Fort Street 2008 3U Maths Trial

Question 1. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\lim_{x \to 0} \frac{\sin 2x}{3x}$ 1

(b) Let A be the point $(8,10)$ and B the point $(-2,4)$. 2

Find the coordinates of the point P which divides the interval AB externally in the ratio 2:5.

(c) Solve $\frac{1}{x+2} \leq 2$ 3

(d) The angle between the line $y=2x$ and the tangent to the curve $y = Ax^2 + Ax$ at $x=1$ is $\frac{\pi}{4}$ radians. Find the values of A. 3

(e) Use the substitution $u = 2x+1$ to evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} x\sqrt{2x+1}\, dx$. 3

Question 2. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Let $f(x) = 4\cos^{-1}\left(\frac{x}{2}\right)$.

(i) State the domain and range of the function $f(x)$. 2

(ii) Sketch the graph of $y = f(x)$, indicating clearly the coordinates of the endpoints of the graph. 1

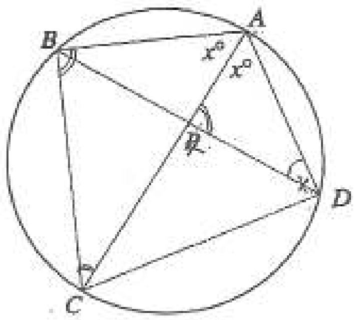
(iii) Find the equation of the tangent to the function $f(x)$ at $x=1$. 3

Leave your answer as exact values in gradient intercept form.

(b) Use the table of standard integrals to evaluate $\int_0^1 \frac{2}{\sqrt{x^2+1}}\, dx$ leaving your answer in exact form. 2

(c) A, B, C and D are points on the circumference of a circle.
AC and BD intersect at P.

$\angle BAC = \angle DAC = x°$.



(i) State why $\angle ACB = \angle ADB$. **1**

(ii) Prove that $\angle ABC = \angle APD$ **1**

(iii) Deduce that $\angle ADC = \angle CPD$. **2**

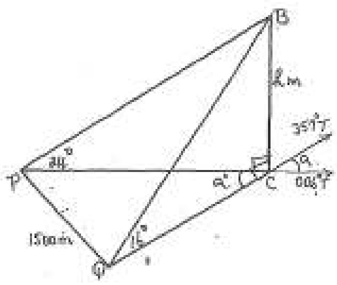
Question 3. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int_0^{\frac{\pi}{4}} 2\cos^2 x \; dx$ **2**

(b) Two observers P and Q are 1500 metres apart.

The bearing of a balloon B from observer P is 006° T while the angle of elevation from P is 24°.

The bearing of balloon B from observer Q is 357° T while the angle of elevation from Q is 16°.



(i) Show that if the height BC is h metres then **3**

$$h = \frac{1500}{\sqrt{\cot^2 24° + \cot^2 16° - 2\cot 24° \cot 16° \cos 9°}}$$

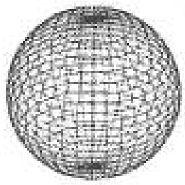
(ii) Hence find h to the nearest metre. **1**

(c) $P(x) = x^3 + 3x^2 + x - 5$.

(i) Show that $x - 1$ is a factor of $P(x)$ **1**

(ii) Hence factorise $P(x)$ **2**

(d) A spherical ball is expanding so that its volume is increasing at the constant rate of $10\,mm^3$ per second.



What is the rate of increase of the radius when the surface area is $400\,mm^2$? **3**

Question 4. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) An aluminium ingot is cooling in a foundry room with a constant temperature of $30^\circ C$.

At time t minutes its temperature T decreases according to the equation

$$\frac{dT}{dt} = -k(T-30) \text{ where k is a positive constant.}$$

The temperature of the aluminium ingot is initially $650^\circ C$ and it cools to $200^\circ C$ after 10 minutes.

(i) Verify that $T = 30 + Ae^{-kt}$ is a solution of this equation, where A is a constant. **1**

(ii) Find the values of A and k correct to two decimal places. **2**

(iii) Most foundry workers can comfortably pick up an ingot by hand when the temperature of the ingot falls to $60^\circ C$ or lower. After how many minutes will most foundry workers first be able to handle the ingot? **2**

Give your answer to the nearest minute.

(b) Use mathematical induction to show $5^n > 3^n + 2^n$ for all integers $n \geq 2$. **3**

(c) (i) Show that $\dfrac{\sin\theta}{1+\cos\theta} = \tan\dfrac{\theta}{2}$ **2**

(ii) Hence, find the general solutions to the equation $\dfrac{\sin\theta}{1+\cos\theta} = \sqrt{3}$ **2**

Question 5. (12 marks) Use a SEPARATE writing booklet. **Marks**

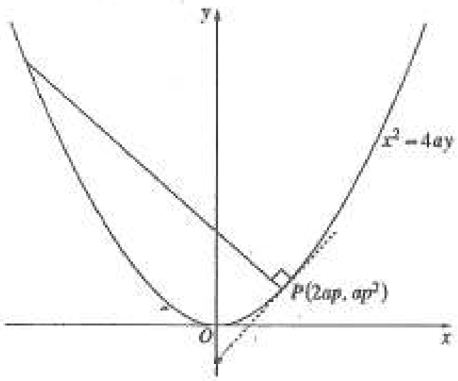
(a) The roots α, β and γ of the equation $x^3 - 2x^2 - 5x + 8 = 0$ are in geometric progression.

(i) Show that $\alpha\gamma = \beta^2$ **1**

(ii) Show that $\beta = 2$ **1**

(iii) Hence find the values of α and γ. **3**

(b)



The diagram shows the normal to the parabola $x^2 = 4ay$ at point $P(2ap, ap^2)$.

(i) Show that the equation of the normal to P is given by **2**

$$x + py = 2ap + ap^3$$

(ii) Find the coordinates of R where the normal at P intersects the y axis. **1**

(iii) Hence find the locus of the midpoint of PR. **2**

(c) If $y = \tan^{-1}(x^2)$, find $\frac{d^2y}{dx^2}$ **2**

Question 6. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The velocity v ms^{-1} of a particle moving in simple harmonic motion according to the equation

$$v^2 = -12 + 8x - x^2 \text{, where } x \text{ is in metres.}$$

(i) Between which two values is the particle oscillating? **1**

(ii) Find the centre of motion. **1**

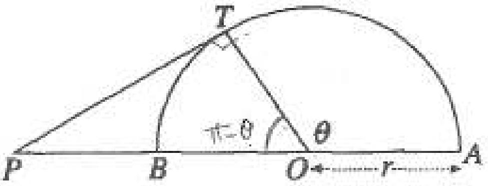
(iii) Find the maximum speed of the particle. **1**

(iv) Find the acceleration of the particle in terms of x. **1**

(v) Find the period of the motion. **1**

(vi) If initially $x = \sqrt{3}$, find a function for displacement in terms of time t. **2**

(b) The point T lies on the circumference of a semicircle, radius r and diameter AB, as shown. The point P lies on AB produced and PT is the tangent at T.



The arc AT subtends an angle of θ at the centre, O, and the area of triangle OPT is equal to that of the sector AOT.

(i) Find an expression for $\tan(\pi - \theta)$ **1**

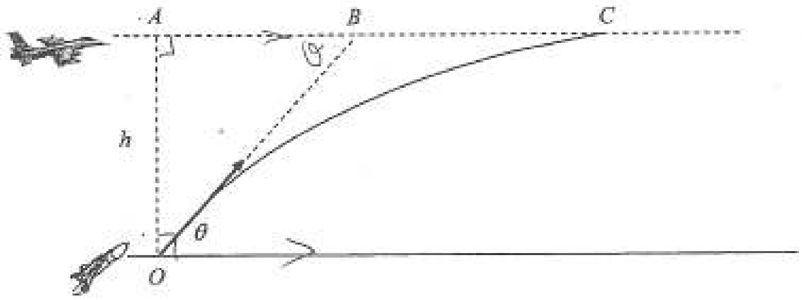
(ii) Hence, or otherwise, show that $\theta + \tan\theta = 0$ **2**

(iii) Taking $\theta = 2$ as an initial approximation, use Newton's method once to find a better approximation for a solution to $\theta + \tan\theta = 0$, correct to 3 significant figures. **2**

Question 7. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Show that the equation $a\sin x + b\cos x = \sqrt{3}$ has real roots if $a^2 + b^2 \geq 3$ **3**

(b) An aircraft is flying with constant velocity $U\ ms^{-1}$ at a constant height h metres above horizontal ground.



When the plane is at A it is directly over a anti-aircraft gun at O.
When the plane is at B a projectile is fired from the gun with velocity $V\ ms^{-1}$ at an angle of elevation θ along OB.
T seconds later the shell hits the aircraft at C. The acceleration due to gravity is $g\ ms^{-2}$.

(i) Assume that the equations of motion of the projectile are **2**

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -g$$

Show that relative to O the horizontal and vertical displacements of the projectile after time t seconds are given by the equations

$$x = Vt\cos\theta \quad \text{and} \quad y = Vt\sin\theta - \frac{1}{2}gt^2$$

(ii) Show that the projectiles path is given by the Cartesian equation **2**

$$y = x\tan\theta - \frac{g\sec^2\theta}{2V^2}x^2$$

(iii) Show that the time until the projectile hits the aircraft is given by **2**

$$T = \frac{h}{(V\cos\theta - U)\tan\theta}$$

(iv) Hence show that $gh = 2U(V\cos\theta - U)\tan^2\theta$ **3**