CRANBROOK SCHOOL

YEAR 12 MATHEMATICS – EXTENSION 1

Term 3 2002 **Time : 2 h / HRK MJB SKB**

All questions are of equal value.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved silent calculators may be used.
Standard Integrals appear at the end of the paper.

Submit your work for Question 1 in a 4 page booklet.
Submit your work for Questions 2 and 3 in the same 8 page booklet.
Submit your work for Questions 4 and 5 in the same 8 page booklet.
Submit your work for Questions 6 and 7 in the same 8 page booklet.

Question 1(12 Marks) (Begin a new 4 page booklet) **HRK**

(a) The nth term of a series is given by $T_n = \dfrac{1}{(2n-1)(2n+1)}$

(i) Find T_5 and T_{k+1} **2**

(ii) Assuming that the sum S_k of the first k terms of this series is given by

$S_k = \dfrac{k}{2k+1}$, prove that $S_{k+1} = \dfrac{k+1}{2k+3}$ **3**

(iii) Using the results of parts (i) and (ii) and the Principle of Mathematical Induction, show that the sum of the first

n terms of the series is $\dfrac{n}{2n+1}$ **3**

(b) Prove by Mathematical Induction that $5^n \geq 1+4n$ **4**

Begin a new 8 page booklet
Question 2 (12 marks) **MJB**

(a) The area of a minor segment of a circle with an angle at the centre of 45^0 is 0.4795cm^2. Find the arc length of this minor segment (2 dp). **3** (b)Sketch the graph of $y = 3\cos(2\theta + \frac{\pi}{4})$ for the domain $0 \le \theta \le 2\pi$ 3

(c) Find the differentials of:

(i) $y = \left(x^2 - \cos(2x-3)\right)^3$ 1

(ii) $y = \ln(\tan 3x)$, expressing your final answer in terms of cosec$3x$. 2

(d) Find the exact area enclosed by the curve $y = \sec^2 2x$, the x-axis in the domain $o \le x \le \frac{\pi}{4}$ 3

Question 3 **(12 marks)** **MJB**

(a) (i) Find the total number of arrangements of 3 red beads and 4 blue beads in a row.

(ii) Find the total number of arrangements for the beads in part (i) when the beads are placed in a circle.

(iii) If the beads were made into a bracelet, what would be the number of arrangements then?

(iv) Comment on the differences between the three arrangements in parts (i), (ii) and (iii) .4

(b) An accountant's files are coded by different arrangements of coloured dots in a row. The colours used are red, white and blue. In an arrangement at most three of the dots are red, at most two of the dots are white and at most one of the dots are blue.

(i) Find the number of different codes possible if six dots are used.

(ii) On some files only five dots are used. Find the number of different codes possible in this case. Justify your answer. 4

(c) The IT meeting room contains a round table surrounded by identical chairs equally spaced around the table.

(i) A committee of ten people includes three students. How many seating arrangements are there in which all three sit together? Give brief reasons for your answer.

(ii) Elections are held for the position of Chairman and Secretary in a second committee of ten people seated around this table. What is the probability that the two people elected are sitting directly opposite each other? Give brief reasons for your answer. 4

Begin a new 8 page booklet

Question 4 (12 marks) **HRK**

(a) The polynomial $\mathrm{P}(x)$ is given by $\mathrm{P}(x) = x^3 - x^2 - 8x + 12$

(i) Factorise $\mathrm{P}(x)$ completely given that $\mathrm{P}(x) = 0$ has a repeated root **3**

(ii) The polynomial $Q(x)$ has the form $Q(x) = P(x)(x+a)$ with $\mathrm{P}(x)$ as given above and where the constant a is chosen so that $Q(x) \geq 0$ for all real values of x.
Find all possible values of a. **3**

(b) (i) By using Newton's Method once find an approximation to the root of the equation $e^{-x} - x - 1.01 = 0$ near $x = 0$ **3**

(ii)
Show that an attempt to find a root of the equation
$e^{-x} + x - 1.01 = 0$
starting with $x = 0$ fails. Explain why this happens. **3**

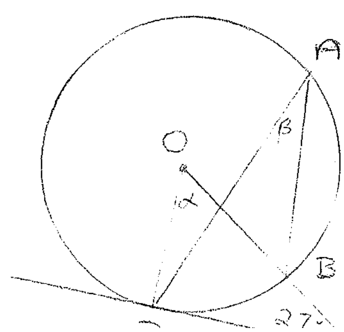
Question 5 (12 marks) **HRK**

(a) (i) Show that the equation of the tangent to the parabola $x^2 = 4ay$ at any point P $(2ap, ap^2)$ is given by $px - y - ap^2 = 0$ **2**

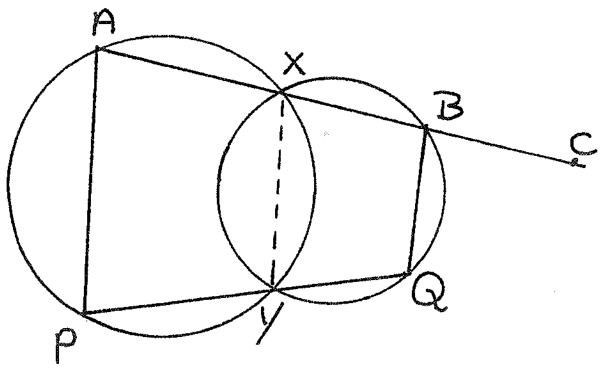
(ii) If S is the focus of the parabola in (i) and T the point of intersection of the tangent and the y - axis, prove that SP = ST **2**

(iii) Hence show that $\angle SPT$ is equal to the acute angle between the tangent and the line through P parallel to the axis of the parabola. **2**

(b) PQ is a tangent to the circle centre O as shown. $\angle PQO = 27^\circ$ **3**
Find α and β giving reasons



(c) Given AXB and PYQ are straight lines, prove that AP is parallel to BQ. 3



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Question 6 (12 marks) **SKB**

(a) Molten plastic at a temperature of $250^0 C$ is poured into moulds to form car parts. After 20 minutes the plastic has cooled to $150^0 C$. If the temperature of the surroundings is $30^0 C$ and the rate of cooling is given by $\frac{dT}{dt} = -k(T-30)$, where k is a constant and the temperature after t minutes is $T^0 C$:

(i) Show that $T = 30 + Ae^{-kt}$, where A is a constant. 2

(ii) Show that the value of A is 220. 1

(iii) Find the value of k to 3 significant figures. 2

(iv) How long, to the nearest minute, will it take for the plastic to cool to $80^0 C$? 2

(b) A boy riding a donkey travels South along a straight dirt road at 6km/h. He is heading for a farm which is 12km along this road and then 8km East into the bush. The donkey is getting tired and cranky and the boy wants to minimise the time to get to the farm. If the donkey can only travel through the bush at 3km/h, how far should the boy ride the donkey down the road (to the nearest metre) before turning into the bush and riding towards the farm? 5

Question 7 **(12 marks)** **SKB**

.(a) (i) Prove that the curve $y = x\sin^{-1} x$ has a minimum turning point at (0,0). **2**

(ii) Hence or otherwise sketch the curve $y = x\sin^{-1} x$ on the x-y plane. **2**

(b) Prove that $\tan^{-1} 2x + \tan^{-1}\left(\frac{1}{2x}\right) = \frac{\pi}{2}$, for all values of x. **3**

(c) (i) Sketch the curve $y = \frac{-1}{\sqrt{1-16x^2}}$ given that it has a maximum turning point at (0, -1). **1**

(ii) Hence, by using the substitution $u = 4x$, find the area bounded by the curve $y = \frac{-1}{\sqrt{1-16x^2}}$, the $x-$ axis and lines $x = \frac{-\sqrt{3}}{8}$ and $x = \frac{-1}{8}$. **4**