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Class: The cool one

FORT STREET HIGH SCHOOL

# 2009

**HIGHER SCHOOL CERTIFICATE COURSE**

## ASSESSMENT TASK 4: TRIAL HSC

# Mathematics Extension 1

TIME ALLOWED: 2 HOURS
(PLUS 5 MINUTES READING TIME)

| Outcomes Assessed | Questions | Marks |
|---|---|---|
| Chooses and applies appropriate mathematical techniques in order to solve problems effectively | 1, 2 | |
| Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry and polynomials | 3, 4, 5 | |
| Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay | 6 | |
| Synthesises mathematical solutions to harder problems and communicates them in appropriate form | 7 | |

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total | % |
|---|---|---|---|---|---|---|---|---|---|
| Marks | /12 9 | /12 11 | /12 5 | /12 9 | /12 9 | /12 8 | /12 4 | /84 55 | |

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started on a new page

**Question 1: (12 marks)** **START A NEW BOOKLET**

a) Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 5x}{\frac{x}{2}}$ **[2]**

b) Find the gradient of the tangent to the curve $y = \tan^{-1}(\sin x)$ at $x = 0$. **[2]**

c) Solve $\dfrac{x}{x^2-1} > 0$ for all real $x$. **[3]**

d) Given that $\log_8 2 = \log_x 5$, find $x$. **[2]**

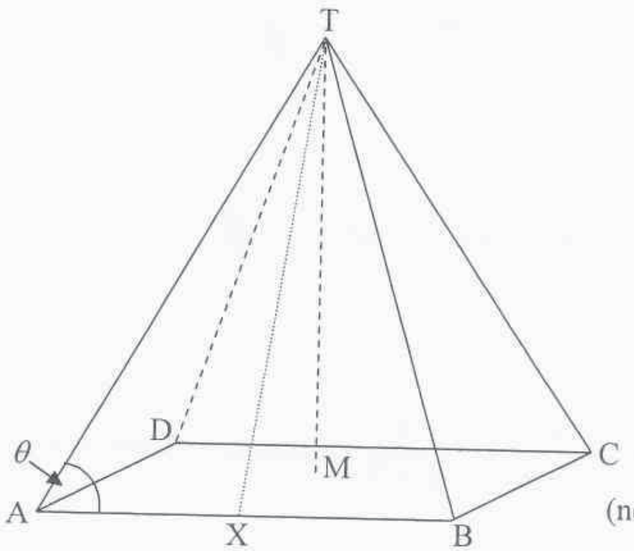
e) Show that $\dfrac{x^2+12}{x^2+9} = 1 + \dfrac{3}{x^2+9}$ and hence evaluate exactly $\displaystyle\int_{\sqrt{3}}^{3} \frac{x^2+12}{x^2+9}\,dx$. **[3]**

**Question 2: (12 marks)** **START A NEW BOOKLET**

a) Using the substitution $u = 2x+1$, evaluate exactly $\displaystyle\int_0^2 \frac{2x}{(2x+1)^2}\,dx$ **[4]**

b) Give the general solution of the equation $\cos(\theta + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$. **[3]**

c)




(not to scale)

The diagram shows a right square pyramid with base $ABCD$, vertex $T$ and height $TM$. It is given that $TM = AB = h$ units. $X$ is the midpoint of $AB$.

i) Show the length of $TX$ is $\frac{h}{2}\sqrt{5}$ **[1]**

ii) Hence show that if $\angle TAB = \theta$, then $\cos\theta = \frac{1}{\sqrt{6}}$ **[2]**

d) Prove that $\cot\frac{\theta}{2} - 2\cot\theta = \tan\frac{\theta}{2}$ **[2]**

**Question 3: (12 marks) START NEW BOOKLET**

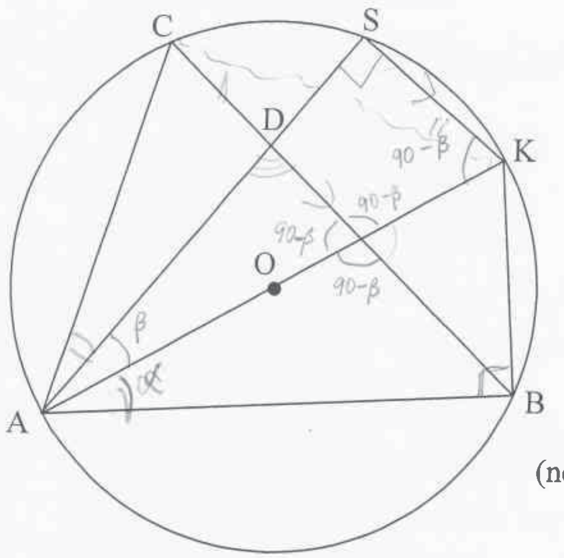
a) $P(6t, 3t^2)$ is a point on the parabola $x^2 = 12y$ with focus $S$. The point $Q$ divides the interval ~~SP~~ PS externally in the ratio 3:2.

i) Find the coordinates of $Q$. **[2]**

ii) Show that the locus of $Q$ is $24y = 216 - x^2$. **[2]**

iii) Hence find the coordinates of the focus and the equation of the directrix of $Q$. **[2]**

b)




(not to scale)

$360 - (90-\beta) - (90-\beta) - 90$

$= 90+\beta$

$90 - \beta + \gamma = \alpha+\beta$

In the diagram, $AK$ is a diameter of the circle centre $O$. $SK \parallel CB$.
Copy or trace the diagram into your answer booklet.
Prove that $\angle BAK = \angle SAC$. **[6]**

**Question 4: (12 marks)** **START A NEW BOOKLET**

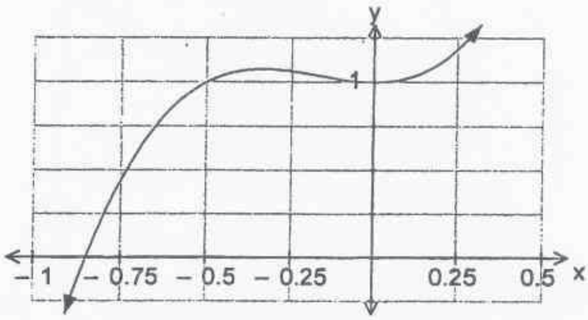
a) Consider the function $f(x) = \frac{1}{2}\cos^{-1}(\frac{2x}{3})$

i) State the domain and range of $y = f(x)$ [2]

ii) Draw the graph $y = f(x)$ [1]

iii) The area bounded by the axes and the curve $y = f(x)$ is rotated about the $y$ axis between $y = 0$ and $y = \frac{\pi}{4}$. Find the volume of the solid of revolution formed. Leave your answer as an exact value. [2]

b) The polynomial $P(x) = 4x^3 + 2x^2 + 1$ has one real root in the interval $-1 < x < 0$.




$y = 4x^3 + 2x^2 + 1$

i) Mr Squiggle used $x = -\frac{1}{4}$ as a first approximation to the root. He then applied Newton's method once to obtain another approximation of the root. What answer did he get? [2]

ii) Explain why Mr Squiggle's application of Newton's method in part (i) was NOT effective in improving the approximation to the root. [1]

c) Use Mathematical Induction to prove that $3^{2n+4} - 2^{2n}$ is divisible by 5 for all positive integral values of $n$. [4]

**Question 5: (12 marks) START A NEW BOOKLET**

a) $P(x) = x^3 + ax^2 + bx + c$ has real roots $\sqrt{k}, -\sqrt{k}, \gamma$.

i) Explain why $\gamma + a = 0$ [1]

ii) Show that $k\gamma = c$ [1]

iii) Show $ab = c$ [2]

b)

i) Show that $\sqrt{12}\sin x + 2\cos x = 4\cos\left(x - \frac{\pi}{3}\right)$. [1]

ii) Hence solve the equation $\sqrt{12}\sin x + 2\cos x = -3$ in the interval $0 \le x \le 2\pi$. Give all answers correct to two decimal places. [3]

c) A particle moves in a straight line so that its displacement, $s$ metres, at time $t$ seconds, is given by

$$s = \sqrt{12}\sin\frac{2t}{5} + 2\cos\frac{2t}{5}$$

Using part (b) or otherwise

i) Show that the motion of the particle is simple harmonic. [1]

ii) Find the smallest positive value of $t$ for which $s = -3$. [1]

iii) Find the distance travelled by the particle in going from its initial position to the position $s = -3$. Justify your answer. [2]

**Question 6: (12 marks) START A NEW BOOKLET**

a) Prove that $\tan^{-1}(2x)+\tan^{-1}\left(\frac{1}{2x}\right)=\frac{\pi}{2}$, for $x>0$. [3]

b) Sketch the curve $y=\frac{x^2+1}{x-1}$

On your graph include any asymptotes, intercepts and coordinates of any turning points. [5]
**Note: the curve does not have any points of inflexion.**

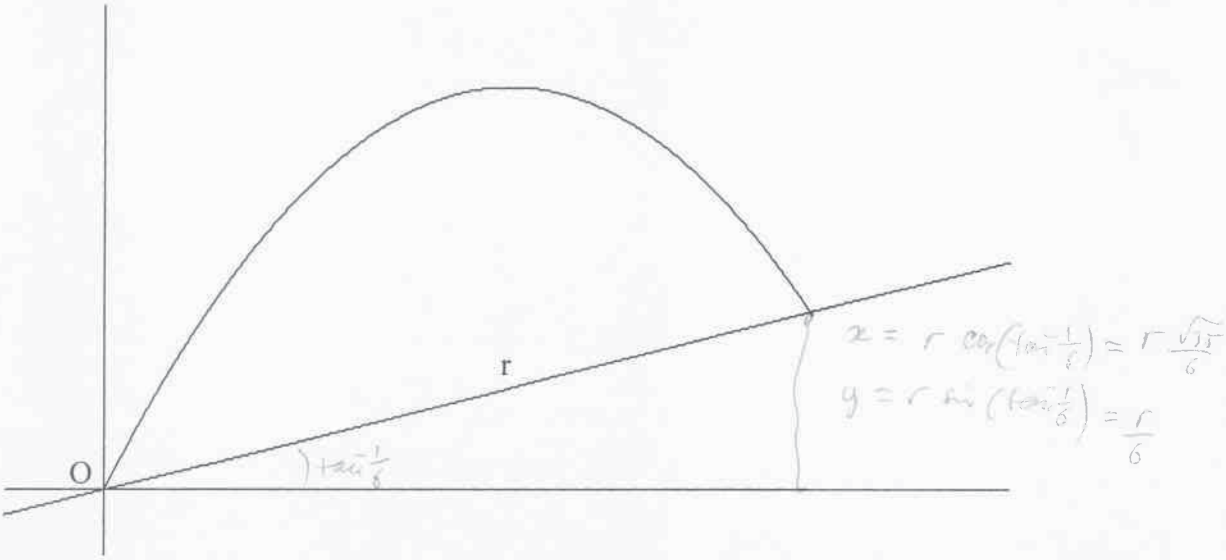
c) A particle with displacement $x$ metres moves in a straight line with velocity $v$ metres per second and acceleration $a$ metres per second per second. It is given that $a=e^{2x-1}$ and that at time $t=0$ seconds, $x=\frac{1}{2}$ and $v=-1$.

(i) Find $v$ as a function of $x$. [2]

(ii) Hence show that $x=\frac{1}{2}-\ln(t+1)$. [2]

## Question 7: (12 marks) START A NEW BOOKLET

a) A golf ball is lying at a point O on an inclined fairway as shown.




The golf ball is hit with an initial velocity of 30m/s at an angle of $\tan^{-1} x = \frac{4}{3}$.
(You may assume that the acceleration due to gravity is 10m/s).

The golf ball's trajectory at time $t$ seconds after being hit may be defined by the equations $x = 18t$ and $y = 24t - 5t^2$, where $x$ and $y$ are the horizontal and vertical displacements, in metres, of the ball from the origin 0 shown in the diagram.

(i) Find the horizontal range of the ball and its greatest height if it had been hit on the horizontal part of the course. **[3]**

(ii) If the fairway is as shown, inclined at an angle of $\tan^{-1}\frac{1}{6}$, show that the time of flight is 4.2 seconds and calculate the distance (r) the ball has been hit up the fairway (correct to 1 decimal place. **[3]**

b) Two straight roads intersect at an angle of $120^0$. A horse and cart starts from one intersection and travels along one road at 20 km/h. One hour later a cyclist starts from the intersection and travels along the other road at 50 km/h. At what rate is the distance between the horse and cart and the cyclist changing three hours after the cyclist starts? (Leave your answer to the nearest km/h.) **[6]**