Name: ______________________

Class Teacher: ______________________

Fort Street
High School

2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics - Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 – 13)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Any work written on this page will not be marked.

Final Marks

Question	Mark
Multiple Choice	**/10**
Question 11	**/15**
Question 12	**/15**
Question 13	**/15**
Question 14	**/15**
Total	**/70**

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

1. $\sqrt{3}\sin x - \cos x$ is equal to

A. $2\sin\left(x - \dfrac{\pi}{3}\right)$

B. $2\sin\left(x - \dfrac{\pi}{6}\right)$

C. $2\cos\left(x - \dfrac{\pi}{3}\right)$

D. $2\cos\left(x - \dfrac{\pi}{6}\right)$

2. Find the vector projection of $\underset{\sim}{p} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ onto $\underset{\sim}{q} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

A. $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

B. $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

C. $\begin{pmatrix} -2\sqrt{5} \\ 4\sqrt{5} \end{pmatrix}$

D. $\begin{pmatrix} 2\sqrt{5} \\ -4\sqrt{5} \end{pmatrix}$

3. Give that $t = \tan\dfrac{\theta}{2}$, then $\tan\theta\sin\theta$ is equal to

A. $\dfrac{4t^2}{1-t^4}$

B. $\dfrac{2t^2}{1-t^4}$

C. $\dfrac{4t^2}{1+t^4}$

D. $\dfrac{2t^2}{1+t^2}$

4. Two forces $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ act on a body. $\underset{\sim}{P}$ acts in the direction $\underset{\sim}{i}$ with magnitude one newton and $\underset{\sim}{Q}$ acts in the direction of $\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j}$ with magnitude four newtons.

The magnitude of the total force acting on the body, in newtons, is

A. 3

B. $\sqrt{15}$

C. $\sqrt{17}$

D. $\sqrt{21}$

5. $P(x)=x^4-3x^3-2x^2-5x$ is to be expressed in the form $P(x)=x(x-4)Q(x)+R(x)$. Then

A. $Q(x)=x^2+x+2$ and $R(x)=3x$

B. $Q(x)=3x$ and $R(x)=x^2+x+2$

C. $Q(x)=x^2-7x-30$ and $R(x)=-150x$

D. $Q(x)=-150x$ and $R(x)=x^2-7x-30$

6. $\int \cos^2 2x dx$ is equal to

A. $\frac{1}{2}(1+\cos 2x)+C$

B. $\frac{1}{2}(1+\cos 4x)+C$

C. $\frac{1}{2}(x+\frac{1}{4}\sin 4x)+C$

D. $\frac{1}{2}(x+\frac{1}{2}\sin 2x)+C$

7. $P(x)$ is an odd function. When $P(x)$ is divided by $(x-1)$ the remainder is 3. The remainder when $P(x)$ is divided by $(x+1)$ will be

A. -3

B. -1

C. 1

D. 3

8. A body of still water has suffered an oil spill and a circular oil slick is floating on the surface of the water.

The area of the oil slick is increasing by $0.1\ \text{m}^2/\text{minute}$.

At what rate is the radius increasing when the radius is 0.3 m ?

A. 0.161 m/min

B. 0.03 m/min

C. 0.0515 m/min

D. 0.0531 m/min

9. A cricket ball is hit from an origin at ground level so that its position vector at time t is given by $\underset{\sim}{\text{r}}(t) = 15t\underset{\sim}{\text{i}} + \left(20t - 5t^2\right)\underset{\sim}{\text{j}}$ for $t \geq 0$, where $\underset{\sim}{\text{i}}$ is the unit vector in the forward direction and $\underset{\sim}{\text{j}}$ is a unit vector vertically up.

When the cricket ball reaches its maximum height, its position vector is

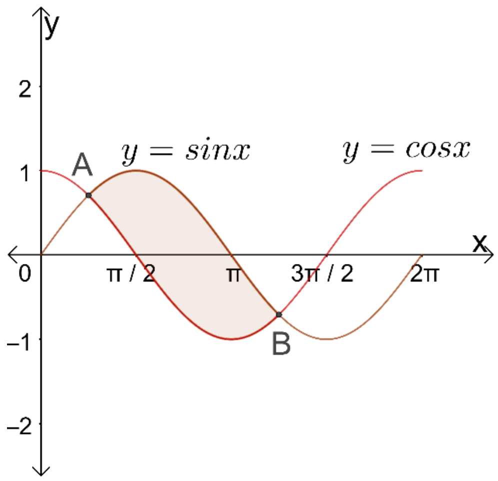
A. $\underset{\sim}{\text{r}} = 30\underset{\sim}{\text{i}} + 20\underset{\sim}{\text{j}}$

B. $\underset{\sim}{\text{r}} = 15\underset{\sim}{\text{i}} + 20\underset{\sim}{\text{j}}$

C. $\underset{\sim}{\text{r}} = 60\underset{\sim}{\text{i}}$

D. $\underset{\sim}{\text{r}} = 30\underset{\sim}{\text{i}} + 10\underset{\sim}{\text{j}}$

10. The area enclosed by the curves $y = \sin x$ and $y = \cos x$ is shaded in the diagram.



Which expression could be used to calculate this area?

A. $$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(\cos(x) - \sin(x)\right) \ dx$$

B. $$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(\sin(x) - \cos(x)\right) \ dx$$

C. $$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(\cos(x) + \sin(x)\right) \ dx$$

D. $$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \left(\cos(x) - \sin(x)\right) \ dx$$

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

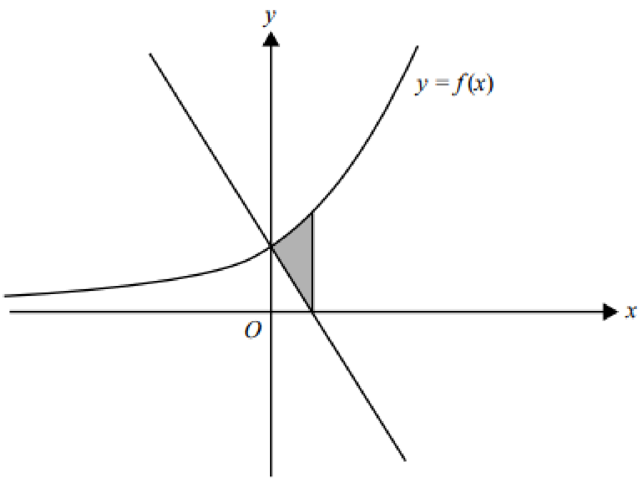
In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet.

(a) Find $\int 2x\cos(x^2-1)dx$, using $u = x^2 - 1$ **2**

(b) Show that the derivative of $y = \frac{1}{2}\cos^{-1}(3x+1)$ is given by $y' = -\frac{1}{2x}\sqrt{\frac{-3x}{3x+2}}$. **2**

(c) The graph of $f(x) = e^{\frac{x}{2}} + 1$ is shown. The normal to the graph of f where it crosses the y-axis is also shown.



i) Find the equation of the **normal** to the graph of f where it crosses the y-axis. **2**

ii) Find the exact area of the shaded region. **2**

Question 11 continues on page 8

Question 11 continued

(d) Use the substitution $u = 1 + 2\tan x$ to evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{(1+2\tan x)^2 \cos^2 x} dx$. **3**

(e) Consider the function $f(x) = 2 - \log_e x$.

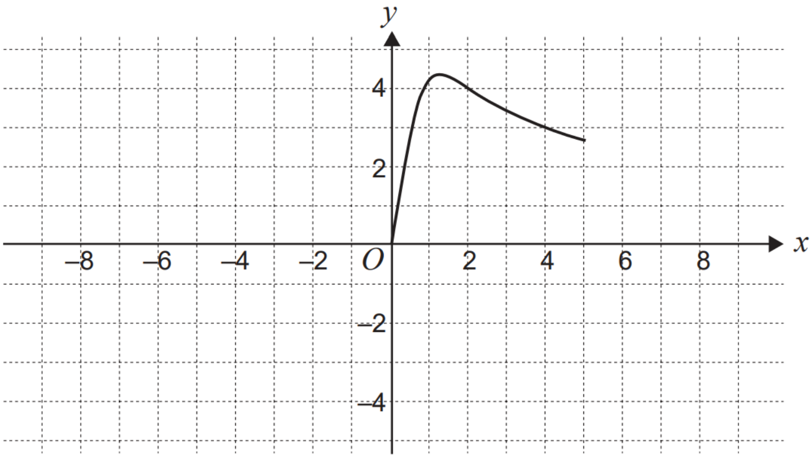
i) Find the equation of the inverse function $f^{-1}(x)$. **1**

ii) Explain why the coordinate of the point of intersection P of the graphs $y = f(x)$ and $y = f^{-1}(x)$ satisfies the equation $e^{2-x} - x = 0$ **3**

End of Question 11

Question 12 (15 marks) Start a new writing booklet.

(a) The top part of a wine glass, while lying on its side, is constructed by rotating the graph of $y = \dfrac{6x}{\sqrt{1+x^3}}$ from $x = 0$ to $x = 5$ about the x-axis as shown below. All lengths are measured in centimetres.



i) Write down a definite integral which represents the volume, V cm^3, of the glass. **1**

ii) Use the substitution $u = 1 + x^3$ to write down a definite integral which represents the volume of the glass in terms of u. **2**

iii) Find the value of V correct to the nearest cm^3. **1**

(b) Malek was asked to prove $\cos(\pi - \theta) = -\cos\theta$ for all real θ. He attempts his proof using mathematical induction. Explain why his method would not yield an appropriate proof. **2**

(c) Use mathematical induction to prove that, for every positive integer n, **3**

$$13 \times 6^n + 2 \text{ is divisible by } 5.$$

Question 12 continues on page 10

(d) i) Differentiate $x\sin^{-1}x+\sqrt{1-x^2}$. **2**

ii) Hence evaluate $\int_0^1 2\sin^{-1}x\,dx$. **2**

(e) The curve with equation $y=f(x)$ passes through the point $\left(\frac{\pi}{8},2\right)$ and has a gradient of -1 at this point. Find the exact gradient of the curve at $x=\frac{\pi}{12}$ given that $y''=-\sec^2(2x)$. **2**

End of Question 12

Question 13 (15 marks) start a new writing booklet.

(a) Given the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} - 5\underset{\sim}{j}$,

i) Calculate the dot product $\underset{\sim}{a} \cdot \underset{\sim}{b}$. **1**

ii) Find $4\underset{\sim}{a} - 3\underset{\sim}{b}$ in column vector form. **2**

(b) Find $\int_0^\pi \sin(mx)\cos(nx)\; dx$ where *m* and *n* are both positive, even integers and $m \neq n$. **3**

(c) A golf ball is hit at a velocity of 55 ms^{-1} at an angle θ to the horizontal, with an acceleration due to gravity of 9.8ms^{-2} being applied to the ball.

The position vector $s(t)$, from the starting point, of the ball after *t* seconds is given by

$$s = 55t\cos\theta\underset{\sim}{i} + (55t\sin\theta - 4.9t^2)\underset{\sim}{j}$$

i) Show that the ball travels $\dfrac{3025\sin 2\theta}{9.8}$ metres before hitting the ground. **2**

ii) To ensure that the ball lands on the green, it must travel between 200 and 250 metres. **2**

What values of θ, correct to the nearest minute, would allow this to happen?

iii) The golfer aims accurately and hits the ball directly towards the green. **1**

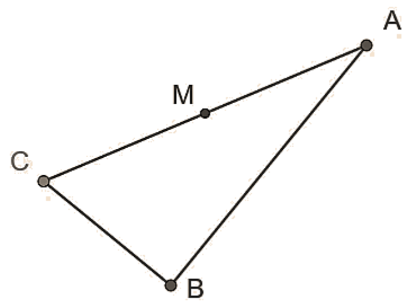
After 3.4 seconds of flight, at a point 8 metres above the ground, the ball hits a low flying TV drone.

If it had not hit the drone or any other obstacles, would the ball have landed on the green?

Question 13 continues on page 12

(d) ΔABC is a right-angled with M being the midpoint of the hypotenuse AC, as shown.

Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



i) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

ii) Prove that M is equidistant from the three vertices of ΔABC **2**

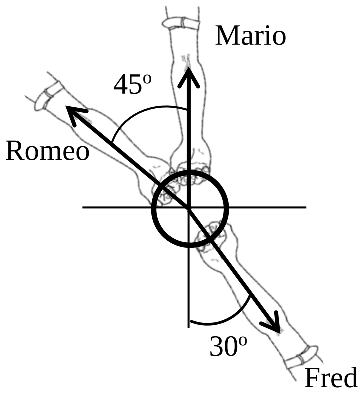
End of Question 13

Question 14 (15 marks) writing booklet.

(a) Use t-formula to solve the equation $\cos x - \sin x = 1$, where $0 \le x \le 2\pi$. **3**

(b) Fred, Mario and Romeo are fighting over a crown. The three of them are holding on to the crown in the formation as shown in the diagram. **3**

If no one manages to pull the crown in their direction (ie, the crown does not move) and Fred is applying a force of 40N, exactly how much force are Mario and Romeo applying to the crown?



(c) Sienna intends to row her boat from the south bank of a river to meet with her friends on the north bank. The river is 100 metres wide. Sienna's rowing speed is 5 metres per second when the water is still. The river is flowing east at a rate of 4 metres per second. Sienna's boat is also being impacted by a wind blowing from the south-west, which pushed the boat at 8 metres per second. She starts rowing across the river by steering the boat such that it is perpendicular to the south bank.

i) Show that the velocity of Sienna's boat can be expressed as the component vector: **2**

$$(4 + 4\sqrt{2})\,\underset{\sim}{\mathbf{i}} + (5 + 4\sqrt{2})\,\underset{\sim}{\mathbf{j}}$$

ii) Calculate the speed of the boat, correct to 2 decimal places. **1**

iii) Determine the distance rowed from Sienna's starting point to her landing point and how long it will take her to reach the north bank. **2**

(d) i) By considering the terms of an arithmetic series, show that **1**

$$(1+2+\cdots+n)^2 = \frac{1}{4}n^2(n+1)^2 \ .$$

ii) By using the Principle of Mathematical Induction prove that **3**

$$1^3+2^3+\cdots+n^3 = (1+2+\cdots+n)^2 \text{ for all } n \ge 1$$

End of Paper

Any work written on this page will not be marked.

Fort Street High School

2020 Trial Higher School Certificate Examination

Mathematics Extension 1

Student Number: ______________________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Solutions

Fort Street
High School

2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics - Extension 1

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Total marks: 70

Section I – 10 marks (pages 3 – 6)

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Section II – 60 marks (pages 7 – 13)

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1. $\sqrt{3}\sin x - \cos x$ is equal to

A. $2\sin\left(x-\frac{\pi}{3}\right)$

B. $2\sin\left(x-\frac{\pi}{6}\right)$

C. $2\cos\left(x-\frac{\pi}{3}\right)$

D. $2\cos\left(x-\frac{\pi}{6}\right)$

$\sqrt{3}\sin x - \cos x = r\sin x\cos\alpha - r\cos x\sin\alpha$
$= r(\sin x - \alpha)$

$\Rightarrow r\cos\alpha = \sqrt{3}$; $r\sin\alpha = 1$

$\therefore (r\cos\alpha)^2 + (r\sin\alpha)^2 = 4$

$\therefore r = 2$

$\frac{r\sin\alpha}{r\cos\alpha} = \frac{1}{\sqrt{3}} \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}$

$\therefore \alpha = \frac{\pi}{6}$

2. Find the vector projection of $\underset{\sim}{p} = \begin{pmatrix}4\\-3\end{pmatrix}$ onto $\underset{\sim}{q} = \begin{pmatrix}-1\\2\end{pmatrix}$.

A. $\begin{pmatrix}-2\\4\end{pmatrix}$

B. $\begin{pmatrix}2\\-4\end{pmatrix}$

C. $\begin{pmatrix}-2\sqrt{5}\\4\sqrt{5}\end{pmatrix}$

D. $\begin{pmatrix}2\sqrt{5}\\-4\sqrt{5}\end{pmatrix}$

$\text{Proj}_{\underset{\sim}{q}}(\underset{\sim}{p}) = \frac{\underset{\sim}{p}\cdot\underset{\sim}{q}}{|\underset{\sim}{q}|^2}\cdot\underset{\sim}{q}$

$|\underset{\sim}{q}|^2 = 1+4$
$= 5$

$\underset{\sim}{p}\cdot\underset{\sim}{q} = (4)(-1)+(-3)(2)$
$= -4-6$
$= -10$

$\therefore \text{Proj}_{\underset{\sim}{q}}(\underset{\sim}{p}) = -\frac{10}{5}\begin{pmatrix}-1\\2\end{pmatrix}$
$= \begin{pmatrix}2\\-4\end{pmatrix}$

3. Give that $t = \tan\frac{\theta}{2}$, then $\tan\theta\sin\theta$ is equal to

A. $\frac{4t^2}{1-t^4}$

B. $\frac{2t^2}{1-t^4}$

C. $\frac{4t^2}{1+t^4}$

D. $\frac{2t^2}{1+t^2}$

$t = \tan\frac{\theta}{2}$; $\tan\theta = \frac{2t}{1-t^2}$, $\sin\theta = \frac{2t}{1+t^2}$

$\therefore \tan\theta\sin\theta = \frac{2t}{1-t^2}\cdot\frac{2t}{1+t^2}$
$= \frac{4t^2}{1-t^4}$

4. Two forces $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ act on a body. $\underset{\sim}{P}$ acts in the direction $\underset{\sim}{i}$ with magnitude one newton and $\underset{\sim}{Q}$ acts in the direction of $\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j}$ with magnitude four newtons.

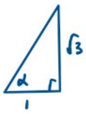
The magnitude of the total force acting on the body, in newtons, is

A. 3

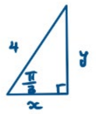
B. $\sqrt{15}$

C. $\sqrt{17}$

D. $\sqrt{21}$



$\Rightarrow \alpha = \frac{\pi}{3} \Rightarrow$



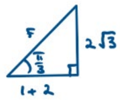
$\Rightarrow x = 4\cos\frac{\pi}{3}$

$= 2$

$y = 4\sin\frac{\pi}{3}$

$= 2\sqrt{3}$

$\Rightarrow$



$\therefore F^2 = 3^2 + (2\sqrt{3})^2$

$= 21$

$\therefore F = \sqrt{21}$

5. $P(x) = x^4 - 3x^3 - 2x^2 - 5x$ is to be expressed in the form $P(x) = x(x-4)Q(x) + R(x)$. Then

A. $Q(x) = x^2 + x + 2$ and $R(x) = 3x$

B. $Q(x) = 3x$ and $R(x) = x^2 + x + 2$

C. $Q(x) = x^2 - 7x - 30$ and $R(x) = -150x$

D. $Q(x) = -150x$ and $R(x) = x^2 - 7x - 30$

$P(x) = x^4 - 3x^3 - 2x^2 - 5x$

$= x(x-4)Q(x) + R(x)$

$$
\begin{array}{r|l}
 & x^2 + x + 2 \\
x^2 - 4x & x^4 - 3x^3 - 2x^2 - 5x \\
 & x^4 - 4x^3 \\
 & x^3 - 2x^2 \\
 & x^3 - 4x^2 \\
 & 2x^2 - 5x \\
 & 2x^2 - 8x \\
 & 3x
\end{array}
$$

$\therefore Q(x) = x^2 + x + 2, \quad R(x) = 3x$

6. $\int \cos^2 2x dx$ is equal to

A. $\frac{1}{2}(1 + \cos 2x) + C$

B. $\frac{1}{2}(1 + \cos 4x) + C$

C. $\frac{1}{2}(x + \frac{1}{4}\sin 4x) + C$

D. $\frac{1}{2}(x + \frac{1}{2}\sin 2x) + C$

$\int \cos^2 2x \, dx = \frac{1}{2}\int (1 + \cos 4x) \, dx$

$= \frac{1}{2}(x + \frac{1}{4}\sin 4x) + c$

7. $P(x)$ is an odd function. When $P(x)$ is divided by $(x-1)$ the remainder is 3. The remainder when $P(x)$ is divided by $(x+1)$ will be

A. -3

B. -1

C. 1

D. 3

Since $P(x)$ is odd, then $P(-x) = -P(x)$

Now, by remainder theorem $P(1) = 3$

$\therefore P(-1) = -P(1)$

$= -3.$

8. A body of still water has suffered an oil spill and a circular oil slick is floating on the surface of the water.

The area of the oil slick is increasing by $0.1\ \text{m}^2$ / minute.

At what rate is the radius increasing when the radius is 0.3 m ?

$\frac{dA}{dt} = 0.1\ m^2/min$; $A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$

$\therefore \frac{dr}{dA} = \frac{1}{2\pi r}$

$\frac{dr}{dt} = \frac{dr}{dA} \cdot \frac{dA}{dt}$

$= \frac{1}{2\pi(0.3)} \times 0.1$

$\doteqdot 0.0531$

A. 0.161 m/min

B. 0.03 m/min

C. 0.0515 m/min

D. 0.0531 m/min

9. A cricket ball is hit from an origin at ground level so that its position vector at time t is given by $\underset{\sim}{r}(t) = 15t\underset{\sim}{i} + (20t - 5t^2)\underset{\sim}{j}$ for $t \geq 0$, where $\underset{\sim}{i}$ is the unit vector in the forward direction and $\underset{\sim}{j}$ is a unit vector vertically up.

When the cricket ball reaches its maximum height, its position vector is

A. $\underset{\sim}{r} = 30\underset{\sim}{i} + 20\underset{\sim}{j}$

B. $\underset{\sim}{r} = 15\underset{\sim}{i} + 20\underset{\sim}{j}$

C. $\underset{\sim}{r} = 60\underset{\sim}{i}$

D. $\underset{\sim}{r} = 30\underset{\sim}{i} + 10\underset{\sim}{j}$

$\underset{\sim}{r}(t) = 15t\ \underset{\sim}{i} + (20t - 5t^2)\ \underset{\sim}{j}$

Max range when $(20t - 5t^2) = 0$

$5t(4 - t) = 0$

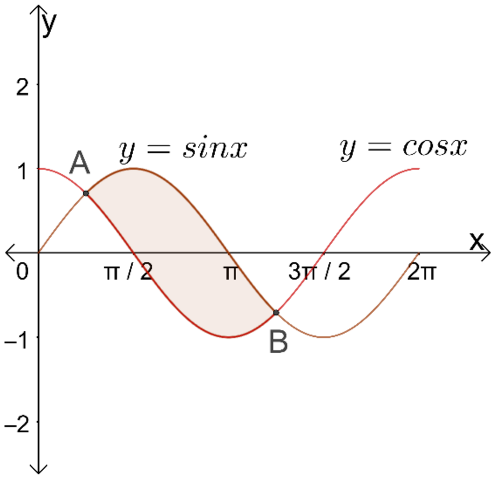
$\therefore$ max range occurs when $t = 4$

$\Rightarrow$ max height when $t = 2$

$\therefore$ Position vector at max height:

$\underset{\sim}{r}(2) = 30\underset{\sim}{i} + 20\underset{\sim}{j}$

10. The area enclosed by the curves $y = \sin x$ and $y = \cos x$ is shaded in the diagram.



Which expression could be used to calculate this area?

A. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\cos(x) - \sin(x))\ dx$

B. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin(x) - \cos(x))\ dx$

C. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\cos(x) + \sin(x))\ dx$

D. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\cos(x) - \sin(x))\ dx$

Points of intersection:

let $\sin x = \cos x$

$\Rightarrow \dfrac{\sin x}{\cos x} = 1 \quad (\cos x \neq 0)$

$\Rightarrow x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

S A T C

$\therefore A = \displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx$

Section II

Question 11 (15 marks) Start a new writing booklet.

(a) Find $\int 2x\cos(x^2-1)dx$, using $u = x^2 - 1$ **2**

Criteria	Marks
• Provides correct solution	2
• Sets up correct integration, using the substitution	1

$u = x^2 - 1$

$du = 2x\,dx$

$\therefore\ I = \int \cos u\, du$

$= \sin u + C$

$= \sin(x^2 - 1) + C$

Markers Comments:

Most students answered correctly. Students are reminded that they must provide their final answer in terms of x and add the constant term to the final answer.

(b) Show that the derivative of $y = \frac{1}{2}\cos^{-1}(3x+1)$ is given by $y' = -\frac{1}{2x}\sqrt{\frac{-3x}{3x+2}}$. **2**

Criteria	Marks
• Provides correct solution	2
• differentiate correctly	1

$y = \frac{1}{2}\cos^{-1}(3x+1)$

$y' = -\frac{1}{2}\frac{3}{\sqrt{1-(3x+1)^2}}$

$= \frac{-3}{2}\frac{1}{\sqrt{1-9x^2-6x-1}}$

$= -\frac{3}{2}\frac{1}{\sqrt{-9x^2-6x}}$

$= -\frac{3}{2}\frac{1}{3x\sqrt{-1-\frac{2}{3x}}}$

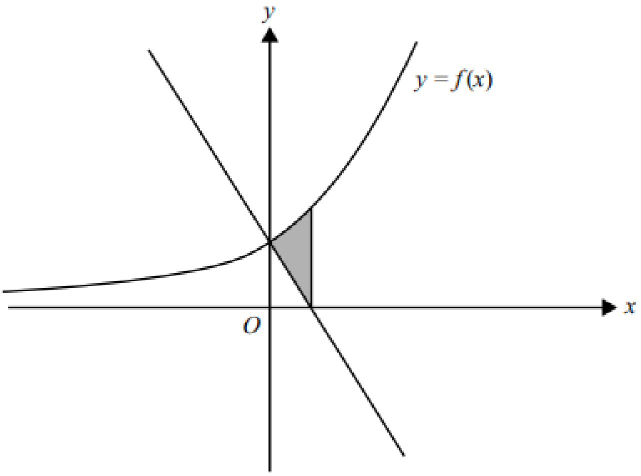
$= -\frac{1}{2x}\frac{1}{\sqrt{\frac{-3x-2}{3x}}}$

$= -\frac{1}{2x}\sqrt{\frac{-3x}{3x+2}}$

Markers Comments:

Most students differentiated correctly with some forgetting the ½. Successful students used various methods to rearrange to achieve the desired outcome.

(c) The graph of $f(x) = e^{\frac{x}{2}} + 1$ is shown. The normal to the graph of f where it crosses the y-axis is also shown.



i) Find the equation of the **normal** to the graph of f where it crosses the y-axis. **2**

Criteria	Marks
• Provides correct solution	2
• Finds the gradient of normal and y-intercept	1

$f(x) = e^{\frac{x}{2}} + 1$

Normal at $x = 0$, $y = 2$

$f'(x) = \frac{1}{2}e^{\frac{x}{2}}$

$\therefore$ Gradient of tangent at $(0,2)$: $m = \frac{1}{2}$

$\therefore$ Gradient of normal at $(0,2)$: $m = -2$

Eqn of normal:

$y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 0)$

$y = 2 - 2x$

Markers comments:

This question was well answered but students setting out needs to improve as many did not clearly communicate their mathematics.

Common errors included incorrect differentiation of the function, substituting in the wrong equation to find the y-value and substituting x=2 (x=0 and y=2) to find the gradient.

Almost all students used the correct strategy to find the equation of the normal.

ii) Find the exact area of the shaded region. **2**

Criteria	Marks
• Provides correct solution	2
• Sets up a correct integration equation for the area	1

Alternate Solution:

x-int of normal:

let $y=0 \Rightarrow x=1$

$\therefore A=\int_0^1 e^{\frac{x}{2}}+1\,dx-\frac{1}{2}\times1\times2$

$=\left[2e^{\frac{x}{2}}+x\right]_0^1-1$

$=(2e^{\frac{1}{2}}+1)-(2e^0+0)-1$

$=2e^{\frac{1}{2}}-2$ units^2

$A=\int_0^1 (e^{\frac{x}{2}}+1)-(2-2x)\,dx$

$=\int_0^1 (e^{\frac{x}{2}}-1+2x)\,dx$

$=\left[2e^{\frac{x}{2}}-x+x^2\right]_0^1$

$=(2e^{\frac{1}{2}}-1+1)-(2e^0-0+0)$

$=2e^{\frac{1}{2}}-2$ units^2

Marker's comments:

Generally answered well by all students.

Students need to take care to make sure they are integrating the exponential function and not differentiating.

(d) Use the substitution $u=1+2\tan x$ to evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{(1+2\tan x)^2\cos^2 x}\,dx$. **3**

Criteria	Marks
• Provides correct solution	3
• Integrating correctly	2
• Taking reasonable steps to develop the integration in terms of u	1

$u=1+2\tan x$

$du=2\sec^2 x\,dx$

$\frac{1}{2}du=\frac{1}{\cos^2 x}\,dx$

when $x=\frac{\pi}{4}\rightarrow u=3$

$x=0\rightarrow u=1$

$\therefore I=\frac{1}{2}\int_1^3 \frac{1}{u^2}\,du$

$=\frac{1}{2}\left[-\frac{1}{u}\right]_1^3$

$=\frac{1}{2}\left(-\frac{1}{3}+1\right)$

$=\frac{1}{3}$

Marker's comments:

Most student answered this question successfully. Students should note that once the integral has been rewritten in terms of u, the limits of the integral must also be written in terms of u, otherwise the solution is incorrect.

(e) Consider the function $f(x) = 2 - \log_e x$.

i) Find the equation of the inverse function $f^{-1}(x)$. **1**

Criteria	Marks
• Provide the correct solution	1

Let $y = 2 - \log_e x$

Then for inverse:

$x = 2 - \log_e y$

$\log_e y = 2 - x$

$y = e^{2-x}$

Marker's comments:

Generally answered well by all students.

ii) Explain why the coordinate of the point of intersection P of the graphs $y = f(x)$ and $y = f^{-1}(x)$ satisfies the equation $e^{2-x} - x = 0$. **3**

Criteria	Marks
• Provide appropriate reasoning and solution	3
• Demonstrate *y* = *x* is a solution	2
• Equate the two equations (substitute one fn into the other fn)	1

To find the point of intersection P, solve simultaneously:

$y = 2 - \log_e x$ —— ①

$y = e^{2-x}$ —— ②

Rearranging ①: $2 = y + \log_e x$

Rearranging ②: $2 = x + \log_e y$

Equating $\Rightarrow$ $y + \log_e x = x + \log_e y$

which has $x = y$ as a solution.

$\therefore$ P must satisfy $e^{2-x} = x$ or $e^{2-x} - x = 0$

Alternate solution:

Since $f^{-1}(x)$ is $f(x)$ reflected about $x = y$

and $f(f^{-1}(x)) = x$,

then any point that lies on the intersection of $f(x)$ and $y = x$

must also lie on $f^{-1}(x)$

$\therefore$ Since P is the point of intersection of $f(x)$ and $f^{-1}(x)$, it must satisfy

$e^{2-x} = x \Rightarrow e^{2-x} - x = 0$

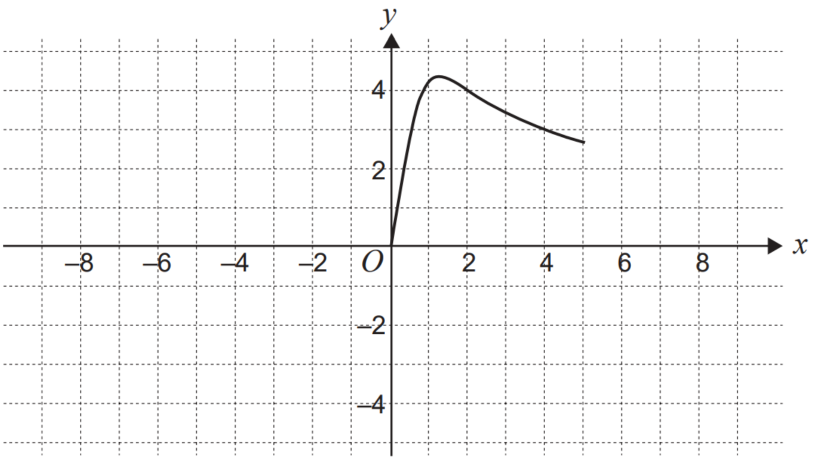
Marker's comments:

Most students struggled to develop a logical reasoning. Students who equated the equations but could not develop this further as well as students who identified that the point of intersection lies on y=x without reasoning achieved 1 mark. Most successful students combined reasoning with support of algebraic manipulation.

End of Question 11

Question 12 (15 marks) Start a new writing booklet.

(a) The top part of a wine glass, while lying on its side, is constructed by rotating the graph of $y = \dfrac{6x}{\sqrt{1+x^3}}$ from $x = 0$ to $x = 5$ about the x-axis as shown below. All lengths are



measured in centimetres.

i) Write down a definite integral which represents the volume, V cm^3, of the glass. **1**

Criteria	Marks
• Provides correct solution	1

$V = \pi \int_a^b y^2 \, dx$

$= \pi \int_0^5 \left(\dfrac{6x}{\sqrt{1+x^3}} \right)^2 dx$

$= \pi \int_0^5 \dfrac{36x^2}{1+x^3} \, dx$

Marker's comments:

Generally answered well by all students.

ii) Use the substitution $u = 1 + x^3$ to write down a definite integral which represents the volume of the glass in terms of u. **2**

Criteria	Marks
• Provides correct solution	2
• Provides steps to lead to the correct solution	1

$u = 1 + x^3$

$du = 3x^2\,dx$

when $x = 0$, $u = 1$

$x = 5$, $u = 126$

$\therefore V = \pi \int_1^{126} \frac{12}{u}\,du$

Marker's comments:

Generally answered well by all students.

iii) Find the value of V correct to the nearest cm^3. **1**

Criteria	Marks
• Provides correct solution	1

$V = 12\pi \left[\ln u\right]_1^{126}$

$= 12\pi(\ln 126 - \ln 1)$

$= 12\pi(\ln 126)$ units^3

Marker's comments:

Generally answered well by all students.

(b) Malek was asked to prove $\cos(\pi - \theta) = -\cos\theta$ for all real θ. He attempts his proof using mathematical induction. Explain why his method would not yield an appropriate proof. **2**

Criteria	Marks
• Provides both the required condition for MI as well as the reason this problem does not meet the condition	2
• Provides either the required condition for mathematical induction **or** the condition not met but not both.	1

To prove by induction, the proof must demonstrate true for all values of θ. Therefore the proof requires θ to be an integer. In this case, since θ is real, the proof is not valid.

Marker's comments:

Generally answered well by all students.

(c) Use mathematical induction to prove that, for every positive integer n, **3**

$13 \times 6^n + 2$ is divisible by 5.

Criteria	Marks
• Provides correct solution	3
• Uses Step 2 towards a solution	2
• Provides Step 1 correctly	1

Marker's comments:

Generally answered well by all students.

Step 1: Prove for $n=1$:

Proof: $13 \times 6^1 + 2 = 80$

which is divisible by 2.

Step 2: Assume true for $n=k$

i.e. $13 \cdot 6^k + 2 = 5N$ where N is an integer

Step 3: Prove true for $n=k+1$

i.e. Prove $13 \cdot 6^{k+1} + 2 = 5M$ where M is an integer

Proof:

$13 \cdot 6^{k+1} + 2 = 13 \times 6^k \times 6 + 2$

$= (5N-2)6 + 2$ (from step 2)

$= 5 \times 6N - 12 + 2$

$= 5(6N+2)$

$= 5M$ (where $M = 6N+2$)

as required

Conclusion:

since true for $n=1$, then by steps 2 and 3 and by induction true for all positive integer n.

(d) i) Differentiate $x\sin^{-1}x + \sqrt{1-x^2}$. **2**

Criteria	Marks
• Provides correct solution	2
• Provides some steps to lead to the simplified solution	1

let $y = x\sin^{-1}x + \sqrt{1-x^2}$

$$y' = x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x + \frac{1}{2}\frac{-2x}{\sqrt{1-x^2}}$$

$$= \sin^{-1}x$$

Marker's comments:

Generally answered well by all students.

ii) Hence evaluate $\int_0^1 2\sin^{-1}x\,dx$. **2**

Criteria	Marks
• Provides correct solution	2
• Uses part (i) appropriately to perform the integration	1

$$\int_0^1 2\sin^{-1}x\,dx = 2\left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^1$$

$$= 2\left[\left(\sin^{-1}1 + 0\right) - \left(0 - 1\right)\right]$$

$$= 2\left(\frac{\pi}{2} + 1\right)$$

$$= \pi + 2$$

Marker's comments:

Generally answered well by all students.

(e) The curve with equation $y = f(x)$ passes through the point $\left(\frac{\pi}{8}, 2\right)$ and has a gradient of -1 at this point. Find the exact gradient of the curve at $x = \frac{\pi}{12}$ given that $y'' = -\sec^2(2x)$. **2**

Criteria	Marks
• Provides correct solution	2
• Provides correct steps to lead to an equation for y'.	1

$$y'' = -\sec^2(2x)$$

$$\Rightarrow y' = -\frac{1}{2}\tan(2x) + c$$

Now at $x = \frac{\pi}{8}$, $y' = -1$

$$\Rightarrow -1 = -\frac{1}{2}\tan\frac{\pi}{4} + c$$

$$\therefore c = \frac{1}{2} - 1$$

$$= -\frac{1}{2}$$

$$\therefore y' = -\frac{1}{2}\tan 2x - \frac{1}{2}$$

$\therefore$ when $x = \frac{\pi}{12}$ $\quad y' = -\frac{1}{2}\tan\frac{\pi}{6} - \frac{1}{2}$

$$\therefore m = -\frac{1}{2\sqrt{3}} - \frac{1}{2}$$

Marker's comments:

Generally answered well by all students.

End of Question 12

Question 13 (15 marks) start a new writing booklet.

(a) Given the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} - 5\underset{\sim}{j}$,

i) Calculate the dot product $\underset{\sim}{a} \cdot \underset{\sim}{b}$. **1**

Criteria	Marks
• Provides correct solution	1

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = (2)(-3) + (3)(-5)$$
$$= -21$$

Marker's comments:

Generally answered well by all students.

ii) Find $4\underset{\sim}{a} - 3\underset{\sim}{b}$ in column vector form. **2**

Criteria	Marks
• Provides correct solution	2
• Sets up the correct step to lead to the correct answer	1

$$4\underset{\sim}{a} - 3\underset{\sim}{b} = 4\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 3\begin{bmatrix} -3 \\ -5 \end{bmatrix}$$
$$= \begin{bmatrix} 17 \\ 27 \end{bmatrix}$$

Marker's comments:

Generally answered well by all students.

(b) Find $\int_0^{\pi} \sin(mx)\cos(nx)\ dx$ where m and n are both positive, even integers and $m \neq n$. **3**

Criteria	Marks
• Provides correct solution	3
• Performs correct integration	2
• Rearranges the initial trig relationship correctly	1

$$\int_0^{\pi} \sin(mx)\cos(nx)\,dx$$

$$= \frac{1}{2}\int_0^{\pi} [\sin(mx+nx) + \sin(mx-nx)]\,dx$$

$$= \frac{1}{2}\int_0^{\pi} [\sin(m+n)x + \sin(m-n)x]\,dx$$

$$= \frac{1}{2}\left[\frac{-\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n}\right]_0^{\pi}$$

Since m and n are even, then $m+n$ and $m-n$ are even $\therefore \cos(m+n)\pi = 1$

& $\cos(m-n)\pi = 1$

$$\therefore I = \frac{1}{2}\left[\left(\frac{-1}{m+n} - \frac{1}{m-n}\right) - \left(\frac{-1}{m+n} - \frac{1}{m-n}\right)\right]$$

$$= 0$$

Marker's comments:

Generally answered well by all students.

(c) A golf ball is hit at a velocity of $55\ \text{ms}^{-1}$ at an angle θ to the horizontal, with an acceleration due to gravity of 9.8ms^{-2} being applied to the ball.

The position vector $s(t)$, from the starting point, of the ball after t seconds is given by

$$s = 55t\cos\theta\underset{\sim}{i} + (55t\sin\theta - 4.9t^2)\underset{\sim}{j}$$

i) Show that the ball travels $\dfrac{3025\sin 2\theta}{9.8}$ metres before hitting the ground. **2**

Criteria	Marks
• Provides correct solution	2
• Correct steps taken to find the time taken for the ball to land.	1

$s(t) = 55t\cos\theta\underset{\sim}{i} + (55t\sin\theta - 4.9t^2)\underset{\sim}{j}$

when the ball hits the ground:

$55t\sin\theta - 4.9t^2 = 0$

$t(55\sin\theta - 4.9t) = 0$

$\Rightarrow t = 0\ ,\quad t = \dfrac{55\sin\theta}{4.9}$ sec

$t = 0$ is at the start of motion

$\therefore$ distance travelled $= 55\left(\dfrac{55\sin\theta}{4.9}\right)\cos\theta$

$= \dfrac{3025 \times 2\sin\theta\cos\theta}{9.8}$

$= \dfrac{3025\sin 2\theta}{9.8}$ m

Marker's comments:

Generally answered well by all students.

ii) To ensure that the ball lands on the green, it must travel between 200 and 250 metres. **2**

What values of θ, correct to the nearest minute, would allow this to happen?

Criteria	Marks
• Provides correct solution	2
• Uses appropriate equation to solve for θ	1

$\dfrac{3025\sin 2\theta}{9.8} = 200$; $\dfrac{3025\sin 2\theta}{9.8} = 250$

$\Rightarrow \sin 2\theta = \dfrac{200 \times 9.8}{3025}$ $\quad \sin 2\theta = \dfrac{250 \times 9.8}{3025}$

$\Rightarrow \theta \doteq 20^\circ\ 12'$ $\quad \theta = 27^\circ\ 3'$

Therefore the angle of projection must be $20^\circ 12' \leq \theta \leq 27^\circ\ 3'$

Marker's comments:

Generally answered well by all students.

iii) The golfer aims accurately and hits the ball directly towards the green. **1**

After 3.4 seconds of flight, at a point 8 metres above the ground, the ball hits a low flying TV drone.

If it had not hit the drone or any other obstacles, would the ball have landed on the green?

Criteria	Marks
• Provides correct solution	1

substituting $t = 3.4$ sec and 8 for vertical displacement into $55t\sin\theta - 4.9t^2$

$\Rightarrow 55(3.4)\sin\theta - 4.9(3.4)^2 = 8$

$\Rightarrow \sin\theta = \dfrac{8 + 4.9(3.4)^2}{55(3.4)}$

$\therefore \theta \doteqdot 20° 13'$

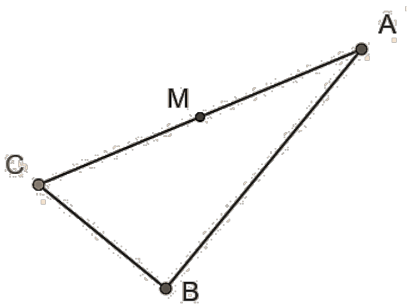
Since θ satisfies the condition in part (ii), the ball would have landed on the green.

Marker's comments:

Generally answered well by all students.

(d) ΔABC is a right-angled with M being the midpoint of the hypotenuse AC, as shown.

Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



i) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

Criteria	Marks
• Provides correct solution	2
•	1

$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB}$
$= \overrightarrow{AM} - \overrightarrow{BM}$
$= \underset{\sim}{a} - \underset{\sim}{b}$

; $\overrightarrow{BC} = \overrightarrow{BM} + \overrightarrow{MC}$
$= \overrightarrow{BM} + \overrightarrow{AM}$
$= \underset{\sim}{a} + \underset{\sim}{b}$

Marker's comments:

Generally answered well by all students.

ii) Prove that M is equidistant from the three vertices of ΔABC **2**

Criteria	Marks
• Provides correct solution	2
• Correct steps leading to the relationship	1

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = (\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b})$$

$$= \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} - \underline{a} \cdot \underline{b} - \underline{b} \cdot \underline{b}$$

$$= |\underline{a}|^2 + 0 + |\underline{b}|^2$$

$$= |\underline{a}|^2 + |\underline{b}|^2$$

Since ΔABC is a right-angled triangle and $\overrightarrow{AB} \perp \overrightarrow{BC}$

then $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0 \Rightarrow |a|^2 - |b|^2 = 0$

$\therefore |a| = |b|$

$\therefore |\overrightarrow{AM}| = |\overrightarrow{BM}| = |\overrightarrow{MC}|$ as required.

Marker's comments:

Generally answered well by all students.

End of Question 13

Question 14 (15 marks) writing booklet.

(a) Use t-formula to solve the equation $\cos x - \sin x = 1$, where $0 \le x \le 2\pi$. **3**

Criteria	Marks
• Provides correct solution	3
• Find correct t values	2
• Correct steps leading to solving for t	1

$t = \tan\frac{x}{2} \Rightarrow \cos x = \frac{1-t^2}{1+t^2}, \sin x = \frac{2t}{1+t^2}$

$\cos x - \sin x = 1$

$\Rightarrow \frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} = 1$

$1 - t^2 - 2t = 1 + t^2$

$\therefore 2t^2 + 2t = 0$

$2t(t+1) = 0$

$\Rightarrow t = 0, \ t = -1$

$\therefore \tan\frac{x}{2} = 0 \Rightarrow x = 0, 2\pi$

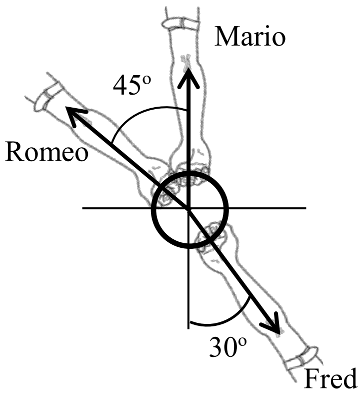
$\tan\frac{x}{2} = -1 \Rightarrow x = \frac{3\pi}{2}$

Marker's comments:

Generally answered well , although the domains were not always taken into account. Most mistakes were due to careless errors.

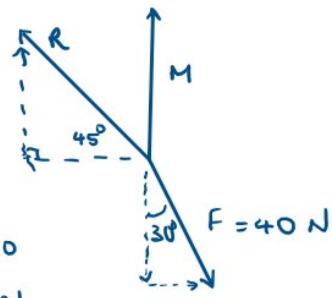
(b) Fred, Mario and Romeo are fighting over a crown. The three of them are holding on to the crown in the formation as shown in the diagram. **3**

If no one manages to pull the crown in their direction (ie, the crown does not move) and Fred is applying a force of 40N, exactly how much force are Mario and Romeo applying to the crown?



Criteria	Marks
• Provides correct solution	3
• Algebraic steps leading to finding one of the forces	2
• Some steps (such as a force diagram) which could lead to the correct solution	1

Let F, M and R be the force applied to the crown by Fred, Mario and Romeo respectively



$F\underset{\sim}{i} = 40 \sin 30^\circ$, $F\underset{\sim}{j} = -40 \cos 30^\circ$

$= 40 \times \frac{1}{2}$ $\qquad = -40 \times \frac{\sqrt{3}}{2}$

$= 20\text{ N}$ $\qquad = -20\sqrt{3}\text{ N}$

Now $R\underset{\sim}{i} = -F\underset{\sim}{i}$ and $R\underset{\sim}{j} = R\underset{\sim}{i}$

$= -20\text{ N}$ $\qquad = 20\text{ N}$

$\therefore\ R = \sqrt{(-20)^2 + (20)^2}$

$= 20\sqrt{2}\text{ N}$

Now $M + R\underset{\sim}{j} = -F\underset{\sim}{j}$

$\Rightarrow\ M = 20\sqrt{3} - 20$

$= 20(\sqrt{3} - 1)\text{ N}$

Marker's comments:

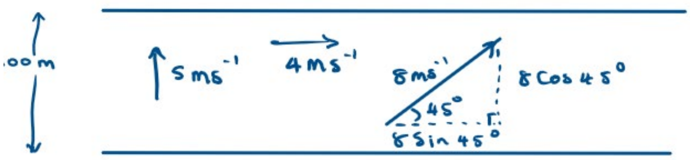
Generally answered well.

(c) Sienna intends to row her boat from the south bank of a river to meet with her friends on the north bank. The river is 100 metres wide. Sienna's rowing speed is 5 metres per second when the water is still. The river is flowing east at a rate of 4 metres per second. Sienna's boat is also being impacted by a wind blowing from the south-west, which pushed the boat at 8 metres per second. She starts rowing across the river by steering the boat such that it is perpendicular to the south bank.

i) Show that the velocity of Sienna's boat can be expressed as the component vector: **2**

$$(4 + 4\sqrt{2})\underset{\sim}{\mathbf{i}} + (5 + 4\sqrt{2})\underset{\sim}{\mathbf{j}}$$

Criteria	Marks
• Provides correct solution	2
•	1



$$\therefore \text{ velocity } = (4 + 8\sin 45°)\underset{\sim}{i} + (5 + 8\cos 45°)\underset{\sim}{j}$$
$$= \left(4 + \frac{8}{\sqrt{2}}\right)\underset{\sim}{i} + \left(5 + \frac{8}{\sqrt{2}}\right)\underset{\sim}{j}$$
$$= (4 + 4\sqrt{2})\underset{\sim}{i} + (5 + 4\sqrt{2})\underset{\sim}{j}$$

Marker's comments:

When direction of the wind was drawn incorrectly the proof was considered incorrect.

As a show question, marks were deducted if not all steps were shown.

ii) Calculate the speed of the boat, correct to 2 decimal places. **1**

Criteria	Marks
• Provides correct solution	1

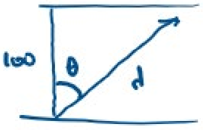
$$\text{speed} = \sqrt{(4+4\sqrt{2})^2 + (5+4\sqrt{2})^2}$$
$$= \sqrt{206.823\ldots}$$
$$\doteqdot 14.38 \text{ ms}^{-1} \quad (2 \text{ dp})$$

Marker's comments:

Generally answered well

iii) Determine the distance rowed from Sienna's starting point to her landing point and how long it will take her to reach the north bank. **2**

Criteria	Marks
• Provides correct solution	2
• Find an expression for the direction of the motion	1



Direction of motion:

$$\tan\theta = \frac{4+4\sqrt{2}}{5+4\sqrt{2}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{4+4\sqrt{2}}{5+4\sqrt{2}}\right)$$

Now $\cos\theta = \frac{100}{d}$

$$\Rightarrow d = \frac{100}{\cos\theta}$$

$$= 100 \div \cos\left(\tan^{-1}\left(\frac{4+4\sqrt{2}}{5+4\sqrt{2}}\right)\right)$$

$$= 134.949\ldots$$

$$\doteqdot 134.95 \text{ m} \quad (2 \text{ dp.})$$

$S = \frac{D}{T} \Rightarrow T = \frac{D}{S}$

$$= \frac{134.95}{14.38}$$

$$\doteqdot 9.38 \text{ seconds} \quad (\text{to } 2 \text{ d.p.})$$

Marker's comments:

Generally answered well. Most mistakes were due to careless errors.

(d) i) By considering the terms of an arithmetic series, show that **1**

$$(1+2+\cdots+n)^2 = \frac{1}{4}n^2(n+1)^2 \ .$$

Criteria	Marks
• Provides correct solution	1

Sum of an arithmetic series is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$1+2+\cdots+n$ is an arith series with $a=1$, $d=1$

$$\Rightarrow S_n = \frac{n}{2}\left[2 + (n-1)\right]$$

$$= \frac{n}{2}(n+1)$$

$$\therefore (1+2+3+\cdots+n)^2 = \frac{n^2}{4}(n+1)^2$$

Marker's comments:

Generally answered well but students were omitting steps in their setting out.

ii) By using the Principle of Mathematical Induction prove that **3**

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2 \text{ for all } n \geq 1$$

Criteria	Marks
• Provides correct solution	3
• Steps leading to proof of step 3	2
• Correct setting out of Step 1 and 2	1

Step 1: Prove for $n=1$

Proof: LHS $=1$; RHS $=1$

$\therefore$ true for $n=1$.

Step 2: Assume true for $n=k$

i.e. $1^3+2^3+\cdots+k^3=(1+2+\cdots+k)^2$

Step 3: Prove true for $n=k+1$

i.e. $1^3+2^3+\cdots+k^3=(1+2+\cdots+(k+1))^2$

Proof:

LHS $= \underbrace{1^3+2^3+\cdots+k^3}+(k+1)^3$

$= (1+2+\cdots+k)^2+(k+1)^3$ (by step 2)

$= \frac{k^2}{4}(k+1)^2+(k+1)^3$ (from (i))

$= (k+1)^2\left(\frac{k^2}{4}+k+1\right)$

$= (k+1)^2\left(\frac{k^2+4k+4}{4}\right)$

$= \frac{1}{4}(k+1)^2(k+2)^2$

$= (1+2+\cdots+k+(k+1))^2$ (from (i))

as required.

Conclusion:

Since true for $n=1$, then by steps 2 and 3, and by induction true for $n \geq 1$.

Marker's comments:

- The setting out was not always in line with the formal setting out.
- When part (i) was used in the process it was not state explicitly.
- Some students provided 2 solutions – in the HSC only the first solution will be marked if one is not crossed out

End of Paper

Fort Street High School

2020 Trial Higher School Certificate Examination

Mathematics Extension 1

Student Number: ______________________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

1.	A ○	B ●	C ○	D ○
2.	A ○	B ●	C ○	D ○
3.	A ●	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ●
5.	A ●	B ○	C ○	D ○
6.	A ○	B ○	C ●	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ●	D ○
9.	A ●	B ○	C ○	D ○
10.	A ○	B ●	C ○	D ○