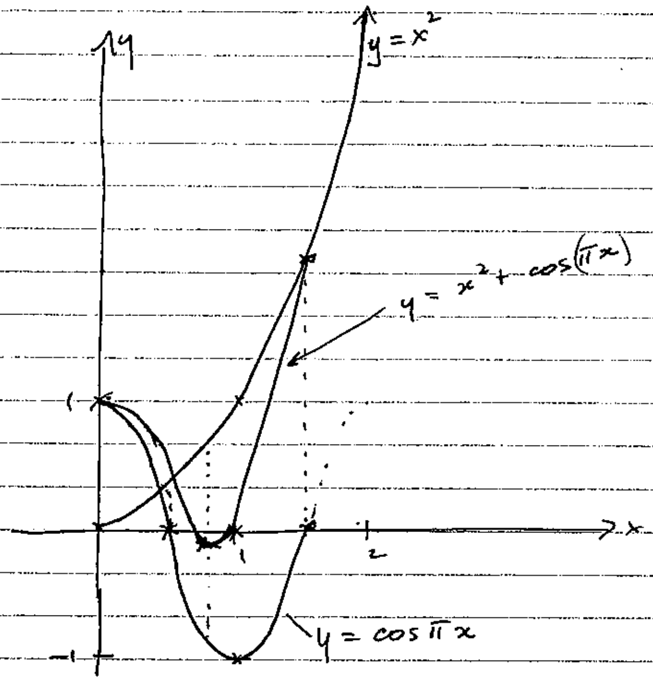
ii $C(t) = 1.3te^{-0.3t}$

(1)

when $t = 17.307$

$C(t) \doteqdot 0.125$ so more applications of newtons method required as concentration still to high

14 a.



1 each graph.

-1 no intercepts labelled

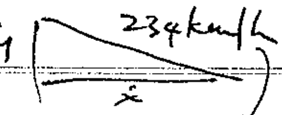
the number of positive Roots for $y = x^2 + \cos \pi x$ is when the curve cross x axis $(y=0)$

$\therefore$ 2 Roots for equation

(1)

Look at $\doteqdot 0.6$ & 1

(14b) i)



$\frac{234000}{60 \times 60} = 65$ m/s

$\ddot{x} = 0$ $\qquad$ $\ddot{y} = -10$

$\dot{x} = c$ $\qquad$ $\dot{y} = -10t + c$

at $t = 0$ $\dot{x} = 65\cos 9°$ $\qquad$ $\dot{y} = -65 \sin 9°$

$\therefore c_1 = 65 \cos 9°$ $\qquad$ $\therefore c_2 = -65 \sin 9°$

$\dot{x} = 65 \cos 9°$ $\qquad$ $\dot{y} = -10t - 65 \sin 9°$

$x = \int 65 \cos 9° \, dt$ $\qquad$ $y = \int -10t - 65 \sin 9° \, dt$

$x = 65t \cos 9° + c_3$ (1) $\qquad$ $y = -5t^2 - 65t \sin 9° + c_4$

at $t = 0$ $x = 0$ $y = 2.6$ (1)

$\therefore C_3 = 0$ $\qquad$ $C_4 = 2.6$

$x = 65t \cos 9°$ $\qquad$ $y = -5t^2 - 65t \sin 9° + 2.6$

$= 2.6 - 5t^2 - 65t \sin 9°$

as req'd.

ii) to clear the net $y > 0.91$ m when $x = 11.9$ m

$t = \frac{x}{65 \cos 9°}$

$= \frac{11.9}{65 \cos 9°}$ (1)

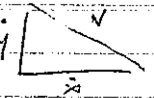
$\therefore y = 2.6 - 5\left(\frac{11.9}{65 \cos 9°}\right)^2 - 65\left(\frac{11.9}{65 \cos 9°}\right) \sin 9°$

$= 0.543435368$ m

$\therefore$ the ball doesn't clear the net. (1)

degree sign?

(14b iii) $V^2 = \dot{x}^2 + \dot{y}^2$ (1)



show substitution.

$$V^2 = (65\cos 9^\circ)^2 + \left(-10\left(\frac{11.9}{65\cos 9^\circ}\right) - 65\sin 9^\circ\right)^2$$

$$= 4266.13$$

$$V = 65.3156$$

$$\doteqdot 65.3 \text{ m/s}.$$ (1)

14c

i) R is on line $x = -a$ ∴ has same height as P.

$R\,(-a, ap^2)$

$Q\,(2ap, 0)$

$$M = \left(\frac{2ap - a}{2}, \frac{ap^2}{2}\right)$$

$$= \left(\frac{a(2p-1)}{2}, \frac{ap^2}{2}\right)$$

1 for x

1 for y

ii)

$$x = \frac{a(2p-1)}{2}$$

$$\frac{2x}{a} = 2p - 1$$

$$\frac{2x}{a} + 1 = 2p$$

$$\frac{x}{a} + \frac{1}{2} = p$$ (1)

$$y = ap^2/2$$

$$= a\left(\frac{x}{a} + \frac{1}{2}\right)^2/2$$ (1)

$$= a\left(\frac{x^2}{a^2} + \frac{x}{a} + \frac{1}{4}\right)/2$$

$$= \frac{\frac{x^2}{a} + x + \frac{a}{4}}{2}$$

$$\therefore y = \frac{x^2}{2a} + \frac{x}{2} + \frac{a}{8}.$$

M locus $y = \frac{x^2}{2a} + \frac{x}{2} + \frac{a}{8}$

axis of symmetry $= \frac{-b}{2a}$

$$= \frac{-\frac{1}{2}}{2\left(\frac{1}{2a}\right)}$$

$$= -\frac{1}{2} \div \frac{1}{a}$$ (1)

$$= \frac{-a}{2}.$$