CRESTWOOD HIGH SCHOOL

MATHEMATICS EXTENSION 1

2011

TRIAL HSC EXAMINATION

Assessor: X Chirgwin

General Instructions:

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Write your student number at the top of every page
- Board approved calculators may be used.
- Draw diagrams using pencil
- A table of Standard Integrals is provided at the back of this paper

Total marks – 84

- Attempt questions 1 – 7
- All questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work

This paper MUST NOT be removed from the examination room

Student Number: ……………………………………………….

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2011 Higher School Certificate Examination.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

a) Solve for x: $\dfrac{x+2}{x-4} \geq 3$ **3**

b) Evaluate $\lim\limits_{x \to 0} \dfrac{10x}{\sin\left(\frac{\pi}{6}x\right)}$ **1**

c) The point P divides the interval joining $A(4, 3)$ and $B(-2, 5)$ externally in the ratio $1 : 3$. Find coordinates of P. **2**

d) For what value of a is the expression $3x^3 + 20x^2 + a$ divisible by $x + 6$? **1**

e) Expand $\left(x - \frac{1}{2}\right)^7$, express each term in its simplest form. **2**

f) Find $\int x^2(7 - 2x^3)^4 dx$, using the substitution $u = 7 - 2x^3$ **3**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

a) For $f(x) = 3\cos^{-1}\frac{x}{2}$

i) State the domain and the range **2**

ii) Sketch $f(x)$ **1**

b) Find $\frac{d}{dx}(e^{4x}\cos 3x)$ **2**

c) Consider the parabola $x = 6t, y = 3t^2$.

i) Express this parabola in cartesian form **1**

ii) Find the gradient of the parabola at the point where $t = -2$ **1**

iii) Find the equation of the normal to the parabola at $t = -2$ **2**

d) Find the size of the acute angle to the nearest minute between the graphs of $y = 3x - 2$ and $y = x^3 - 4$ at the point of intersection where $x = 2$ **3**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

a) Given $xy = 2$, evaluate $\int_1^8 x\,dy$ to 4 significant figures. **2**

b) Let $f(x) = \sin x - \ln x$

i) Show that a root to $f(x) = 0$ lies between 2 and 2.5 **1**

ii) Starting with a value of $x = 2$, use 1 application of Newton's method to find a better approximation to this root of $f(x) = 0$, correct your answer to 3 decimal places **3**

c) How many 12 letter combinations can be made using the letters of the word SUBSTITUTION? **1**

d) The polynomial $3x^3 + 5x^2 - 4x - 2 = 0$ has 3 roots, namely α, β and γ. Find the values of:

i) $\alpha + \beta + \gamma$ **1**

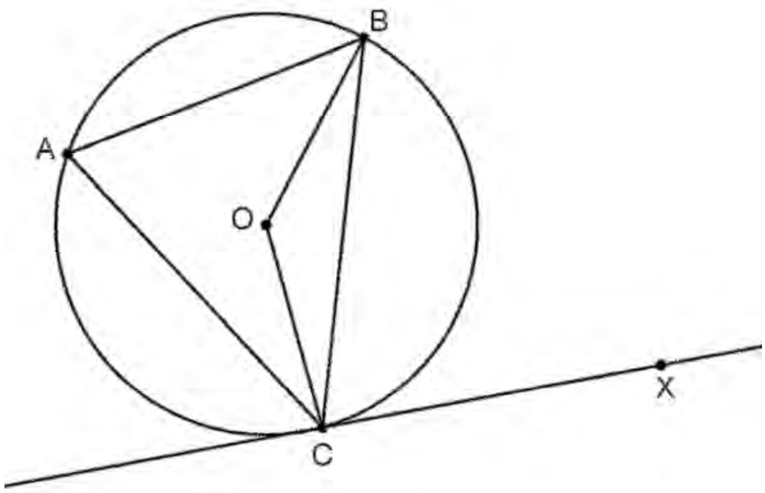
ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ **2**

iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

a) CX is a tangent to the circle centre O. Let $\angle CAB = \alpha$



i) Find $\angle COB$ with reasons **1**

ii) Find $\angle OCB$ with reasons **1**

iii) Show that $\angle BCX = \angle BAC$ **1**

b) i) Find $\dfrac{d}{dx}\left(x\sin^{-1}\dfrac{x}{4} + \sqrt{16 - x^2}\right)$ **3**

ii) Hence, evaluate $\displaystyle\int_0^4 \sin^{-1}\frac{x}{4}\,dx$ **2**

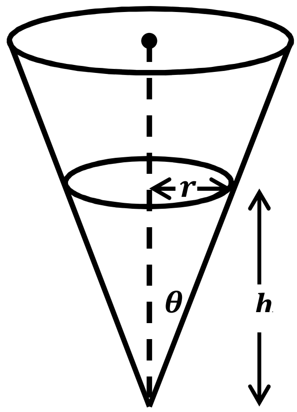
c) Use the process of mathematical indunction to show that for all $n \geq 1$, **4**

$$1 + 15 + 65 + \ldots\ldots\ldots\ldots + (4n^3 - 6n^2 + 4n - 1) = n^4$$

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

a) The diagram below shows a container in the shape of a right circular cone.



The semi-vertical angle $\theta = \tan^{-1}\frac{1}{2}$

Water is poured in at the constant rate of 15 cm^3 per minute.
Let the height of the water at time t seconds be h cm, let the radius of the water surface be r cm, and let the volume of water be V cm^3.

i) Show that $r = \frac{1}{2}h$ **1**

ii) Show that $V = \frac{1}{12}\pi h^3$ **1**

iii) Find the exact rate at which h is increasing when the height of the water in the cone is 35 cm. Leave your answer in exact value. **2**

b) The point $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

i) Show that the coordinates of the mid-point M, of the chord PQ are **1**

$$\left[a(p+q), \frac{a}{2}(p^2+q^2)\right]$$

ii) If the chord PQ is a focal chord. Find the equation of the locus of M and describe the locus of M geometrically. **3**

Question 5 Continued

c) An ice cube tray is filled with water at a temperature of $22°\mathrm{C}$ and placed in a freezer that has a constant temperature of $-21°\mathrm{C}$. The cooling rate of the water is proportional to the difference between the temperature of the freezer and the temperature of the water, T.

That is, T satisfies the equations

$$\frac{dT}{dt} = -k(T + 21) \qquad \& \qquad T = -21 + Ae^{-kt}$$

i) Show that $A = 43$ **1**

ii) After 8 minutes in the freezer the temperature of the water is $2°\mathrm{C}$. Find the time to the nearest minute for the water to reach $-20.9°\mathrm{C}$. **3**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

a) A particle is oscillating in simple harmonic motion such that its displacement x metres from the origin is given by the equation

$$\frac{d^2x}{dt^2} = -25x$$

where t is time in seconds.

i) Show that $x = a\cos(5t + \alpha)$ is a solution of motion for this partciple. (a and α are constants) **1**

ii) When $t = 0$, $v = 5$ m/s and $x = 6$ m. Show that the amplitude of the oscillation is $\sqrt{37}$. **2**

iii) What is the maximum speed of the particle? **1**

b) A particle moves along a straight line such that its distance from the origin at time t in seconds is x metres and its velocity is v.

i) Prove that $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \dfrac{d^2x}{dt^2}$ **2**

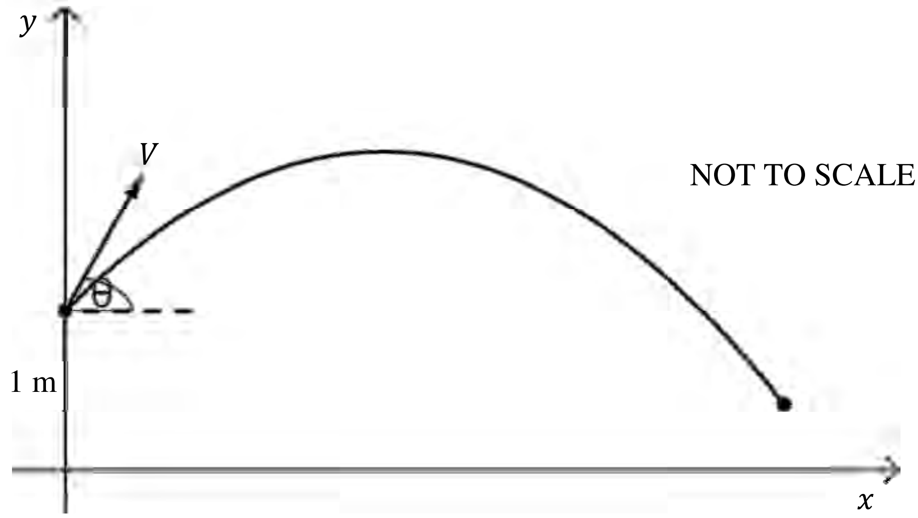
ii) If the acceleration satisfies $\dfrac{d^2x}{dt^2} = -9\left(x + \dfrac{81}{x^3}\right)$ and the particle is initially at rest when $x = 3$, show that $v^2 = 9\left(\dfrac{81 - x^4}{x^2}\right)$ **3**

c) Find the area under the curve $y = \dfrac{1}{\sqrt{9 - 4x^2}}$ bounded by the x-axis on the domain $0 \le x \le \dfrac{3}{4}$. **3**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

a) A boy throws a ball and projects it with a speed of V m/s from a point 1 metre above the ground. The ball lands on top of a flowerpot in a neighbour's yard.



The angle of projection is θ as indicated in the diagram.
The equations of motion of the ball are:

$$\ddot{x} = 0 \text{ and } \ddot{y} = -10$$

where x and y are shown on the axes on the diagram. The position of the ball t seconds after it is thrown by the boy is described by the co-ordinates $(x\,,\ y)$.

It has been found that $y = Vt\sin\theta - 5t^2 + 1$.

i) Show that $x = Vt\cos\theta$ **2**

ii) When the ball is at its maximum height above the ground, it passes directly above a 1.5 metre high fence and clears the fence by 0.5 metres. **3**
Find an expression for V in terms of θ.

iii) Find the value of V given that $\theta = \tan^{-1}\dfrac{9}{40}$ **2**

Give your answer in m/s, correct to 2 decimal places.

Question 7 Continued

b) Assume that tides rise and fall in Simple Harmonic Motion. A ship needs 11 metres of water to pass down a channel safely. At low tide, the channel is 8 metres deep and at high tide 12 metres deep. Low tide is at 9:00 am and high tide is at 3:00 pm. **5**

Find the first time period during which the ship can safely proceed through the channel.

-End of Paper-

2011 Trials Mathematics Extension 1 Solution

Q1.

a)

$\dfrac{x+2}{x-4} \geq 3$

$(x+2)(x-4) \geq 3(x-4)^2 \quad x \neq 4$

$x^2 - 2x - 8 \geq 3(x^2 - 8x + 16)$

$x^2 - 2x - 8 - 3x^2 + 24x - 48 \geq 0$

$-2x^2 + 22x - 56 \geq 0$

$x^2 - 11x + 28 \leq 0$

$(x-4)(x-7) \leq 0$

$\therefore 4 < x \leq 7$

b)

$\lim\limits_{x\to 0} \dfrac{10x}{\sin\left(\frac{\pi}{6}x\right)} = \dfrac{10}{\frac{\pi}{6}} = \dfrac{60}{\pi}$

c)

$A(4,3)$ and $B(-2,5)$

divides externally – $m : n$ or $m : -n$

$-1 : 3$ or $1 : -3$

$x = \dfrac{-2 \times (-1) + 4 \times 3}{-1+3}$

$x = 7$

$y = \dfrac{5 \times (-1) + 3 \times 3}{-1+3}$

$y = 2$

$\therefore P(7,2)$

d)

Let $P(x) = 3x^3 + 20x^2 + a$

If $P(x)$ is divisible by $x + 6$

$P(-6) = 3(-6)^3 + 20 \times (-6)^2 + a = 0$

$\therefore a = -72$

e)

$\left(x - \frac{1}{2}\right)^7$

$= x^7 + 7x^6 \times \left(-\frac{1}{2}\right) + 21x^5 \times \left(-\frac{1}{2}\right)^2$

$+35x^4 \times \left(-\frac{1}{2}\right)^3 + 35x^3 \times \left(-\frac{1}{2}\right)^4$

$+21x^2 \times \left(-\frac{1}{2}\right)^5 + 7x \times \left(-\frac{1}{2}\right)^6$

$+\left(-\frac{1}{2}\right)^7$

$= x^7 - \frac{7}{2}x^6 + \frac{21}{4}x^5 - \frac{35}{8}x^4 + \frac{35}{16}x^3$

$-\frac{21}{32}x^2 + \frac{7}{64}x - \frac{1}{128}$

f)

$u = 7 - 2x^3 \qquad du = -6x^2dx$

$\int x^2(7-2x^3)^4dx$

$= -\frac{1}{6}\int -6x^2(7-2x^3)^4dx$

$= -\frac{1}{6}\int u^4du$

$= -\frac{1}{6}\left[\frac{u^5}{5}\right] + C$

$= -\frac{(7-2x^3)^5}{30} + C$

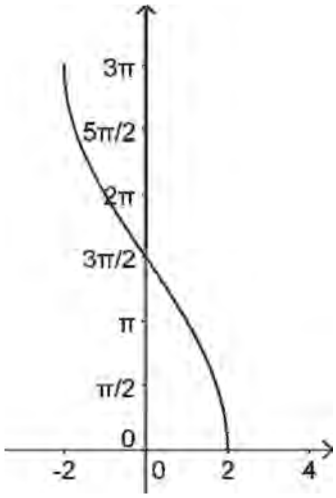
Q2.

a) i)

$$f(x) = 3\cos^{-1}\frac{x}{2}$$

Domain: $-2 \le x \le 2$
Range: $0 \le y \le 3\pi$

ii)



b)

$$\frac{d}{dx}(e^{4x}\cos 3x)$$
$$= 4e^{4x} \times \cos 3x - e^{4x} \times 3\sin 3x$$
$$= e^{4x}(4\cos 3x - 3\sin 3x)$$

c) i)

$$x = 6t, \qquad t = \frac{x}{6}$$
$$y = 3t^2 = 3 \times \left(\frac{x}{6}\right)^2$$
$$y = \frac{x^2}{12}$$
$$x^2 = 12y$$

ii)

$$\frac{dy}{dx} = \frac{2x}{12} = \frac{12t}{12} = t$$
$$\frac{dy}{dx} = -2 \qquad (t = -2)$$

iii)

$$m_T = -2, \quad m_N = \frac{1}{2}$$
$$t = -2, \ \ x = -12, \ \ y = 12$$
$$y - 12 = \frac{1}{2}(x + 12)$$
$$2y - 24 = x + 12$$
$x - 2y + 36 = 0$ (Equation of Normal)

d)

$$y = 3x - 2 \qquad \frac{dy}{dx} = m_1 = 3$$
$$y = x^3 - 4 \qquad \frac{dy}{dx} = 3x^2$$
$$at\ x = 2 \qquad \frac{dy}{dx} = m_2 = 12$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$$\tan\theta = \left|\frac{3 - 12}{1 + 3 \times 12}\right|$$
$$\tan\theta = \frac{9}{37}$$
$\theta = 13°40'$ (nearest minute)

Q3.

a)

$$xy = 2, \quad x = \frac{2}{y}$$

$$\int_1^8 x\,dy = \int_1^8 \frac{2}{y}dy$$
$$= 2[\ln y]_1^8$$
$$= 2(\ln 8 - \ln 1)$$
$= 4.159$ (4 sig fig)

b) i) (remember to calculate in radian)

$$f(x) = \sin x - \ln x$$
$$f(2) = \sin 2 - \ln 2 = 0.216 \ldots$$
$$f(2.5) = \sin 2.5 - \ln 2.5 = -0.317 \ldots$$

Since there is a sign change between $f(2)$ and $f(2.5)$, and the function is continuous between these two points, therefore there is a root lies between 2 and 2.5.

ii)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$x_1 = 2 - \frac{f(2)}{f'(2)}$$
$$f'(x) = \cos x - \frac{1}{x}$$
$$f'(2) = \cos 2 - \frac{1}{2}$$
$$x_1 = 2 - \frac{\sin 2 - \ln 2}{\cos 2 - \frac{1}{2}}$$
$x_1 = 2.236$ (3 d.p.)

c) SUBSTITUTION

2 S's, 2 U's, 3 T's, 2 I's

$$\frac{12!}{2!\,2!\,3!\,2!} = 9979200$$

d)

i) $3x^3 + 5x^2 - 4x - 2 = 0$

$$\alpha + \beta + \gamma = -\frac{5}{3}$$

ii)

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \alpha\gamma}{\alpha\beta\gamma}$$
$$= \frac{-\frac{4}{3}}{\frac{2}{3}} = -2$$

iii)

$$\alpha^2 + \beta^2 + \gamma^2$$
$$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$
$$= \left(-\frac{5}{3}\right)^2 - 2\left(-\frac{4}{3}\right)$$
$$= \frac{49}{9}$$

Q4.

a) i) $\angle COB = 2\alpha$ (angle at the centre is twice the angle at the circumference)

ii) $\angle OCB = \frac{(180-2\alpha)}{2} = 90 - \alpha$
(angle sum of a triangle)

iii) $\angle OCX = 90°$ (radius meets tangent)
$\angle BCX = 90° - \angle OCB = 90° - (90° - \alpha)$
$\therefore \angle BCX = \alpha = \angle BAC$

b) i)

$$\frac{d}{dx}\left(x\sin^{-1}\frac{x}{4} + \sqrt{16-x^2}\right)$$
$$= \frac{x}{\sqrt{16-x^2}} + \sin^{-1}\left(\frac{x}{4}\right)$$
$$+\frac{1}{2}(16-x^2)^{-\frac{1}{2}} \times (-2x)$$
$$= \frac{x}{\sqrt{16-x^2}} + \sin^{-1}\left(\frac{x}{4}\right) - \frac{x}{\sqrt{16-x^2}}$$
$$= \sin^{-1}\left(\frac{x}{4}\right)$$

ii)
$$\int_0^4 \sin^{-1}\left(\frac{x}{4}\right)$$
$$= \left[x\sin^{-1}\frac{x}{4} + \sqrt{16-x^2}\right]_0^4$$
$$= \left(4\sin^{-1}1 + \sqrt{16-16}\right) - \left(0 + \sqrt{16}\right)$$
$$= 2\pi - 4$$

c)
Step1: Prove statement is true for $n = 1$
$$LHS = (4 \times 1^3 - 6 \times 1^2 + 4 \times 1 - 1)$$
$$= 1$$
$$RHS = 1^4 = 1$$
$$LHS = RHS$$
$\therefore$ Statement is true for $n = 1$

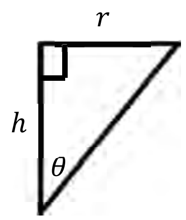
Step2: Assume statement is true for $n = k$
$$1 + 15 + 65 + \ldots\ldots + (4k^3 - 6k^2 + 4k - 1)$$
$$= k^4$$

Step3: Prove statement is true for $n = k + 1$

$$1 + 15 + 65 + \ldots\ldots + (4k^3 - 6k^2 + 4k - 1)$$
$$+(4(k+1)^3 - 6(k+1)^2 + 4(k+1) - 1)$$
$$= (k+1)^4$$
$$LHS = k^4 + (4(k+1)^3 - 6(k+1)^2 + 4(k+1) - 1)$$
$$LHS = k^4 + 4k^3 + 12k^2 + 12k + 4$$
$$-6k^2 - 12k - 6 + 4k + 4 - 1$$
$$LHS = k^4 + 4k^3 + 6k^2 + 4k + 1$$
$$LHS = (k+1)^4$$
$$LHS = RHS$$
$\therefore$ Statement is true for all $n \geq 1$ by mathematical induction.

Q5.

a) i)



$$\tan\theta = \frac{r}{h}$$
$$\frac{1}{2} = \frac{r}{h}$$
$$r = \frac{1}{2}h$$

ii)

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi \times \left(\frac{1}{2}h\right)^2 \times h$$
$$V = \frac{1}{12}\pi h^3$$

iii)

$$\frac{dV}{dt} = 15 \text{ cm}^3/\text{min}$$
$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$
$$\frac{dV}{dh} = \frac{\pi h^2}{4} \qquad \frac{dh}{dV} = \frac{4}{\pi h^2}$$
$$\frac{dh}{dt} = \frac{4}{\pi h^2} \times 15$$

When $h = 35$ cm
$$\frac{dh}{dt} = \frac{4}{\pi \times 35^2} \times 15$$
$$\frac{dh}{dt} = \frac{12}{245\pi} \text{ cm/min}$$

b) i) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$
$$M = \left(\frac{2ap + 2aq}{2} , \frac{ap^2 + aq^2}{2}\right)$$
$$M = \left(a(p + q) , \frac{a}{2}(p^2 + q^2)\right)$$

ii) PQ is a focal chord, so $pq = -1$
$$x = a(p + q)$$
$$p + q = \frac{x}{a}$$
$$y = \frac{a}{2}(p^2 + q^2)$$
$$y = \frac{a}{2}((p + q)^2 - 2pq)$$
$$y = \frac{a}{2}\left(\left(\frac{x}{a}\right)^2 + 2\right)$$
$$y = \frac{a}{2}\left(\frac{x^2}{a^2} + 2\right)$$
$$y = \frac{x^2}{2a} + a$$
$$x^2 = 2a(y - a)$$

This is a prabola with vertex $(0, a)$ and focal length $\frac{1}{2}a$

c) i) $t = 0, T = 22$
$$22 = -21 + Ae^0$$
$$A = 43$$

ii) $t = 8, T = 2$
$$2 = -21 + 43e^{-8k}$$
$$e^{-8k} = \frac{23}{43}$$
$$k = -\frac{1}{8}\ln\frac{23}{43}$$
when $T = -20.9$
$$-20.9 = -21 + 43e^{-kt}$$
$$t = \frac{\ln\frac{1}{430}}{-k}$$
$$t = 78 \text{ minutes}$$

Q6.

a) i) $x = a\cos(5t+\alpha)$

$\dot{x} = -5a\sin(5t+\alpha)$

$\ddot{x} = -25a\cos(5t+\alpha)$

$\ddot{x} = -25x$

$\therefore x = a\cos(5t+\alpha)$ is a solution

ii) Method 1

when $t = 0, v = 5$

$\dot{x} = -5a\sin(5t+\alpha)$

$5 = -5a\sin(\alpha)$

$a\sin\alpha = -1$ ………………………①

when $t = 0, x = 6$

$6 = a\cos(\alpha)$

$a\cos\alpha = 6$ ………………………②

①^2 + ②^2

$a^2\sin^2\alpha + a^2\cos^2\alpha = (-1)^2 + 6^2$

$a^2(\sin^2\alpha + \cos^2\alpha) = 37$

$a^2 = 37$

$a = \sqrt{37}$

Method 2

$v^2 = n^2(a^2 - x^2)$

$25 = 25(a^2 - 36)$

$a^2 = 37$

$a = \sqrt{37}$

iii) Method 1

Maximum speed occurs when

$\sin(5t+\alpha) = 1$

$\dot{x} = -5a\sin(5t+\alpha)$

$\dot{x} = -5\times\sqrt{37}\sin(5t+\alpha)$

$\dot{x} = -5\sqrt{37}$

Speed is always positive

$\therefore$ Maximum speed is $5\sqrt{37}$

Method 2 maximum velocity when $x = 0$

$v^2 = 25(37-0)$

$v = \pm 5\sqrt{37}$

$\therefore$ Maximum speed is $5\sqrt{37}$

b) i)

$\frac{d}{dv}\left(\frac{1}{2}v^2\right) = \frac{1}{2}\times 2\times v\times\frac{dv}{dx}$

$\frac{d}{dv}\left(\frac{1}{2}v^2\right) = v\frac{dv}{dx}$

$\frac{d}{dv}\left(\frac{1}{2}v^2\right) = \frac{dx}{dt}\times\frac{dv}{dx}$

$\frac{d}{dv}\left(\frac{1}{2}v^2\right) = \frac{dv}{dt}$

$\therefore \frac{d}{dv}\left(\frac{1}{2}v^2\right) = \frac{d^2x}{dt^2}$

ii)

$\frac{d^2x}{dt^2} = \frac{d}{dv}\left(\frac{1}{2}v^2\right) = -9\left(x + \frac{81}{x^3}\right)$

$\frac{1}{2}v^2 = \int -9\left(x + \frac{81}{x^3}\right)dx$

$\frac{1}{2}v^2 = -\frac{9x^2}{2} - \frac{729x^{-2}}{-2} + C$

when $t = 0, x = 3, v = 0$

$0 = -81 + 81 + C \quad \rightarrow C = 0$

$v^2 = -9x^2 + \frac{729}{x^2}$

$\therefore v^2 = 9\left(\frac{81 - x^4}{x^2}\right)$

c)

$A = \int_0^{\frac{3}{4}} \frac{1}{\sqrt{9-4x^2}}dx$

Let $u = 2x$

$du = 2dx$

$x = \frac{3}{4} \qquad u = \frac{3}{2}$

$x = 0 \qquad u = 0$

$A = \frac{1}{2}\int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-u^2}}du$

$A = \frac{1}{2}\left[\sin^{-1}\frac{u}{3}\right]_0^{\frac{3}{2}}$

$A = \frac{1}{2}\left(\frac{\pi}{6}\right)$

$A = \frac{\pi}{12}$ units^2

Q7.

a) i)

$\ddot{x} = 0$

$\dot{x} = \int 0\, dt$

$\dot{x} = C$

when $t = 0,\ \dot{x} = V\cos\theta$

$C = V\cos\theta$

$\therefore \dot{x} = V\cos\theta$

$x = \int V\cos\theta\, dt$

$x = Vt\cos\theta + C$

when $t = 0, x = 0$

$C = 0$

$\therefore x = Vt\cos\theta$

ii) From the question, the maximum height reached by the ball is 2 metres. The maximum height occurs when $\dot{y} = 0$

$y = Vt\sin\theta - 5t^2 + 1$

$\dot{y} = V\sin\theta - 10t$

$V\sin\theta - 10t = 0$

$t = \dfrac{V\sin\theta}{10}$

Height of the ball given by

$y = Vt\sin\theta - 5t^2 + 1$

$2 = \dfrac{V^2\sin^2\theta}{10} - \dfrac{5V^2\sin^2\theta}{100} + 1$

$\dfrac{V^2\sin^2\theta}{20} = 1$

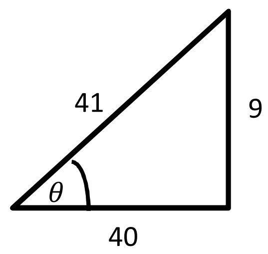
$V^2 = \dfrac{20}{\sin^2\theta}$

$V = \dfrac{\sqrt{20}}{\sin\theta} \quad (V > 0)$

iii)

$\theta = \tan^{-1}\dfrac{9}{40}$

$\sin\theta = \dfrac{9}{41}$

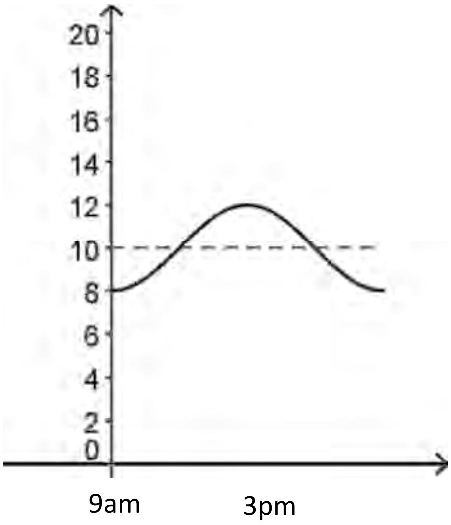


$V = \dfrac{\sqrt{20}}{\sin\theta}$

$V = \dfrac{2\sqrt{5}\times 41}{9}$

$V = 20.37$ m/s (2 d.p)

b)



Centre of the motion is 10

Amplitute is 2 m

Between 9am to 3pm, there are 6 hours.

So a full period is 12 hours.

$12 = \frac{2\pi}{n}$

$n = \frac{\pi}{6}$

$x = -2\cos\left(\frac{\pi}{6}t\right) + 10$

when $x = 11$ m

$11 = -2\cos\left(\frac{\pi}{6}t\right) + 10$

$-\frac{1}{2} = \cos\left(\frac{\pi}{6}t\right)$

$\frac{\pi}{6}t = \frac{2\pi}{3}\ ,\ \frac{4\pi}{3}$

$t = 4 \text{ hours}\ ,\ 8 \text{ hours}$

9:00 + 4 hours = 13:00 = 1pm

9:00 + 8 hours = 17:00 = 5pm

The first time period the ship can safely pass through would be between 1pm and 5pm.