STANDARD INTEGRALS

$\int x^n \, dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$

$\int e^{ax} \, dx = \frac{1}{a}e^{ax}, \quad a \neq 0$

$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$

$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$

$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$

$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$

$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$

$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$

$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$

$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE: $\ln x = \log_e x, \quad x > 0$

QUESTION 1 *(12 marks)*

(a) Find

i) $\int \sec^2 2x \, dx$ (1)

ii) $\int \frac{dx}{9 + x^2}$ (1)

iii) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}} \, dx$ (2)

(b) Let $f(x) = \sin x \cos x$

Find

i) $f'(x)$ (2)

ii) $\int_0^{\frac{\pi}{2}} f(x) \, dx$ (2)

(c) Differentiate

i) $\sin^{-1} 2x$ (1)

ii) $\tan^{-1} \frac{x}{5}$ (1)

(d) Find the exact value of (2)

$\sin^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

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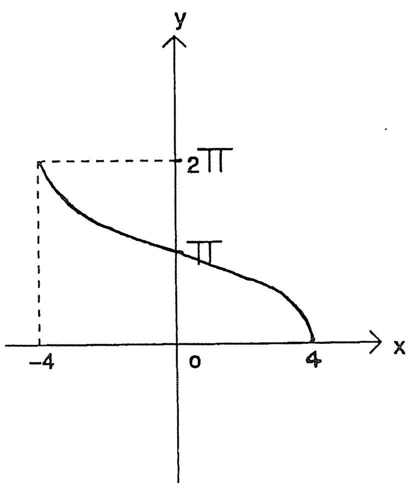
QUESTION 2 *(12 marks)*

(a) Solve $\dfrac{2x+1}{x-2} > 1$ (3)

(b) Use the substitution $u = \log_e x$ to evaluate

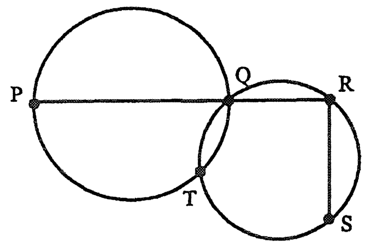
$$\int_1^e \frac{\log_e x}{x}\,dx$$ (3)

(c) If the graph below represents $y = A\cos^{-1}Bx$ where A & B are constants, find the value of A & B. (2)



Question 2 cont.....

(d)



(4)

PQ is a diameter of the larger circle & PQR is in a straight line. PR ⊥ RS. Prove that P, T and S are collinear.

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Question 3 *(12 marks)*

(a) A spherical ball is being inflated at the rate of π cubic cm per minute. At what rate is the radius of the balloon increasing when the radius is 2cm? (2)

(b) In how many ways can 5 men & 2 ladies be seated in a line if the 2 ladies are not to sit together? (4)

What is the probability that the ladies are separated by two men?

(c) In a colony of organisms, it is known that the natural growth rate of the colony is given by $\dfrac{dN}{dt} = kN - r$, where N is the number of organisms at the time t (minutes) and r is the constant rate per minute at which the organisms die per minute, k being a growth factor

i) Verify that $N = \dfrac{r}{k} - \dfrac{A}{k}e^{kt}$ is a solution of the equation $\dfrac{dN}{dt} = kN - r$ (6)

ii) Find the time when the population of the colony is reduced to zero, given that when $t = 0$, $N = 200$, $k = 2$, $r = 500$. Give your answer to one decimal place.

iii) Find N when $t = 0.5$ min.

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QUESTION 4 *(12 marks)*

(a) If x is an obtuse angle and $\tan 2x = \frac{12}{5}$ find the exact value of $\tan x$. (3)

(b) Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $A\sin(\theta + \alpha)$ where $A > 0$ & $0^o < \alpha^o < 360^o$. Hence solve $\sqrt{3}\cos\theta - \sin\theta = 1$ if $0^o < \theta^o < 360^o$. (3)

(c) Solve $\tan x = \sin 2x$ for $-\pi \le x \le \pi$. Hence find the general solution. (3)

(d) If α, β and γ are the roots of the equation $x^3 + 4x^2 + 8x + 16 = 0$, find the value of $\alpha^2 + \beta^2 + \gamma^2$. (3)

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QUESTION 5 *(12 marks)*

(a) The equation giving the acceleration of a particle moving in a straight line is $\frac{d^2x}{dt^2} = 8x^3$ where x is the displacement at time t. If when $t = 0$, $x = 1$ & $\frac{dx}{dt} = 2$ find an expression for v in terms of x and show that $x = \frac{1}{1-2t}$. (4)

(b) P is a point on the parabola $x^2 = 8y$ and S is the focus. The tangent to the parabola at P meets the Y axis in M. The perpendicular from focus S to the tangent PM meets the tangent in N. (8)

i) Find the equation of the tangent at P.

ii) Find the equation of SN.

iii) Show that the co-ordinates of N are (2p,0)

iv) Find the equation of the locus of the midpoint of MN as P varies.

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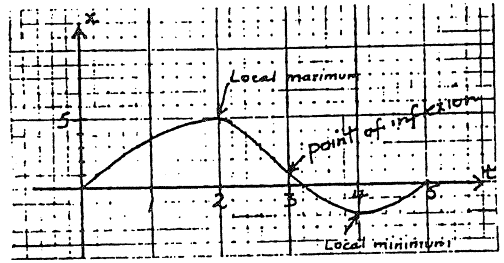
QUESTION 6 *(12 marks)*

(a) A sequence, (known as the Fibonacci sequence) is defined by $U_1 = 1$, $U_2 = 2$ and $U_n = U_{n-1} + U_{n-2}$ for $n > 2$. Use the principle of induction to prove that $U_1 + U_2 + U_3 + \ldots + U_n = U_{n+2} - 2$. (6)

(b) A particle moves in a straight East-West line. Take East as the positive direction. The curve below is part of the displacement-time curve for the motion. x indicates the displacement (in centimetres) from 0, a fixed point on the line, and t the time (in hours) after 12 midday.



You must justify your answers to each of the following by reference to the graph: (6)

i) In which direction is the particle travelling after 2.30pm?

ii) When does the particle first turn around?

iii) In which intervals of time is the particle speeding up?

iv) State an interval of time when the particle is slowing down?

v) Where is the particle at its fastest speed?

vi) When is the particle's velocity negative?

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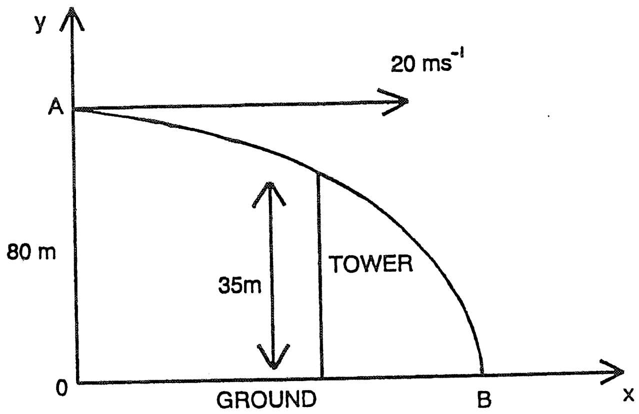
QUESTION 7 *(12 marks)*

(a) The speed v ms^{-1} of a particle moving in a straight line is given by $v^2 = 84 + 16x - 4x^2$, where the displacement of the particle relative to a fixed point is x cm.

Find i) an expression for the particle's acceleration in terms of x and hence show that the motion of the particle is Simple Harmonic Motion. (6)

ii) the period, the centre and the amplitude of the motion.

(b) In the diagram below, an object is projected horizontally with velocity of $20\ ms^{-1}$ from a point A, 80 m above the ground and strikes the ground at point B.



i) Derive expressions for x and y after t seconds, taking the acceleration due to gravity as $10\ ms^{-2}$. (6)

ii) Find the time taken to reach the point B and the distance OB.

iii) If the object just clears a vertical tower of height 35 m, find the distance from 0 to the base of the tower.

End of Examination