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FORT STREET HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2002

MATHEMATICS

EXTENSION I (ADDITIONAL)

Time allowed : Two Hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- A table of standard integrals is included.
- Board-approved calculators may be used.
- Each question is to be started on a NEW PAGE and solutions are to be written on ONE side only.
- You may ask for extra Writing Paper if required.
- You must STAPLE each question in a separate bundle clearly labeled with your Name.

Name :

Q	1	2	3	4	5	6	7	Total /84
Mark /12								

Question 1: *(start a new sheet of paper)* **[12 marks]**

a) For the polynomial $P(x) = 2x^3 - 3x^2 - 3x + 2$,

i) Show that $(x+1)$ is a factor. [1]

ii) Hence, or otherwise, find all the factors of $P(x)$. [3]

b) Determine the ratio in which the point C(6,9) divides the interval AB, given A is the point (-1,-5) and B is (3,3). [2]

c) Find the acute angle between the lines $2x + y = 2$ and $y - x = 4$. [2]

d) Evaluate $\int_1^4 y dx$ if $xy = 1$. [2]

e) Factorise $2^{n+1} + 2^n$, and hence write $\frac{2^{1001} + 2^{1000}}{3}$ as a power of 2. [2]

Question 2: *(start a new sheet of paper)* **[12 marks]**

a) Given $f(x) = x \sin 2x$,

i) Differentiate $f(x)$. [1]

ii) Hence find $\int (x \cos 2x) dx$. [2]

b) Solve the inequality $x + 1 \geq \frac{1}{x-1}$. [3]

c) Using the substitution $u = 2 + x$, find $\int x(2+x)^4 dx$. [3]

d) The arc of the curve $y = \cos 2x$ between $x = 0$ and $x = \frac{\pi}{6}$ is rotated a full revolution about the x-axis.

Find the exact volume of the solid formed. [3]

Question 3: *(start a new sheet of paper)* **[12 marks]**

a) Use the Principle of Mathematical Induction to prove that $2^{3n} - 3^n$ is divisible by 5 for all positive integers n. [4]

b) Use one iteration of Newton's method to find an approximation to the root of the equation $x\ln x - 2x = 0$ near $x = 7$ (answer to 1 dp) [3]

c) If $t = \tan\frac{\theta}{2}$,

i) Show that $\sin\theta = \frac{2t}{1=t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$. [2]

ii) Using these results, show that $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$. [2]

iii) Hence find the exact value of $\tan 15°$. [1]

<u>Question 4</u>: *(start a new sheet of paper)* **[12 marks]**

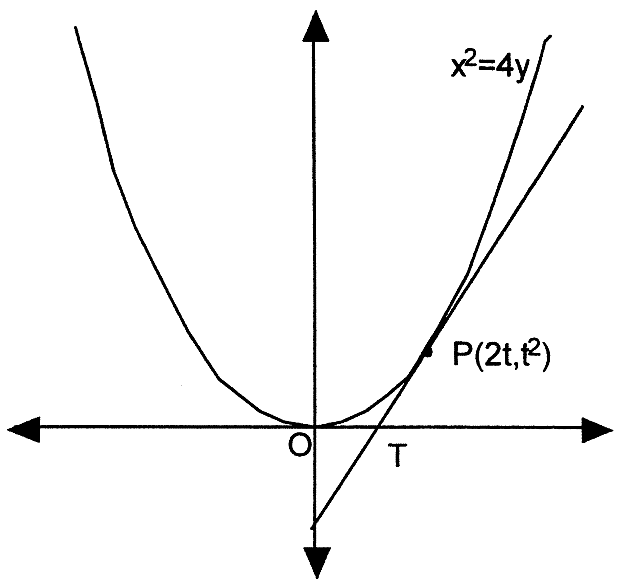
a) Newton's Law of Cooling states that the rate of change in the temperature, T, of a body is proportional to the difference between the temperature of the body and the surrounding temperature, P.

i) If A and k are constants, show that the equation $T = P + Ae^{kt}$ satisfies Newton's Law of Cooling. [2]

ii) A cup of tea with a temperature of $100°C$ is too hot to drink. Two minutes later, the temperature has dropped to $93°C$. If the surrounding temperature is $23°C$, calculate A and k. [2]

iii) The tea will be drinkable when the temperature has dropped to $80°C$. How long, to the nearest minute, will this take? [2]

b) $P(2t, t^2)$ is a variable point which moves on the parabola $x^2 = 4y$. The tangent to the parabola at P cuts the x-axis at T. M is the midpoint of PT.



i) Show that the tangent PT has equation $tx - y - t^2 = 0$. [2]

ii) Show that M has the coordinates $\left(\frac{3t}{2}, \frac{t^2}{2}\right)$. [2]

iii) Hence find the Cartesian equation of the locus of M as P moves on the parabola. [2]

Question 5: *(start a new sheet of paper)* **[12 marks]**

a) Given $f(x) = \sqrt{2x - 3}$,

i) Draw a neat sketch of $f(x)$, stating its domain and range. [3]

ii) Find the inverse function $f^{-1}(x)$, and sketch it on the same diagram as $f(x)$, stating its domain and range. [3]

b) A pebble is projected from the top of a vertical cliff with velocity $20ms^{-1}$ at an angle of elevation of $\frac{\pi}{6}$ radians. The cliff is 40m high and overlooks a lake. (Assume $g = -10m/s$)

i) Taking the origin O to be at the base of the cliff immediately below the point of projection, draw a neat diagram showing the path of the pebble. Mark all data given above on the diagram. [1]

ii) Derive expressions for the horizontal component $x(t)$ and the vertical component $y(t)$ of the pebbles displacement from O after t seconds. [2]

iii) Calculate the time that elapses before the pebble hits the lake and the distance of the point of impact from the foot of the cliff. Give your answer to one decimal place. [3]

Question 6: *(start a new sheet of paper)* **[12 marks]**

a) If α, β, γ are the roots of $x^3 - 2x^2 + 3x + 7 = 0$, find the values of

i) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma}$ [2]

ii) $\alpha^2 + \beta^2 + \gamma^2$ [2]

b) The elevation of a hill at a point P due east of it is 38° and at a point Q south of it the elevation is 23°. If the distance from P to Q is 420m, find the height of the hill. [4]

c)

i) Express $2cos(2x) + 2\sqrt{3}\,sin(2x)$ in the form $R\,cos(2x + \alpha)$, where $0 < \alpha < \frac{\pi}{2}$. [2]

ii) Hence or otherwise find all positive solutions of $2cos(2x) + 2\sqrt{3}\,sin(2x) = 0$. [2]

Question 7: *(start a new sheet of paper)* **[12 marks]**

a) A particles motion is defined by the equation $v^2 = 16 + 6x - x^2$, where x is its displacement from the origin in metres and v its velocity in ms^{-1}. Initially, the particle is 8 metres to the right of the origin.

i) Show that the particle is moving in Simple Harmonic Motion. [1]

ii) Find the centre, the period and the amplitude of the motion. [3]

iii) The displacement of the particle at any time t is given by the equation $x = a\sin(nt + \theta) + b$. Find the values of θ and b, given $0 \le \theta \le 2\pi$. [2]

b) Without using a calculator, show that $\tan^{-1}\left(\frac{2}{3}\right)+\tan^{-1}\left(\frac{1}{5}\right)=\frac{\pi}{4}$. [3]

c) The acceleration of a particle moving in a straight line is given by $\ddot{x}=e^{-2x}$. The particle is at rest at the origin.

i) Prove that $v^2=1-e^{-2x}$. [2]

ii) What is the maximum speed the particle can reach? [1]

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$