Name: ____________________________

Teacher: ____________________________

Class: ____________________________

FORT STREET HIGH SCHOOL

2010

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

TIME ALLOWED: 2 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1, 2	
Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry and polynomials	3, 4, 5	
Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	6	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	7	

Question	1	2	3	4	5	6	7	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/84	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 (12 marks)

a) Differentiate $\cos^{-1}\left(\frac{3x}{2}\right)$ with respect to *x*. **2**

b) Find the acute angle between the lines $2x+y-3=0$ and $x=1$. **2**

Correct your answer to the nearest degree.

c) Find the coordinates of the point *P* which divides the interval *AB* with endpoints **3**

$A(2,3)$ and $B(7,-7)$ externally in the ratio 4:9.

d) Evaluate $\int_0^1 \frac{x+1}{x^2+1}\,dx$ **3**

e) Find all the real values of *a* for which ax^3-8x^2-9 is divisible by $(x-a)$ **2**

Question 2 **(12 marks)** *Start a separate booklet*

a) Solve the inequality $\dfrac{2x-3}{x+2} \geq 3$ **3**

b) i) Show that a solution for $3\sin x - x = 0$ lies between x = 2·2 and x = 2·4. **1**

ii) Taking x = 2·3 as an initial approximation for a solution to $3\sin x - x = 0$, **3**

apply Newton's method once to find a better approximation correct to three decimal places.

c) Find the values for a for which $f\left(x\right) = e^{-ax}\left(x-a\right)$ has a stationary point at $x = \dfrac{5}{2}$. **2**

d) Use the substitution $x = \log_e u$ to find $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx$ **3**

Question 3 **(12 marks)** *Start a separate booklet*

a) Let $f(x) = 2x - x^2$ for $x \leq 1$.

This function has an inverse $f^{-1}(x)$.

i) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. **2**

ii) Find an expression for $f^{-1}(x)$ **2**

iii) Evaluate $f^{-1}\left(\frac{3}{4}\right)$. **1**

b) i) Express $\sin x - \cos x$ in the form $A\sin(x - \alpha)$ where $0 < \alpha < \frac{\pi}{2}$ **1**

ii) Determine $\lim_{x \to \frac{\pi}{4}} \left[\frac{\sin x - \cos x}{x - \frac{\pi}{4}} \right]$ **2**

c) Use the method of mathematical induction to prove that if $y = xe^x$ then **4**

$$\frac{d^n y}{dx^n} = e^x(x + n)\text{, for all positive integers } n.$$

Question 4 **(12 marks)** *Start a separate booklet*

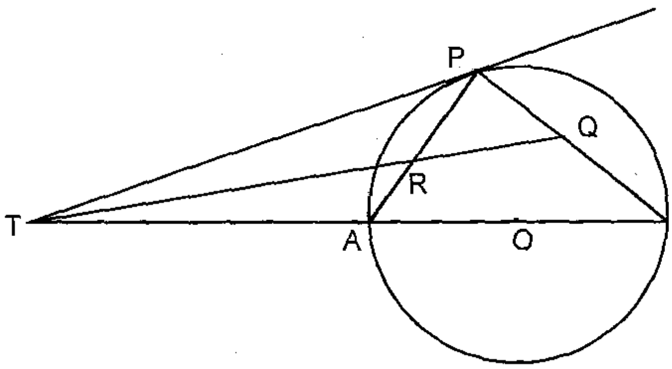
a) i) A particle moves from the origin with initial velocity $u \ \ ms^{-1}$ and experiences a retardation of magnitude $v+2v^2$, where v is the velocity of the particle at time t. **1**

Show that when it is at position x, $\dfrac{dv}{dx}=-(1+2v)$

ii) Find its distance from the origin when it comes to rest. **3**

b) O is the centre of the circle and TQ bisects $\angle OTP$. **4**

TB, AP and BP are straight lines and TP is a tangent to the circle at P.



Let $\angle PTQ=\alpha$ and $\angle TBQ=\beta$

Show $\angle TQP=45^\circ$

c) $P\left(2at,at^2\right)$ is a variable point on the parabola $x^2=4ay$ whose focus is S.

$Q(x,y)$ divides the interval from P to S in the ratio $t^2:1$.

i) Find the coordinates of Q in terms of a and t. **1**

ii) Verify that $\dfrac{y}{x}=t$ **1**

iii) Prove that as P moves on the parabola, Q moves on a circle and state its centre and radius. **2**

Question 5 **(12 marks)** *Start a separate booklet*

a) Solve the equation $x^3 - 3x + 2 = 0$, given it has a double root. **2**

b) i) Show that $\cos 3x = 4\cos^3 x - 3\cos x$ **2**

ii) Show that the solution of $\cos 3x - \sin 2x = 0, \;\; \mathit{for} \;\; 0 < x < \frac{\pi}{2}$ is given by **3**

$$\sin x = \frac{\sqrt{5}-1}{4}$$

iii) Verify that $x = \frac{\pi}{10}$ is a solution to $\cos 3x = \sin 2x$. **2**

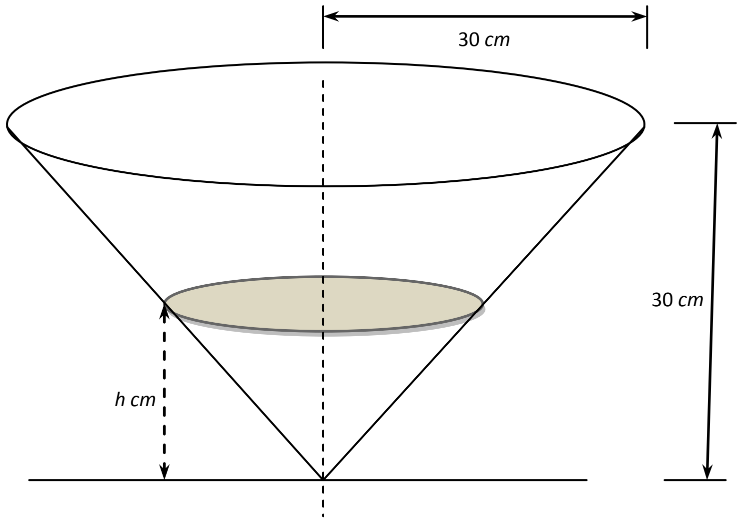
iv) Using the results obtained in parts (ii) and (iii) prove **3**

$$\sin\frac{\pi}{5}\cos\frac{\pi}{10} = \frac{\sqrt{5}}{4}$$

Question 6 **(12 marks)** *Start a separate booklet*

a) On a certain day the depth of water in a bay at high tide is 11 metres. At low tide, 6·25 hours later, the depth of the water is 7 metres. If high tide is due at 3:20 pm, what is the earliest time at which a ship needing a depth of 10 metres of water can enter the bay? (It may be assumed that the rise and fall of the water level is in simple harmonic motion). **4**

b) **4**



Water is poured into a conical vessel at a constant rate of $24\ cm^3$ *per second.*

The depth of the water is h cm at time t seconds.

What is the rate of increase of the area of the surface of the liquid when the depth is 16 *cm*?

c) A particle is projected in a straight line from an origin with velocity $2\ ms^{-1}$.
When x metres from the origin, its acceleration is $\left(2 - e^{-\frac{x}{2}}\right)\ ms^{-2}$.

i) Show that, when x metres from the origin, its velocity, $v\ ms^{-1}$, is given by **2**

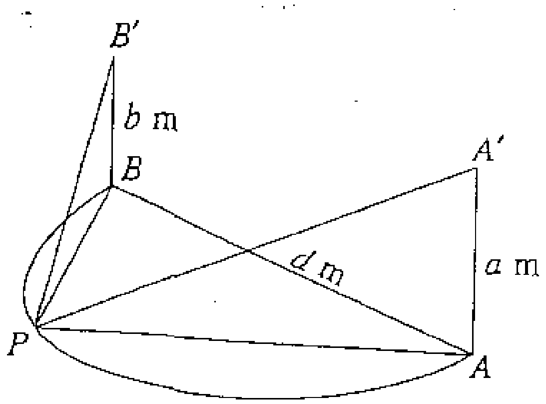
$$v^2 = 4x + 4e^{-\frac{x}{2}}$$

ii) Explain why, for large positive values of x, $v \approx 2\sqrt{x}$. **1**

iii) Prove that the particle will move from x = 100 to x = 121 in approximately 1 second. **1**

Question 7 **(12 marks)** *Start a separate booklet*

a) *APB* is a horizontal semicircle, diameter *d* metres. At A and B are vertical posts of height *a* m and *b* m. From *P*, the angle of elevation of the tops of both posts is θ



i) Prove that $d^2 = \dfrac{a^2}{\tan^2\theta} + \dfrac{b^2}{\tan^2\theta}$. **2**

ii) From *B*, the angle of elevation of A' is α and from *A*, the angle of elevation of B' is β. **2**

Prove that $\tan^2\alpha + \tan^2\beta = \tan^2\theta$.

b) Two particles are projected at different times from the same point with speed *V*. The angles of projection of the two particles are α° and $(90-\alpha)^\circ$ respectively. The greatest heights they reach above the horizontal plane through the point of projection are h_1 and h_2 respectively.

i) Show that for any angle α, $h_1 + h_2 = \dfrac{R}{2}$, where *R* is the maximum range. **4**

ii) If $\tan\alpha = \dfrac{3}{4}$ and $v = 196\ m/s$, what time must elapse between the instants of projection if the particles collide as they hit the horizontal plane? (Take $g = 9{\cdot}8\ ms^{-2}$). **4**

SOLUTIONS

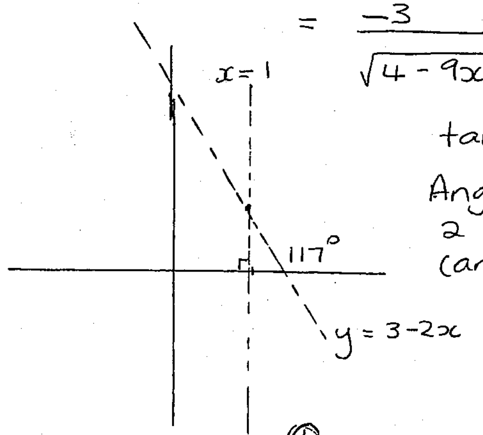
Question 1

COMMENTS

a) $\dfrac{d}{dx}\cos^{-1}\dfrac{3x}{2} = \dfrac{-1}{\sqrt{1-\frac{9x^2}{4}}} \times \dfrac{3}{2}$ (1)

$= \dfrac{-3}{\sqrt{4-9x^2}}$ $\dfrac{-1}{\sqrt{\frac{4}{9}-x^2}}$ (1)

b)



$\tan^{-1} -2 = 117°$ (1)

Angle between 2 lines is $27°$ (angle sum Δ) (1)

c) $P = \left(\dfrac{-9\times 2 + 4\times 7}{-5}, \dfrac{-9\times 3 + 4\times -7}{-5}\right)$ (1) (1)

$= (-2, 11)$ (1)

d) $\displaystyle\int_0^1 \frac{x+1}{x^2+1}\,dx = \int_0^1 \frac{x\,dx}{x^2+1} + \int_0^1 \frac{1\,dx}{x^2+1}$

$= \frac{1}{2}\ln(x^2+1)\Big]_0^1 + \tan^{-1}x\Big]_0^1$ (1) (1)

$= \frac{1}{2}\ln 2 + \frac{\pi}{4}$ (1)

e) $P(a) = a^4 - 8a^2 - 9$

$= (a^2-9)(a^2+1)$ (1)

If $(x-a)$ is a factor $P(a) = 0$

$\therefore\ a = \pm 3$ (1)

TRIAL HSC 2010 : EXTENSION 1 SOLUTIONS

Question 2.

COMMENTS

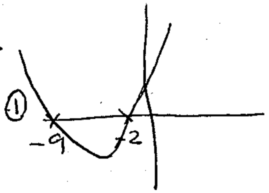
a) $\dfrac{(2x-3)}{(x+2)} \times (x+2)^2 \geq 3(x+2)^2$

$(2x-3)(x+2) \geq 3(x+2)^2$ (1)

$(x+2)\left[(2x-3) - 3(x+2)\right] \geq 0.$

$-(x+2)(x+9) \geq 0.$

$(x+2)(x+9) \leq 0$ (1)



$\therefore\ -9 \leq x < -2$ (1)

Mostly well done. Students should realise that $x \neq -2$

b) (i) When $x = 2.2$ $\quad 3\sin x - x = 0.2254\ldots$

$x = 2.4$ $\quad 3\sin x - x = -0.373$

Since the sign changes, a solution must lie between these two values. (1)

(ii) $x_2 = 2.3 - \dfrac{f(2.3)}{f'(2.3)}$ (1)

$= 2.3 - \dfrac{3\sin(2.3) - 2.3}{3\cos(2.3) - 1}$ (1)

$= 2.279$ (to 3 d. pl). (1)

* Everything (values + conclusion) had to be right for 1 mark.
Parts (i) & (ii) were poorly done because
1. Students did not know Newton's Formula (-2 marks)
2. Students do not seem to know how to work with radians ~ many calculater errors +
3. $\dfrac{d(\sin x)}{dx} = \cos x$ & not $-\cos x$

c) $f'(x) = e^{-ax} \times 1 + (x-a) \times -a e^{-ax}$

$= e^{-ax}\left(1 - ax + a^2\right)$

If stationary point exists at $\frac{5}{2}$ then

$f'\left(\frac{5}{2}\right) = 0$

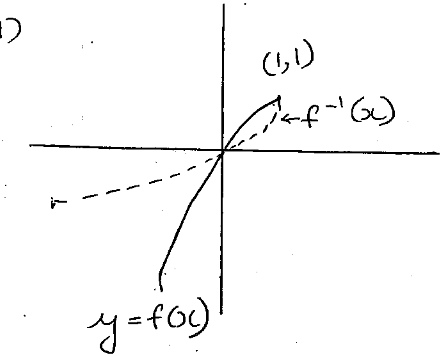
TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 2 (cont'd)

	COMMENTS
$f'\left(\frac{5}{2}\right) = e^{-\frac{5a}{2}}\left(1-\frac{5a}{2}+a^2\right)$ $= \frac{e^{-\frac{5a}{2}}}{2}\left(2a^2-5a+2\right)$ ① $=0$ if $= \frac{e^{-\frac{5a}{2}}}{2}(2a-1)(a-2)$ $=0$ if $a = \frac{1}{2}$ or 2. ①	* Should use product rule to differentiate. * realise that $e^{-\frac{5a}{2}} \neq 0$, $e^{-\frac{5}{2}a} > 0$ } marks were not deducted if this was not written
d) $x = \log_e u$ $\frac{dx}{du} = \frac{1}{u}$ $\int \frac{e^x\,dx}{\sqrt{1-e^{2x}}} = \int \frac{u \times \frac{du}{u}}{\sqrt{1-u^2}}$ ① $= \int \frac{du}{\sqrt{1-u^2}}$ ① $= \sin^{-1} u + c$. $= \sin^{-1}\left(e^x\right) + c$ ①	← Answer should be given in terms of 'x'. Lost 1 mark if integration constant was forgotten.
OR. $x = \ln u \rightarrow u = e^x \quad \therefore du = e^x dx$ $\therefore \int \frac{e^x\,dx}{\sqrt{1-e^{2x}}} = \int \frac{du}{\sqrt{1-u^2}}$ $= \sin^{-1} u + C$ $= \sin^{-1}(e^x) + c$	

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 3.

	COMMENTS
a) (i)  ②	
(ii) For $y = 2x - x^2$ inverse is $x = 2y - y^2$ $x+1 = -(y^2-2y+1)+1$ $x+1 = 1-(y-1)^2$. $(y-1)^2 = 1-x$. ① $y = 1 \pm \sqrt{1-x}$ From graph we see $y = 1 - \sqrt{1-x}$. ①	Try to make y subject ①
(iii) $f^{-1}\left(\frac{3}{4}\right) = 1 - \sqrt{1-3/4}$ ① $= \frac{1}{2}$.	
b) (i) $\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$ ① (ii) $\lim_{x \to \frac{\pi}{4}} \left(\frac{\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}\right) = \sqrt{2}$ ②	$A = \sqrt{1^2+1^2} = \sqrt{2}$ $\alpha = \tan^{-1}(-1)$ $\alpha = -\frac{\pi}{4}$

Question 3 (contd) SOLUTIONS	COMMENTS

c) $y = xe^x$

Test $n=1$

$$\frac{dy}{dx} = xe^x + e^x \quad ①$$
$$= e^x(x+1) \quad \therefore \text{ true for } n=1$$

Assume true for $n=k$

$$\frac{d^k y}{dx^k} = e^x(x+k) \quad ①$$

Consider $n=k+1$

$$\frac{d^{k+1}y}{dx^{k+1}} = e^x + (x+k)e^x$$
$$= e^x(1+x+k)$$
$$= e^x(x+(k+1)) \quad ①$$

This is of the same form as for $n=k$, therefore if true for $n=k$ it is also true for $n=k+1$. Since it is true for $n=1$, it is true for $n=2$ and hence all following positive integers. ①

Question 4 SOLUTIONS	COMMENTS

a) (i) $a = -(v+2v^2)$

$$v\frac{dv}{dx} = -(v+2v^2)$$
$$\therefore \frac{dv}{dx} = -(1+2v) \quad ①$$

This needed to be demonstrated not just written down.

(ii)
$$\frac{dx}{dv} = \frac{-1}{1+2v}$$
$$x = -\int \frac{1}{1+2v}\,dv$$
$$x = -\frac{1}{2}\ln(1+2v) + c.$$

- many students didn't '+c' & did not use conditions to evaluate 'c'

when $x=0$, $v=u$

$$0 = -\frac{1}{2}\ln(1+2u) + c.$$
$$\therefore c = \frac{1}{2}\ln(1+2u)$$
$$x = \frac{1}{2}\ln(1+2u) - \frac{1}{2}\ln(1+2v)$$
$$= \frac{1}{2}\ln\left(\frac{1+2u}{1+2v}\right)$$

* some students evaluated a definite integral which was fine

When comes to rest $v=0$

→ some students ignored this part

$$\therefore x = \frac{1}{2}\ln(1+2u)$$

b) $\angle PTQ = \angle QTB = \alpha$ (given)

$\angle APB = 90^\circ$ (angle in a semi-circle) ①

$\angle PBT = \beta$ given.

$\angle TPA = \beta$ (angle between chord and tangent equals angle in alternate segment). ①

O'all poorly done. Many made proof unnecessarily lengthy & convoluted

$\therefore \angle PRQ = \alpha+\beta$ (exterior angle to ΔTPR) ①

$\angle PQR = \alpha+\beta$ (" " " ΔTBQ)

In ΔPRQ both base angles are $(\alpha+\beta)$

$\therefore \Delta PRQ$ is isosceles so each must be 45°. ①

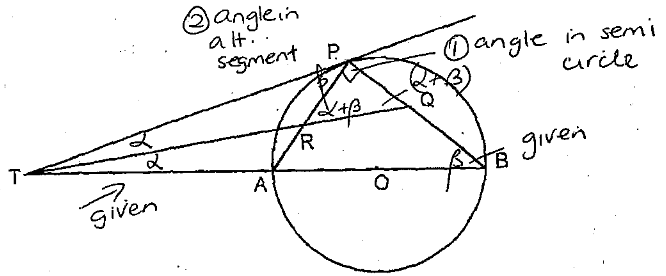
$\therefore \angle RQP = 45^\circ$ as req'd.

Note students MUST LEARN proper geometric reasoning & state them succinctly.

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 4 (contd)

COMMENTS



I was extremely liberal, but you will not be so lucky in the HSC

c) (i) $P = (2at, at^2)$ $\quad S = (0, a)$ $\quad k : l$, $t^2 : 1$

$x = \dfrac{kx_2 + lx_1}{k+l}$ $\quad y = \dfrac{ky_2 + ly_1}{k+l}$

$\therefore Q = \left(\dfrac{2at}{t^2+1}, \dfrac{2at^2}{t^2+1}\right)$ ①

← learn formula

(ii) $y = \dfrac{2at^2}{t^2+1}$

$= \left(\dfrac{2at}{t^2+1}\right) t$

$= xt.$

$\therefore \dfrac{y}{x} = t$ ①

← many used x and y showed it was t. (must use L.H.S. / R.H.S setting out)

(iii) At Q

$x = \dfrac{2at}{t^2+1}$

$= \dfrac{2a\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)^2 + 1}$ ①

$= \dfrac{\frac{2ay}{x}}{\frac{y^2+x^2}{x^2}}$

$= \dfrac{2ay}{x} \cdot \dfrac{x^2}{y^2+x^2}$

$= \dfrac{2ayx}{y^2+x^2}$

poorly done.

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 4 (contd)

COMMENTS

$1 = \dfrac{2ay}{y^2+x^2}.$

$x^2 + y^2 - 2ay = 0.$

$x^2 + (y^2 - 2ay + a^2) = a^2$

$x^2 + (y-a)^2 = a^2.$

$\therefore$ Q lies on circle whose centre is $(0, a)$ and radius is a units. ①

← some students got to here but did not complete the square to find the true centre + radius

SOLUTIONS

Question 5.

SOLUTIONS	COMMENTS
a) Roots of $x^3 - 3x + 2 = 0$ are α, α and β.	
$\therefore 2\alpha + \beta = 0. \Rightarrow \beta = -2\alpha$	Many Ext 1 students used factor theorem & polynomial div.
$\alpha^2 + 2\alpha\beta = -3$ (1)	Many Ext 1 students used $f(x)=0$ $f'(x)=0$ for double root.
$\alpha^2\beta = -2$	
$\alpha^2 \times -2\alpha = -2$	
$\alpha^3 = 1$	
$\therefore \alpha = 1$ and $\beta = -2$. (1)	
$\therefore$ Roots are 1, 1 and -2.	
b) (i) $\cos 3x = \cos(2x + x)$	well done
$= \cos 2x \cos x - \sin 2x \sin x$	
$= (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x$ (1)	
$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$	
$= 4\cos^3 x - 3\cos x$. (1)	
(ii) $\cos 3x - \sin 2x = 0$ $\quad 0 < x < \frac{\pi}{2}$	A good number of students ignored the soln to $\cos x = 0$ and lost a mark.
$4\cos^3 x - 3\cos x - 2\sin x \cos x = 0$.	
$\cos x(4\cos^2 x - 3 - 2\sin x) = 0$.	
$\cos x[4(1 - \sin^2 x) - 3 - 2\sin x] = 0$	
$\cos x(4\sin^2 x + 2\sin x - 1) = 0$. (1)	
$\cos x = 0$ when $x = \frac{\pi}{2}$ which is not in domain	
$\therefore 4\sin^2 x + 2\sin x - 1 = 0$ ie	
$\sin x = \frac{-2 \pm \sqrt{4+16}}{8}$ (1)	
$= \frac{\pm\sqrt{5} - 1}{4}$	

SOLUTIONS

Question 5 (contd)

SOLUTIONS	COMMENTS
$\sin x = \frac{-\sqrt{5} - 1}{4}$ is also outside required range	
$\therefore \sin x = \frac{\sqrt{5} - 1}{4}$. (1).	
(iii) If $x = \frac{\pi}{10}$, $\cos 3x = \cos \frac{3\pi}{10}$	Many students used calculator approximations rather than properly showing this simple trig result, and lost a mark
$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$ (1)	
$= \sin \frac{2\pi}{10}$. (1)	
$\therefore x = \frac{\pi}{10}$ is a solution.	
(iv) $\sin \frac{\pi}{5} \cos \frac{\pi}{10} = \sin \frac{2\pi}{10} \cdot \cos \frac{\pi}{10}$.	Part (iv) not attempted by many students, only completed by most able. (1)
$= 2\sin \frac{\pi}{10} \cos \frac{\pi}{10} \cos \frac{\pi}{10}$	
$= 2\sin \frac{\pi}{10} \cos^2 \frac{\pi}{10}$. (1)	
$= 2\sin \frac{\pi}{10}\left(1 - \sin^2 \frac{\pi}{10}\right)$	
$= 2 \times \left(\frac{\sqrt{5}-1}{4}\right)\left(1 - \left(\frac{\sqrt{5}-1}{4}\right)^2\right)$	
$= \frac{\sqrt{5}-1}{2}\left(\frac{16 - (5 + 1 - 2\sqrt{5})}{16}\right)$	
$= \frac{\sqrt{5}-1}{2} \times \frac{10 + 2\sqrt{5}}{16}$	
$= \frac{\sqrt{5}-1}{2} \times \frac{5 + \sqrt{5}}{8}$	
$= \frac{5\sqrt{5} + 5 - 5 - \sqrt{5}}{16}$	
$= \frac{4\sqrt{5}}{16}$ (1).	
$= \frac{\sqrt{5}}{4}$.	

Question 6. SOLUTIONS | COMMENTS.

b) Using similar triangles radius of water surface $= h$.

$$\frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt} \qquad \left[\begin{array}{l}\text{vol of cone} \\ = \frac{1}{3}\pi r^2 h \\ = \frac{1}{3}\pi h^3\end{array}\right]$$

$$\boxed{24 = \pi h^2 \frac{dh}{dt}} \quad (1)$$

$$\therefore \frac{dh}{dt} = \frac{24}{\pi h^2}$$

At $h = 16$ $\quad \frac{dh}{dt} = \frac{24}{\pi \times 16^2}$

$$\boxed{\frac{dh}{dt} = \frac{3}{32\pi}\ \text{cm/s.}} \quad (1)$$

$$S = \pi h^2$$

$$\frac{dS}{dh} = 2\pi h.$$

$$\frac{dS}{dt} = \frac{dS}{dh}\cdot\frac{dh}{dt}$$

$$\boxed{\frac{dS}{dt} = 2\pi h \times \frac{dh}{dt}} \quad (1)$$

At $h = 16$

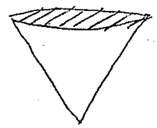
$$\frac{dS}{dt} = 2\pi \times 16 \times \frac{3}{32\pi}$$

$$\boxed{\frac{dS}{dt} = 3\ \text{cm}^2/\text{s}.} \quad (1)$$

Comments:

(1) students did not change $\frac{1}{3}\pi r^2 h$ to $\frac{1}{3}\pi h^3$ instead they used $\frac{dV}{dr} = \frac{d(\frac{1}{3}\pi r^2 h)}{dr}$ eventually this gave an answer $\frac{dS}{dt} = 4.5$.

(2) Students found the surface area of a cone. They should have concentrated on the surface of the area



Question 6 SOLUTIONS | COMMENTS.

a) If high tide is at 3.20pm. Low tide would occur at 9.05 am

($6\frac{1}{4}$ hours earlier)

$$\frac{2\pi}{n} = \frac{25}{2}.$$

$$25n = 4\pi$$

$$n = \frac{4\pi}{25} \quad (1)$$

$$a = 2 \quad (1)$$

$$\therefore x = -2\cos\frac{4\pi}{25}t.$$

When $x = 1$ $\quad \boxed{10 = 9 - 2\cos\frac{4\pi}{25}t}$

$$1 = -2\cos\frac{4\pi}{25}t$$

$$\cos\frac{4\pi t}{25} = -\frac{1}{2}$$

$$\frac{4\pi t}{25} = \frac{2\pi}{3} \quad (1)$$

$$t = \frac{25}{6}$$

$= 4$ hours 10 mins

$\therefore$ Ship can safely enter at

1:15 pm. (1)

Comments:

(1) $T = 13$ hrs

$6.25 + 6.25 = 12.5$

(2) students did not calculate the first time that $x=10$ but the second time

(2) students made $T = 6.25$

Question 6 (contd) — SOLUTIONS — COMMENTS

c) (i) $\frac{d}{dx}\frac{1}{2}v^2 = 2 - e^{-x/2}$

$$\boxed{\frac{1}{2}v^2 = 2x + 2e^{-x/2} + C} \quad (1)$$

$x = 0 \quad v = 2$

$2 = 2 + C \quad \therefore \quad C = 0$

$$\boxed{v^2 = 4x + 4e^{-x/2}} \quad (1)$$

Comment: (1) students forgot to write the constant and lost 1 mark.

(ii) As $x \to \infty \quad e^{-x/2} \to 0$

$\therefore \; v^2 \to 4x$

$$\boxed{v \doteq 2\sqrt{x}} \quad (1)$$

Comment: Well done

(iii) when $x = 100 \quad v = 20$

$x = 121 \quad v = 22$

$\therefore \; t = \frac{d}{s}$

$t = \frac{21}{21.5}$

$$\boxed{t \doteq 1}$$

Comment: Some students seemed completely stumped and did not even attempt this question.

(1) For the ones that did, it was mostly well done.

Question 7. — SOLUTIONS — COMMENTS

a) (i) Since $\angle BPA = 90°$ (angle in a semi-circle)

$d^2 = BP^2 + AP^2 \quad (1)$

$\tan\theta = \frac{b}{PB} \qquad \tan\theta = \frac{a}{PA}$

$\therefore \; d^2 = \frac{b^2}{\tan^2\theta} + \frac{a^2}{\tan^2\theta}. \quad (1)$

Comment: Need to state this to show why you can use Pythagoras' theorem.

(ii) $\tan\alpha = \frac{a}{d}, \quad \tan\beta = \frac{b}{d}. \quad (1)$

$a^2 = d^2\tan^2\alpha, \quad b^2 = d^2\tan^2\beta.$

$\therefore \; d^2 = \frac{d^2\tan^2\beta}{\tan^2\theta} + \frac{d^2\tan^2\alpha}{\tan^2\theta}.$

$\tan^2\theta = \tan^2\beta + \tan^2\alpha. \quad (1)$

b) (i) For the first particle

$\ddot{x} = 0 \qquad \ddot{y} = -g$

$\dot{x} = V\cos\theta \qquad \dot{y} = -gt + V\sin\alpha$

$x = Vt\cos\theta \qquad y = Vt\sin\alpha - \frac{gt^2}{2}.$

Max. height occurs when $\dot{y} = 0.$

$V\sin\alpha = gt$

$t = \frac{V\sin\alpha}{g}. \quad (1)$

Question 7 (cont'd)

COMMENTS:

$$\therefore y = V.\frac{V\sin\alpha}{g} - \frac{1}{2}g.\frac{V^2\sin^2\alpha}{g^2}$$

$$= \frac{V^2\sin^2\alpha}{2g}.$$

$$\therefore h_1 = \frac{V^2\sin^2\alpha}{2g}. \quad (1)$$

Similarly

$$h_2 = \frac{V^2\sin^2(90-\alpha)}{2g}.$$

Many failing to recognise $\sin(90-\alpha) = \cos\alpha$.

$$h_1 + h_2 = \frac{V^2\sin^2\alpha}{2g} + \frac{V^2\cos^2\alpha}{2g} \quad (1)$$

$$= \frac{V^2}{2g}.$$

Maximum range is when $y=0$.

$$\text{ie } 0 = Vt\sin\alpha - \frac{gt^2}{2}$$

$$= t\left(V\sin\alpha - \frac{gt}{2}\right)$$

Need to derive these outcomes

$$\therefore t = \frac{2V\sin\alpha}{g}$$

$$x = V.\frac{2V\sin\alpha}{g}.\cos\theta$$

$$= \frac{V^2\sin 2\alpha}{g} \quad (1)$$

Question 7 (cont'd)

COMMENTS.

Max value of $\sin 2\alpha = 1$.

$$\therefore \max R = \frac{V^2}{g}$$

$$\frac{R}{2} = \frac{V^2}{2g} \quad (1)$$

$$= h_1 + h_2$$

(ii) Particle 1 - hits horizontal plane when $y=0$. Since $\tan\alpha = 3/4$, $\sin\alpha = 3/5$, $\cos\alpha = \frac{4}{5}$.

Many making this more complicated than really is

$$\text{ie } 0 = 196t \times \frac{3}{5} - 9.8 \times \frac{t^2}{2} \quad (1)$$

$$= t\left(196 \times \frac{3}{5} - 4.9t\right)$$

$$\therefore t = 0 \text{ or } 24 \text{ seconds}. \quad (1)$$

Particle 2 - hits horizontal plane when $y=0$

$$\text{ie } y = 196t \times \frac{4}{5} - 9.8\frac{t^2}{2} \quad (1)$$

$$= t\left(196 \times \frac{4}{5} - 4.9t\right)$$

$$\therefore t = 0 \text{ or } 32 \text{ s}.$$

$$\text{Time lapse} = 32 - 24 \quad (1)$$

$$= 8 \text{ s}.$$