FORT STREET HIGH SCHOOL

Name: ______________________

Teacher: ______________________

Class: ______________________

2012

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

Time allowed: 2 hours
(plus 5 minutes reading time)

Outcomes Assessed	Questions
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry and polynomials	11,12
Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	14
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	13

Total Marks 70
Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section
Section II 60 Marks
Attempt Questions 11-14,
Allow about 1 hour 45 minutes for this section

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 60	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
	Percent	

General Instructions:

- Questions 11-14 are to be started in a new booklet
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

SECTION I MULTIPLE CHOICE (10 MARKS)

CIRCLE CORRECT ANSWER

Question 1

Find $\int \frac{dx}{\sqrt{16-9x^2}}$

a) $sin^{-1}(\frac{3x}{4})$

b) $cos^{-1}(\frac{3x}{4})$

c) $\frac{1}{3}sin^{-1}(\frac{3x}{4})$

d) $\frac{1}{4}sin^{-1}(\frac{3x}{4})$

Question 2

Find $\int \frac{dx}{25+16x^2}$

a) $\frac{4}{5}tan^{-1}\left(\frac{5x}{4}\right)$

b) $\frac{1}{40}tan^{-1}\left(\frac{4x}{5}\right)$

c) $\frac{1}{5}tan^{-1}(\frac{5x}{4})$

d) $\frac{1}{20}tan^{-1}(\frac{4x}{5})$

Question 3

Find the domain of $y = cos^{-1}(\frac{2x}{3})$

a) $-3 \leq x \leq 3$

b) $-\frac{2}{3} \leq x \leq \frac{2}{3}$

c) $-2 \leq x \leq 2$

d) $-\frac{3}{2} \leq x \leq \frac{3}{2}$

Question 4

Find $\lim_{x\to 0}(\frac{sin3x}{2x})$

a) 1

b) $\frac{3}{2}$

c) $\frac{2}{3}$

d) $\frac{1}{2}$

Question 5

In how many ways can a team of 8, 4 men and 4 women be formed from a group of 6 men and 8 women?

a) 604800
b) 85
c) 1050
d) 3003

Question 6

If $t = tan\frac{\theta}{2}$, $then\ tan\theta =$

a) $\frac{2t}{1-t^2}$

b) $\frac{1+t^2}{1-t^2}$

c) $\frac{1-t^2}{1+t^2}$

d) $\frac{2t+1}{1+t^2}$

Question 7

Given $P(x) = 3x^3 - 2x^2 - 4x$

What is the remainder when P(x)

Is divided by (x + 3).

a) 78

b) -87

c) -78

d) 87

Question 8

Evaluate $\int_1^3 \frac{dx}{2x+1}$ to 3 decimal places.

a) 0.424

b) 4.236

c) 0.242

d) 0.538

Question 9

In how many ways can 6 people

be arranged in a circle?

a) 720

b) 120

c) 6

d) 500

Question 10

The solution to $(2x-5)(x+3)(6-x) \leq 0$ is

a) $-2.5 \leq x \leq 3\,, x \geq 6$

b) $3 \leq x \leq 6\,, x \leq -2.5$

c) $-3 \leq x \leq 2.5\,, x \geq 6$

d) $2.5 \leq x \leq 3\,, x \geq -6$

Spare Working Area

SECTION II (60 Marks)

Question 11 (15 Marks) Use a SEPARATE writing booklet **Marks**

a) Solve the inequality $\frac{3x+4}{x-5} \geq 2$ 3

b) Evaluate $\int_1^5 \frac{xdx}{\sqrt{4x+5}}$ using $u = \sqrt{4x+5}$ 3

c) Captain Barbossa is walking along a straight pier and observes a mast Bearing 040°T with an angle of elevation of 15°, after walking 100 metres along the pier the same mast is on a bearing of 300°T and an angle of elevation of 18°. Find the height of the mast to the nearest metre. 4

d) Draw a neat half page sketch of the $y = 3\sin^{-1}\left(\frac{x}{2} - 1\right)$ 3
Stating the domain and range.

e) The curves $y = x^2$ *and* $y = x^3$ intersect at A(0,0) and B(1,1). 2
Find the size of the acute angle between these curves at the Point B, to the nearest minute.

Question 12 (15 Marks) Use a SEPARATE writing booklet **Marks**

a) Express $\sqrt{3}\, sinx + cosx$ in the form $Asin(x{+}\propto)$ 2
Where $\propto$ is in radians and $A > 0$.
Hence or otherwise,

(i) Sketch the graph of $y = \sqrt{3}\, sinx + cosx$ for $0 \leq x \leq 2\pi$. 2

(ii) Solve the equation $\sqrt{3}\, sinx + cosx = \frac{\pi}{3}$ 2
Correct to 4 decimal places.

b) At the Cafe at the end of the pier Captain Barbossa ordered a Cup of coffee, the cooling of which follows the following Differential equation

$$\frac{dT}{dt} = -k(T - E)$$

Where T is temperature of the coffee and E is the temperature of the environment . Temperature is measured in degrees Celsius and the time t is in minutes. The environment temperature is a cool 16°C

(i) The coffee at 92°C was left to stand on the table for 3 minutes after which it had cooled to 72°C, Derive a solution to this differential equation and calculate the value of k Correct to 4 decimal places. 4

(ii) If Captain Barbossa can drink the coffee at 55°C, how Long does he have to wait? (Correct to the nearest minute) 2

c) Consider the equation 3

$$x^3 + 6x^2 - x - 30 = 0$$

One of the roots of this equation is equal to the sum of the other two roots. Find the values of the three roots.

Question 13 (15 Marks) Use a SEPARATE writing booklet **Marks**

a) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Derive the the equation of the tangent to the parabola at P. 1

(ii) The tangents at P and Q intersect at 60° , 1
Show $p - q = \sqrt{3}\,(1 + pq)$

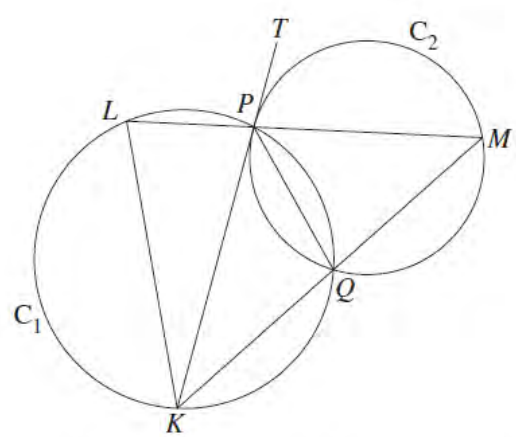
(iii) Given the tangents intersect at $T(\ a(p+q),\ apq\ )$
Find the locus of T as P moves on the parabola 2
$x^2 = 4ay$

b) At the Aquarium in the middle of the pier there is a tank of 2
8 Clownfish, and another tank of 7 Blue tang.
Captain Jack Sparrow wants his fish tank to contain 6 fish.
Fish are selected at random from both tanks.
What is the probability Jack's tank will contain at least 4 clownfish?

c) Prove by mathematical induction 3
$47^n + 53 \times 147^{n-1}$ is divisible by 100 for all integers $n \geq 1$.

d) One solution of the equation $2cos2x = x + 1$ is close to $x = 0.4$ 3
Use one application of Newton's Method to find another approximation to this solution. Give your answer correct to 4 decimal places.

e) 3



Two circles C_1 and C_2 intersect at P and Q as shown in the diagram. The tangent TP to C_2 at P meets C_1 at K. The line KQ meets C_2 at M. The line MP meets C_1 at L.

Copy or trace the diagram into your writing booklet.

Prove that ΔPKL is isosceles.

Question 14 (15 Marks) Use a SEPARATE writing booklet **Mark**

a) The temperature in the captain's cabin obeys the laws of simple harmonic motion. The door can only be opened when the temperature reaches $22^{o}C$ in the cabin, below this temperature the door expands and cannot be opened.
The temperature in the cabin was at a minimum of $10^{o}C$ at 6am by noon it was at a maximum of $30^{o}C$.

(i) Find an expression for the rise and fall of the temperature in the cabin. 1

(ii) At what time, to the nearest minute, can Jack Sparrow first enter the captain's cabin (after 6am)? 2

(iii) How long can Jack remain in the cabin, to the nearest minute, before the door jams shut? 2

b) In the captain's cabin is a sea water spa, containing 800 litres, it was leaking 10 litres per minute. To keep the water in the spa level, Jack decides to pump in sea water at 10 litres per minute; the local sea water has a salinity of 950grams/litre.
The spa initially contained 750kgs of salt.

(i) Is the amount of salt in the spa increasing or decreasing over time? 1

(ii) Set up a differential equation and derive a solution to the amount of salt in the spa at any time, 2

(iii) How much salt is in the spa after 5 hours? Answer to the nearest gram. 2

c) Where is captain Barbossa? Jack wandered the deck of the Black Pearl; There's Barbossa 30 metres up the perpendicular mast, in the crow's nest. Barbossa throws a gold coin upwards at 20m/sec at 45^{o} to the horizontal, from where he is in the crow's nest 30m above the ship's deck. Simultaneously a parrot was released from the base of the mast. The parrot flew in a straight line at an angle of 30^{o} to the horizontal and caught the coin.

(i) Derive the projectile motion equations using $g = 10ms^{-2}$ for the x and y coordinates of the coin. 2

(ii) What was the speed the parrot needed to fly, to the nearest km/hr, to catch that gold coin? 3

SECTION I | Solutions Trial 2012 Ext 1 | FSHS

M/C Q1–10 1 each

Q1 Let $u = 3x$ $du = 3dx$

$= \frac{1}{3}\int \frac{du}{\sqrt{4^2 - u^2}} = \frac{1}{3}\sin^{-1}\frac{u}{4}$ $\boxed{C}$

Q2 Let $u = 4x$ $\frac{1}{4}du = dx$

$= \frac{1}{4}\int \frac{du}{5^2 + u^2} = \frac{1}{4}\cdot\frac{1}{5}\tan^{-1}\frac{u}{5}$

$= \frac{1}{20}\tan^{-1}\frac{4x}{5}$ $\boxed{D}$

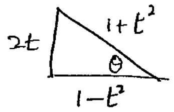
Q3 $-1 \le \frac{2x}{3} \le 1$ $\therefore$ $-\frac{3}{2} \le x \le \frac{3}{2}$ $\boxed{D}$

$-3 \le 2x \le 3$

Q4 $\lim_{x \to 0} \frac{\sin 3x}{3x}\cdot\frac{3}{2} = \frac{3}{2}$ $\boxed{B}$

Q5 ${}^6C_4 \times {}^8C_4 = 1050$ $\boxed{C}$

$\boxed{\frac{\ }{10}}$

Q6 

$\tan\theta = \frac{2t}{1-t^2}$ $\boxed{A}$

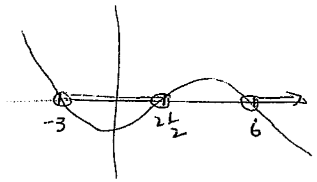
Q7 $P(-3) = 3(-3)^3 - 2(-3)^2 - 4\times -3$

$= -81 - 18 + 12 = -87$ $\boxed{B}$

Q8 $\frac{1}{2}\ln(2x+1)\Big]_1^3 = \frac{1}{2}\ln\frac{7}{3}$

$= 0.424$ $\boxed{A}$

Q9 $5! = 120$ $\boxed{B}$

Q10 

$\boxed{C}$

SECTION II $\frac{\ }{60}$

Question 11 15 Marks

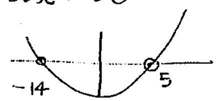
a) Solve $\frac{3x+4}{x-5} \ge 2$ $\times$ bs by $(x-5)^2 \ge 0$

✓ $\therefore (x-5)(3x+4) \ge 2(x-5)^2$

$3x^2 + 4x - 15x - 70 \ge 2x^2 - 20x + 50$

✓ $\therefore x^2 + 9x - 70 \ge 0$

$(x-5)(x+14) \ge 0$



$x \le -14$

$x \ge 5$ but $x \ne 5$

ie $x > 5$

✓ Soln $x > 5,\ x \le -14$

b) $\int_1^5 \frac{x\,dx}{\sqrt{4x+5}}$ $u = \sqrt{4x+5}$ $\therefore u^2 = 4x+5 \to x = \frac{u^2-5}{4}$

$\therefore 2u\,du = 4dx$

Bounds $x = 5$ $u = 5$

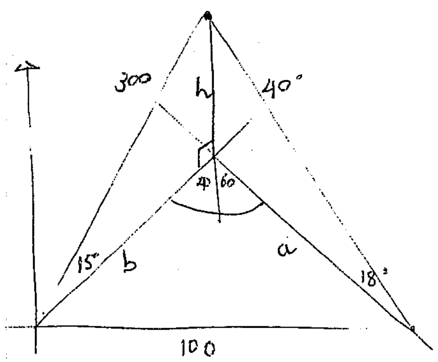
$x = 1$ $u = 3$

Integral transform

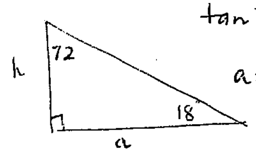
✓ relation between du + dx $\int_3^5 \frac{\frac{u^2-5}{4}\cdot\frac{u}{2}}{u}\,du = \int_3^5 \frac{u^2-5}{8}\,du$

✓ correct integral transform $= \frac{1}{8}\left[\frac{u^3}{3} - 5u\right] = \frac{1}{8}\left[41\tfrac{2}{3} - 25 - [9 - 15]\right]$

✓ answer $= \frac{1}{8}\left[16\tfrac{2}{3} + 6\right] = \boxed{2\tfrac{5}{6}}$

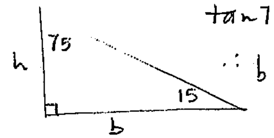
c) 

✓ diagram



$\tan 72 = \frac{a}{h}$

$a = h\tan 72$



$\tan 75 = \frac{b}{h}$

$\therefore b = h\tan 75$

✓ correct rule, correct values

Using Cosine rule $100^2 = a^2 + b^2 - 2ba\cos 100°$

✓ correct use of cosine rule $100^2 = m^2\left[\tan^2 75 + \tan^2 72 - 2\tan 75\tan 72\cos 100\right]$

$m \doteqdot 19.1077$

✓ $\boxed{m \approx 19m}$

d) $y = 3\sin^{-1}\left(\frac{x}{2} - 1\right)$

$-1 \le \frac{x}{2} - 1 \le 1$

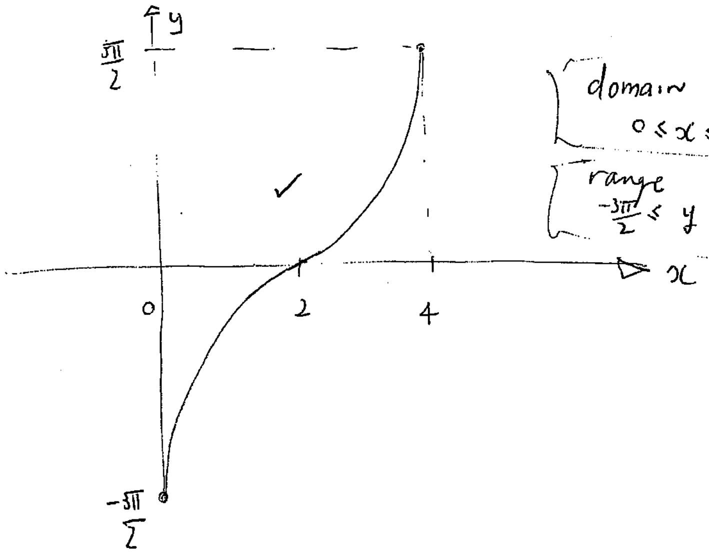
$0 \le \frac{x}{2} \le 2$

$0 \le x \le 4$

$x = 0 \quad y = 3\sin^{-1}(-1) = -\frac{3\pi}{2}$

$x = 2 \quad y = 0$

$x = 4 \quad y = 3\sin^{-1}(1) = \frac{3\pi}{2}$



domain $0 \le x \le 4$ ✓

range $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$ ✓

e) $y = x^2 \quad y = x^3$

$y' = 2x \quad y' = 3x^2$

at $x = 1$

$m_1 = 2 \quad m_2 = 3$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{2-3}{1+6}\right| = \frac{1}{7}$ ✓

$\theta = \tan^{-1}\left(\frac{1}{7}\right)$

$\theta \doteqdot 8^\circ 8'$ ✓

Question 12 15 Marks

a) $\sqrt{3}\sin x + \cos x = A[\sin x\cos\alpha + \cos x\sin\alpha]$

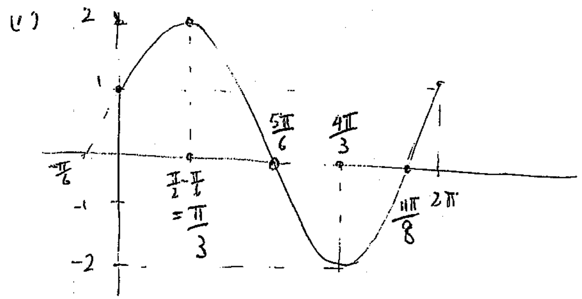
$A\cos\alpha = \sqrt{3}$ $\therefore A = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ ✓

$A\sin\alpha = 1$

$\tan\alpha = \frac{1}{\sqrt{3}} \quad \therefore \alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ ✓

$\therefore \sqrt{3}\sin x + \cos x = 2\sin\left(x + \frac{\pi}{6}\right)$

(i)



✓ shape using amplitude

✓ correct graph, with some intersection

$\boxed{\frac{T}{6}}$

(ii) Solving $2\sin\left(\frac{\pi}{6} + x\right) = \frac{\pi}{3}$

✓ $\therefore \sin\left(x + \frac{\pi}{6}\right) = \frac{\pi}{6} \quad (\approx 0.52)$

$\therefore x + \frac{\pi}{6} = \sin^{-1}\left(\frac{\pi}{6}\right) \quad (\approx 31^\circ)$

$= 0.551\ldots, 2.5905\ldots$

✓ $\therefore x = 0.0275, 2.0669$

b) $\frac{dT}{dt} = -k(T - E)$ $\quad \frac{dT}{T-E} = -k\,dt$ integrate b.s

✓ $\therefore \ln(T - E) = -kt + C \longrightarrow T - E = e^{-kt+C}$ let $e^C = A$

$\therefore T - E = Ae^{-kt}$ $\quad t = 0 \quad T = 92^\circ C \quad E = 16^\circ$

$\therefore \overset{92}{T} - 16 = Ae^0 \quad \therefore A = 76$ ✓

$\boxed{\frac{T}{6}}$

✓ $\therefore$ Soln $T = 16 + 76e^{-kt}$ Now $t = 3$ mins $T = 72$

(i) $72 = 16 + 76e^{-3k} \quad \therefore \frac{56}{76} = e^{-3k} \quad k = -\frac{1}{3}\ln\frac{56}{76}$

✓ $\boxed{\therefore k = 0.1018}$ (4 DP)

(ii) $55 = 16 + 76e^{\frac{1}{3}\ln\frac{56}{76}t} \quad \therefore t = \frac{\ln\frac{39}{76}}{\frac{1}{3}\ln\frac{56}{76}} = 6.554\ldots$ mins

✓ $\frac{39}{76} = e^{\frac{1}{3}\ln\frac{56}{76}t}$

c) $x^3+6x^2-x-30=0$

one root is sum of other 2

Let roots be α, β, γ

$\therefore \alpha = \beta+\gamma$

$\sum \alpha_i = \frac{-b}{a} = -6 \quad \therefore \alpha+\beta+\gamma = -6$

$2\alpha = -6 \quad \therefore \alpha = -3$

$\sum \alpha_i\alpha_j = \frac{c}{a} = -1 \quad \therefore \alpha\beta+\alpha\gamma+\beta\gamma = -1$

$\alpha(\beta+\gamma)+\beta\gamma = -1$

$\alpha^2+\beta\gamma = -1$

$\therefore 9+\beta\gamma = -1$

$\therefore \beta\gamma = -10$

$\sum \alpha_i\alpha_j\alpha_k = -\frac{d}{a} = 30 \quad \therefore \alpha\beta\gamma = 30$

$\therefore \beta\gamma = -10$

Product -10 try $\gamma = -2, \beta = 5$

no as $\alpha \neq \beta+\gamma$

$\therefore \quad \alpha=-3, \ \beta=-5, \ \gamma=+2$

Extension 1 Trial HSC 2012 Markers Notes - Q12.

a) (i) · generally well done

(ii) · many did not read the domain requirement of $0 \le x \le 2\pi$.

· many could not interpret the phase shift of $\frac{\pi}{6}$ correctly; frequent errors were the graph of $y = 2\sin(x-\frac{\pi}{6})$ or the graph of $y = 2\cos x$.

· several student had the correct starting point of $(0,1)$, but then had the max/min points still at $x=\frac{\pi}{2}, x=\frac{3\pi}{2}$ (while managing to get $y=0$ when $x=\frac{5\pi}{6}$ and $x=\frac{11\pi}{6}$)!

· many sketches were very poor in terms of neatness

· frequently, intercepts were not shown (x-axis). It is not difficult to find - ie when is $\sin x = 0$? ans: when $x = 0, \pi, 2\pi$

so when is $\sin(x+\frac{\pi}{6})=0$ ans: when $x+\frac{\pi}{6} = 0, \pi, 2\pi$

ie $x = -\frac{\pi}{6}, \pi-\frac{\pi}{6}, 2\pi-\frac{\pi}{6}$

$= -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

out of domain $0 \le x \le 2\pi$

similarly, when is $\sin x = 1$? ans: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

so when is $\sin(x+\frac{\pi}{6})=1$ ans $x+\frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{2}-\frac{\pi}{6}, \frac{3\pi}{2}-\frac{\pi}{6}$

$= \frac{\pi}{3}, \frac{4\pi}{3}$ (for max/min pts)

(iii) · many only found one solution - there are 2 solutions on $0 \le x \le 2\pi$!

· common error was $\sin^{-1}(\frac{\pi}{6}) = \frac{1}{2}$!! and $\sin^{-1}(\frac{\pi}{6})$ calculated with degrees set on the calculator.

b) (i) · you <u>must</u> derive from the given equation when asked (you can only state $T = 16 + Ae^{-kt}$ as a soln when asked to show it satisfies!)

· only one or 2 students were able to show $E=16$ (ie $T \to 16$ as $t \to \infty$)

· the solution to the differential equation is of the form $T = f(t)$ (too many left t the subject), which also caused many errors with how the $+c$ was handled.

(ii) · generally well done

c) (i) · many left $2\alpha+2\beta=-6$, instead of $\alpha+\beta=-3$ (caused issues in later substitutions)

· several alternative solutions were presented (quadratic, poly division) apart from the one given - marked for equivalence.

Question 13 15 Marks

a) (i) $P(2ap, ap^2)$ $x^2 = 4ay$ $\therefore y = \frac{x^2}{4a}$ $\therefore y' = \frac{2x}{4a} = \frac{x}{2a}$

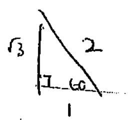
$\left.\frac{dy}{dx}\right]_{x=2ap} = \frac{2ap}{2a} = p$ ie $\boxed{m = p}$

T/1

Eqn of tangent $y - y_1 = m(x - x_1)$ $\therefore y - ap^2 = p(x - 2ap)$

$\therefore \boxed{y = px - ap^2}$ ✓ eqn of tangent at P.

b) (ii) tangent at Q is q (similarly)



$\tan 60 = \sqrt{3}$

$\therefore \tan 60 = \frac{p-q}{1+pq}$ using $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

T/1 $\therefore \sqrt{3} = \frac{p-q}{1+pq}$ ie $\sqrt{3}(1+pq) = p - q$ ✓

(iii) $T(a(p+q), apq)$ given $\sqrt{3}(1+pq) = p - q$

Locus

$x = a(p+q)$, $y = apq$ $\therefore pq = \frac{y}{a}$

$\therefore \sqrt{3}\left(1 + \frac{y}{a}\right) = p - q$

✓ from $\frac{x}{a} = p + q$ $(p-q)^2 = p^2 + q^2 - 2pq$

$\left(\frac{x}{a}\right)^2 = p^2 + q^2 + 2pq$

$-4pq + \left(\frac{x}{a}\right)^2 = p^2 + q^2 + 2pq - [4pq]$ T/2

$\therefore (p-q)^2 = \left(\frac{x}{a}\right)^2 - 4pq = \left(\frac{x}{a}\right)^2 - 4 \cdot \frac{y}{a}$

✓ $\therefore 3\left(1 + \frac{y}{a}\right)^2 = \left(\frac{x}{a}\right)^2 - \frac{4y}{a}$

$3a^2 + 10ay + 3y^2 = x^2$] locus messy.

$x^2 - y(y - 10a) = 3a^2$

b) Fish Tank

8C 7B

$^8C_4 \times {}^7C_2$ 1470

$+ {}^8C_5 + {}^7C_1$ 392

$+ {}^8C_6$ 28

1890 ✓

$^{15}C_6 = 5005$

$P(C \geq 4) = \frac{1890}{5005} = \boxed{\frac{54}{143}}$ ✓

c) Divisibility

$47^n + 53 \times 147^{n-1} = 100 \times R$ (R integer)

S1 $n = 1$ $47^1 + 53 \times 147^0 = 100$ which is divisible by 100

S2 Assume true for $n = k$

ie $47^k + 53 \times 147^{k-1} = 100P$ $\therefore 47^k = 100P - 53 \times 147^{k-1}$ (P integer)

S3 Prove true for $n = k+1$

ie $47^{k+1} + 53 \times 147^k = 100Q$

LHS $47 \times 47^k + 53 \times 147^k$ using assumption

$\therefore 47[100P - 53 \times 147^{k-1}] + 53 \times 147^k$

$47 \cdot 100P - 47 \times 53 \times 147^{k-1} + 53 \times 147 \times 147^{k-1}$

$47 \cdot 100P + 100 \times 53 \times 147^{k-1}$

$= 100[47P + 53 \times 147^{k-1}]$ integer

$= 100Q$ where $Q = 47P + 53 \times 147^{k-1}$ (integer)

S4 Since it is true for $n = 1$ and since $n = k$ and $n = k+1$ are true, then it will be true for $n = 1+1 = 2$, and so on for all positive integers n.

Question 13 cont.

d) $f(x) = 2\cos 2x - x - 1$ Newtons Method

$f'(x) = -4\sin 2x - 1$ $\quad x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

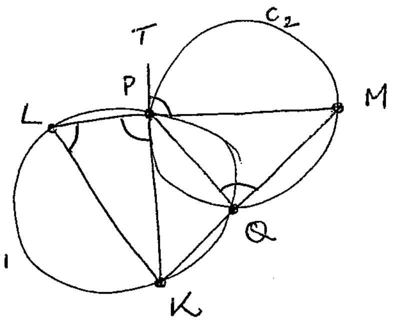
Now $x_1 = 0.4$

$$x_2 = 0.4 - \frac{f(0.4)}{f'(0.4)} = 0.4 - \frac{(2\cos(0.8) - 0.4 - 1)}{-4\sin(0.8) - 1}$$

$$x_2 = 0.4 + \frac{2\cos(0.8) - 0.4 - 1}{4\sin(0.8) + 1}$$

$$x_2 \doteqdot 0.3983 \quad (4\ \text{DP})$$

e)



$\angle TPM = \angle LPK$ (vertically opposite)

$\angle TPM = \angle PQM$ (angles in alternate segment C_2)

$\therefore \angle LPK = \angle PQM$

Now $\angle PQM = \angle PLK$

(opposite angles of a cyclic quad are supplementary)

$\therefore \angle LPK = \angle PLK$

$\therefore \triangle PKL$ is isosceles (base angles equal).

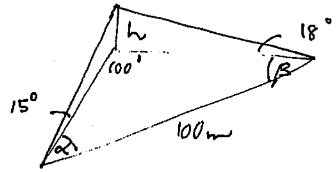
Question 11 comments

a) Most people $\times (x-5)^2 \geq 0$ but forgot $x \neq 5$

b) Integral tranformed reasonably well some people forgot to change bounds, some mistakes with substitutions

c) Diagram mostly ok, Many people tried to use the sine rule, however cosine rule needed to be used.

$h \approx 19$m



d) Inverse trig graph overall well done some domain errors.

e) Angle between lines $m_1 = 2$, $m_2 = 3$ well done some errors with formula

Question 13 comments

a)(i) Some people only found gradient $m = p$, not the tangents eqn.

(ii) Well done overall

(iii) Locus not well done

$3a^2 + 10ay + 3y^2 = x^2$ (refer answers)

Needed to use part (ii) $\sqrt{3}(1+pq) = p - q$

b) Fish tank reasonably well done some people used permutations?

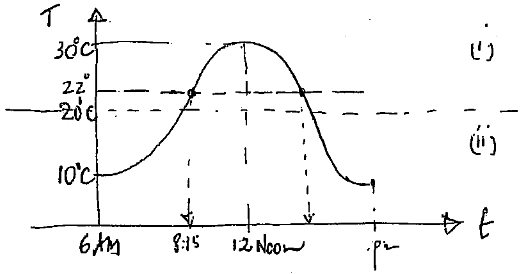
c) Induction middle part messed up, assumption not well used, some conclusions not correct.

d) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ well done, some substitutions poor (calc wrong mode)

e) Circle geometry fairly well done.

Question 14   15 Marks

a)



(i) $T = -10\cos\left(\frac{\pi}{6}t\right) + 20$ ✓

(ii) Solve

$2 = -10\cos\left(\frac{\pi}{6}t\right)$ ✓

$\therefore \frac{\pi}{6}t = \cos^{-1}\left(-\frac{1}{5}\right)$

$t = \frac{6}{\pi}\cos^{-1}\left(-\frac{1}{5}\right) = 3.38456\ldots$ ✓

$t = 3$ hrs : 23 min 5 sec

$n = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$

amplitude = 10 ✓

T/5

(iii) Jack can first enter cabin 9:23 am

Jack can remain in cabin for 5 hr + 14 mins ✓
(time subtraction or symmetry).

This entire question was very badly done!

- students got the formula wrong.
- There is a tide question which resemble this in the ext1 Fitzpatrick.
- many students did not solve this by looking at the symmetry.

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b) (i) Original Salinity $\frac{750\,kg}{800}$ /L   0.9375 kg /L
Inflow 950 gm/L   i.e. 937.5 gm/L.
✓ ∴ Salinity gradually increasing over time.

(ii) $\frac{dQ}{dt}$ = inflow − outflow

$= 10 \times .950 - \frac{10 \times Q}{800} = 9.5 - \frac{Q}{80}$ ✓

T/5

$\therefore \frac{dQ}{dt} = \frac{760 - Q}{80}$ $\therefore dt = \frac{80\,dQ}{760 - Q}$ integrating both sides.

$t = -80\ln(760 - Q) + c$ $t = 0$ $Q = 750$

$\therefore 0 = -80\ln(760 - 750) + c$ $\therefore c = +80\ln 10$

$\therefore t = -80\ln(760 - Q) + 80\ln 10$

$t = 80\ln\left(\frac{10}{760 - Q}\right)$ $\therefore \frac{t}{80} = \ln\left(\frac{10}{760 - Q}\right)$

$\therefore e^{\frac{t}{80}} = \frac{10}{760 - Q}$ or $e^{-\frac{t}{80}} = \frac{760 - Q}{10}$ ✓

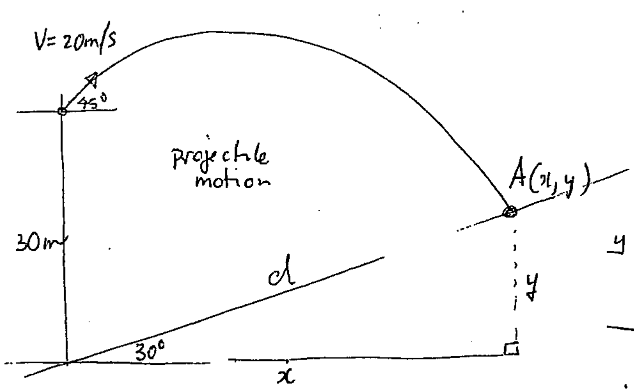
$\therefore 10e^{-\frac{t}{80}} = 760 - Q$ $\therefore \boxed{Q = 760 - 10e^{-\frac{t}{80}}}$

(iii) 5 hrs → 5×60 = 300 mins

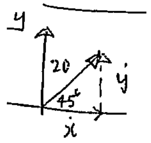
✓ $Q = 760 - 10e^{-\frac{300}{80}} = 760 - 0.235 = \boxed{759.76\ \text{kg}}$ ✓

→ students did not set up the equation which was very dissapointing since there was a question in the textbook.

c)



$t=0$



$\dot{x} = \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 10\sqrt{2}\ m/s$

$\dot{y} = \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 10\sqrt{2}\ m/s$

$t=0$
$x=0$
$y=30$

$\boxed{\frac{T}{2}}$

$\ddot{y} = -10$ | $\ddot{x} = 0$

$\therefore \dot{y} = -10t + C_1$ | $\dot{x} = C_2$

$t=0 \quad \dot{y} = C_1 = 10\sqrt{2}$ | $\dot{x} = 10\sqrt{2}\ m/s$

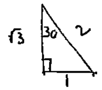
$\therefore \dot{y} = -10t + 10\sqrt{2}$ | $\dot{x} = 10\sqrt{2}$

$y = -5t^2 + 10\sqrt{2}t + C_3$ | $x = 10\sqrt{2}t + C_4$

$t=0 \quad y=30 \quad \therefore C_3 = 30$ | $x=0 \therefore C_4 = 0$

(i) $\therefore \underline{y = -5t^2 + 10\sqrt{2}t + 30}$ ✓ $\underline{x = 10\sqrt{2}t}$ ✓

(ii) $\tan 30 = \frac{y}{x} = \frac{1}{\sqrt{3}} \quad \therefore \boxed{x = \sqrt{3}y}$ ✓



time for parrot is same at time to get to A(x, y) (coin) Projectile

$\therefore t = \frac{x}{10\sqrt{2}} = \frac{\sqrt{3}y}{10\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}y}{20}$

$\boxed{\frac{T}{3}}$

$\therefore y = -5\left(\frac{\sqrt{6}y}{20}\right)^2 + 10\sqrt{2} \times \frac{\sqrt{6}y}{20} + 30 = \frac{-30y^2}{400} + \sqrt{3}y + 30$

$40y = -3y^2 + 40\sqrt{3}y + 1200$

$\therefore 3y^2 + 40(1-\sqrt{3})y - 1200 = 0 \quad \therefore y =$ using pos case

take +ve case

$y = \frac{-40(1-\sqrt{3}) \pm \sqrt{(40(1-\sqrt{3}))^2 - 4 \times 3 \times -1200}}{6} \quad \therefore y = \frac{29.28 \pm 123.52099}{6}$

$y = 25.46717\ldots$ ✓

$t = \frac{\sqrt{6}y}{20} \doteqdot 3.11907$ secs

$d = \sqrt{x^2 + y^2} = 50.9343\ m$ in 3.11907 secs

$v = 16.329\ m/s \ldots \longrightarrow 58.7877\ km/hr \quad \boxed{59\ km/hr}$ ✓

- students derived the formulae, but did not sub in the values of $v = 20$, $\alpha = 45$

- once the initial formulae are incorrect, it becomes very hard to continue