Student Name: ______________________

FRENSHAM

2015

YEAR 12

TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 Simplify $\dfrac{x^3-1}{x^2-1}\times\dfrac{x^2-4x-5}{4x^2+4x+4}$

(A) $\dfrac{(x-5)}{4}$

(B) $\dfrac{(x-1)}{4}$

(C) $\dfrac{(x+1)}{4}$

(D) $\dfrac{(x^2+x+1)}{4}$

2 Consider $f(x)=\dfrac{6}{x}$ and $g(x)=2x+4$. What are the values of x for which $f(x)=g(x)$?

(A) $x=-1$ or $x=3$

(B) $x=-1$ or $x=-3$

(C) $x=1$ or $x=3$

(D) $x=1$ or $x=-3$

3 Let α and β be roots of the equation $3x^2-7x+12=0$. What is the value of $\alpha+\beta$?

(A) $-\dfrac{7}{3}$

(B) $\dfrac{7}{3}$

(C) 4

(D) 7

4 What is the value of $\int_0^{\frac{\pi}{4}} (\sec^2 x - x)\, dx$?

(A) $1-\dfrac{\pi^2}{32}$

(B) $1-\dfrac{\pi^2}{16}$

(C) $1-\dfrac{\pi}{8}$

(D) $1-\dfrac{\pi}{4}$

5 A circular metal plate of area A cm^2 is being heated. It is given that

$$\frac{dA}{dt}=\frac{\pi t}{32}\ \text{cm}^2/\text{h}.$$

What is the exact area of the plate after 8 hours, if initially the plate had a radius of 6 cm?

(A) π (B) 0.25π

(C) 36π (D) 37π

6 The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term?

(A) ±2

(B) ±4

(C) $\dfrac{5}{16}$

(D) $\dfrac{5}{1024}$

7 What are the coordinates of the focus of the parabola $x^2 = 6y + 2x + 11$?

(A) $\left(-\dfrac{3}{2}, 1\right)$

(B) $\left(-\dfrac{1}{2}, 1\right)$

(C) $\left(1, -\dfrac{3}{2}\right)$

(D) $\left(1, -\dfrac{1}{2}\right)$

8 An infinite geometric series has a first term of 3 and a limiting sum of 1.8. What is the common ratio?

(A) –0.3

(B) –0.6

(C) –1.5

(D) –3.75

9 When simplified fully: $\cos^2\left(\dfrac{\pi}{2}-\theta\right)\cot\theta$ is:

(A) $\cos^2\theta\cot\theta$

(B) $\sin\theta\cos\theta$

(C) $\dfrac{\sin^3\theta}{\cos\theta}$

(D) $\sin^2\theta\cot\theta$

10 Find the value of $\log_5 200 - 3\log_5 2$

(A) 1.4

(B) 2.0

(C) 3.2

(D) 2.5

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

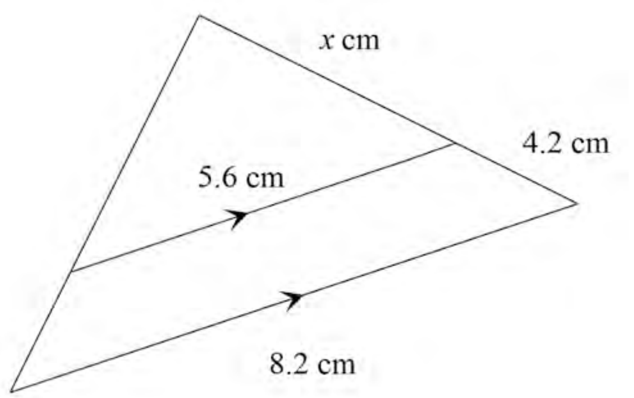
Question 11 (15 marks) **Marks**

(a) Simplify $\dfrac{y}{y^2-4}-\dfrac{2}{y-2}$ **1**

(b) Find $\displaystyle\int \frac{3x}{x^2+1}\,dx$ **2**

(c) Simplify fully: $\dfrac{2}{\sqrt{7}+3}-\dfrac{3\sqrt{7}}{\sqrt{7}-3}$ **2**

(d) Find the value of x (correct to the nearest mm). **2**



(e) Differentiate with respect to x

(i) $\tan 5x$ **1**

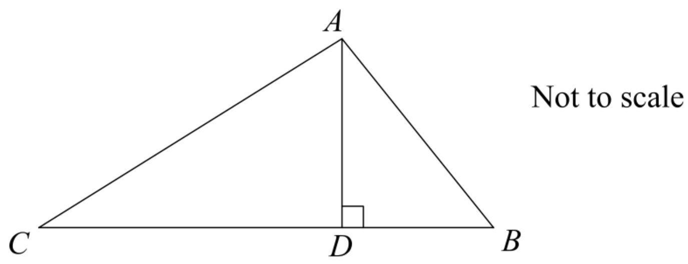
(ii) $\dfrac{\log_e x}{x}$ **1**

(iii) $x\cos x$ **1**

(f) Shade the following regions bounded by the curves: **2**

$y < \sqrt{4-(x-2)^2}$ and $y > \dfrac{x^2}{2}$.

(g) In the triangle *ABC*, $\angle ACB = 30$, $\angle ABC = 50$ and *BC* = 10 cm.
The foot of the perpendicular from *A* to *BC* is *D*.

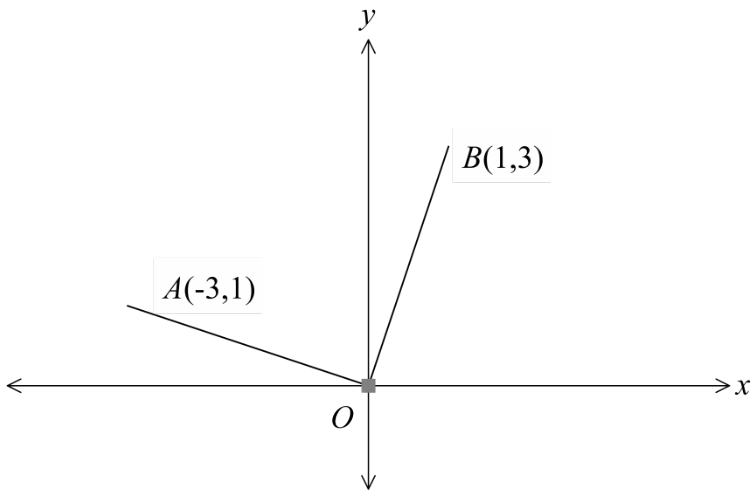


(i) Use the sine rule to find an *expression* for the length of *AB*. **2**

(ii) Hence or otherwise, find the length of *AD*. **1**
Answer correct to two decimal places.

Question 12 (15 marks)

(a) Points $A(-3,1)$ and $B(1,3)$ are on a number plane. Copy the diagram into your writing booklet.



(i) Find the gradient of the line *OA* **1**

(ii) Show that *OA* is perpendicular to *OB* **1**

(iii) *OACB* is a quadrilateral in which *BC* is parallel to *OA*. Show that the equation of *BC* is $x+3y-10=0$ **2**

(iv) The point *C* lies on the line $x=-2$. What are the co-ordinates of the point *C*? **1**

(v) Show that the length of the line *BC* is $\sqrt{10}$ **1**

(vi) Find the area of *OACB* **1**

(b) The table shows the values of a function $f(x)$ for five values of *x*. **2**

x	1	1.5	2	2.5	3
f(x)	4	1.5	2	2.5	8

Use Simpson's Rule with these five values to estimate $\int_1^3 f(x)dx$

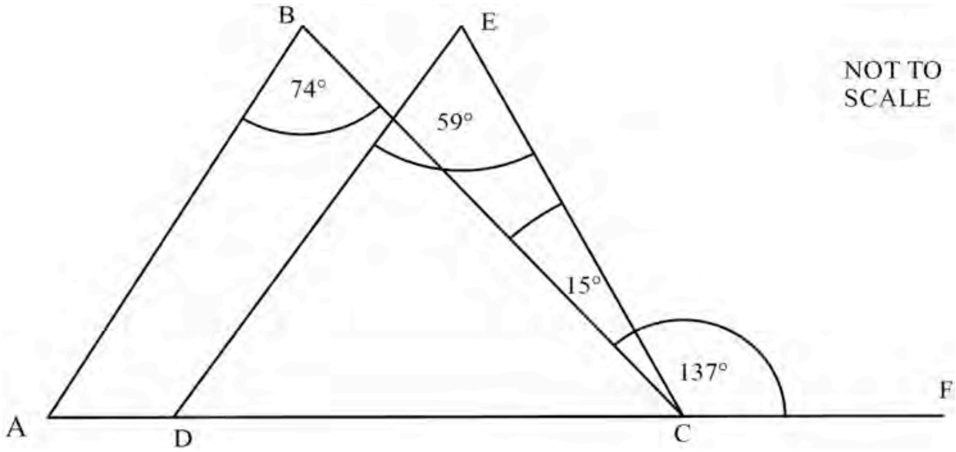
(c) Solve $\sqrt{3}\cos x = \sin x,\ 0 \le x \le 2\pi$ **2**

(d) Find the values of *A*, *B* and *C* if $3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$ **2**

(e) A curve has the equation $y = x\cos x$. Given that $P\left(\dfrac{\pi}{2},0\right)$ is the first point to the right of the origin where the curve crosses the *x* axis, find the equation of the tangent at point *P*. **2**

Question 13 (15 marks)

(a)



In the diagram above AF is a straight line, $\angle ABC = 74$, $\angle DEC = 59$, $\angle BCF = 137$ and $\angle BCE = 15$.

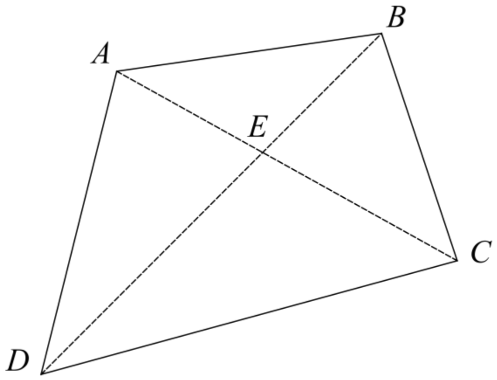
Prove that AB||DE **2**

(b) Jack drops a super bouncy ball from the top of a 56 m building on to a concrete surface below. Its first rebound is 42 m, and each subsequent rebound is three quarters the height of the previous one.

(i) How high will it rise on the fifth rebound? **2**

(ii) How far will it travel in total? **1**

(c) In quadrilateral *ABCD* the diagonals *AC* and *BD* intersect at *E*.
Given $AE = 3$, $EC = 6$, $BE = 4$ and $ED = 8$.



Not to scale

(i) Show that $\Delta ABE \parallel\!| \Delta DEC$ **3**

(ii) What type of quadrilateral is *ABCD*? Geometrically justify your answer. **2**

(d) Find the shortest distance between the point $(0,5)$ and the line $3x - y + 1 = 0$. **2**

(e) The parabola $y = ax^2 + bx + c$ has a vertex at (3, 1) and passes through (0, 0).

(i) Find the other *x*-intercept of the parabola. **1**

(ii) Find *a*, *b* and *c*. **2**

Question 14 (15 marks)

(a) The displacement of a object at time (t) seconds is given by: **3**

$$x = 3e^{-2t} + 10e^{-t} + 4t$$

Find the time(s) the object comes to rest.

(b) For the curve $y = x^3(3 - x)$

(i) Find any stationary points and determine their nature. **3**

(ii) Draw a sketch of the curve showing the stationary points, inflexion points and intercepts on the axes. **3**

(c) Georgina borrows \$650 000 to purchase her first home. She takes out a loan over 30 years, to be repaid in equal monthly instalments. The interest rate is 5.4% per annum reducible, calculated monthly.

(i) Show that the amount, \$$A_n$, owing after the nth repayment is given by the formula: **1**

$$A_n = 650\,000(1.0045)^n - M\left(1 + 1.0045 + 1.0045^2 + \ldots\ldots + 1.0045^{n-1}\right)$$

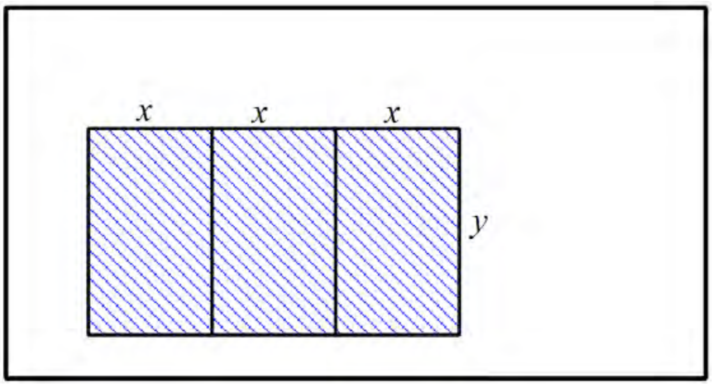
(ii) Find the monthly repayment required to repay the loan in 30 years. **2**

(iii) Georgina wants pay the loan off in less than 30 years. If she can afford to pay \$5 000 per month, how many months will it take her to pay off the home loan? **2**

(iv) How much will Georgina save in interest if she pays \$5 000 per month? **1**

Question 15 (15 marks)

(a) Greg has a one hectare (Ha) block of land. He is going to fence off three identical rectangular plots within his block for his three children. Each plot will measure *x* m by *y* m as shown in the diagram below. He will retain the remainder of the block for himself and his wife. Greg can only afford 300 m of fencing to go around the children's plots.



(i) Show $y = 75 - \dfrac{3x}{2}$. **1**

(ii) Find the value of *x* for which the area will be a maximum. **3**

(iii) Find the maximum area of one of the children's blocks. **1**

(iv) How much of Greg's 1 Ha block is left for him and his wife? **1**

(b) The acceleration, after *t* seconds, of a particle moving in a straight line is given by $\ddot{x} = \dfrac{-14}{(t+4)^3}$.

Initially the particle is located $\dfrac{3}{4}$ metres to the left of the origin and the initial velocity is $\dfrac{7}{16}$ m/s.

(i) Find the velocity *v* and the displacement *x* at any time *t*. **2**

(ii) What is the velocity of the particle when it passes through the origin? **2**

(iii) Sketch a graph of the displacement as a function of time. **2**

(c) Find the value of *n* such that: **3**

$$\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$$

Question 16 (15 marks)

(a) The radiation in a rock after a nuclear accident was 8,000 becquerel (bq). One year later, the radiation in the rock was 7,000 bq. It is known that the radiation in the rock is given by the formula:

$$R = R_0 e^{-kt}.$$

where R_0 and k are constants and t is the time measured in years.

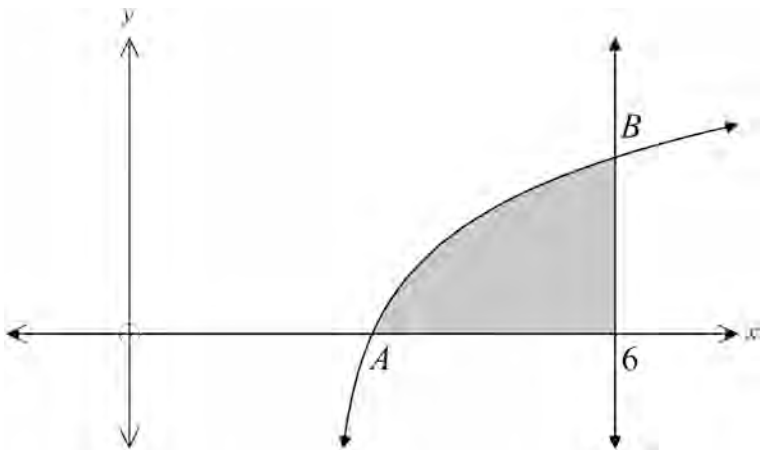
(i) Evaluate the constants R_0 and k. **2**

(ii) What is the radiation of the rock after 10 years? Answer correct to the nearest whole number. **1**

(iii) The region will become safe when the radiation of the rock reaches 50 bq. During which year will the region become safe? **2**

(b) The diagram shows a shaded region which is bounded by the curve $y = \ln(2x - 5)$, the x axis and the line $x = 6$.

The curve $y = \ln(2x - 5)$ intersect the x axis at A and the line $x = 6$ at B.



(i) Show that the coordinates of points A and B are (3, 0) and (6, ln7) respectively. **1**

(ii) Show that if $y = \ln(2x - 5)$, then $x = \dfrac{e^y + 5}{2}$. **1**

(iii) Hence find the exact area of the shaded region. **3**

QUESTION 16 CONTINUES ON THE NEXT PAGE

(c) A triangle *ABC* is right-angled at *C*. *D* is the point on *AB* such that *CD* is perpendicular to *AB*. Let $\angle BAC = \theta$.

(i) Draw a diagram showing this information. **1**

(ii) Given that $8AD + 2BC = 7AB$, show that $8\cos\theta + 2\tan\theta = 7\sec\theta$ **2**

(iii) Find θ **2**

END OF PAPER

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STANDARD INTEGRALS

$\int x^n dx \qquad = \frac{1}{n+1}x^{n+1}, \ n \neq -1; x \neq 0, \text{if } n < 0$

$\int \frac{1}{x} dx \qquad = \ln x, \ x > 0$

$\int e^{ax} dx \qquad = \frac{1}{a}e^{ax}, \ a \neq 0$

$\int \cos ax \, dx \qquad = \frac{1}{a}\sin ax, \ a \neq 0$

$\int \sin ax \, dx \qquad = -\frac{1}{a}\cos ax, \ a \neq 0$

$\int \sec^2 ax \, dx \qquad = \frac{1}{a}\tan ax, \ a \neq 0$

$\int \sec ax \tan ax \, dx \qquad = \frac{1}{a}\sec ax, \ a \neq 0$

$\int \frac{1}{a^2 + x^2} dx \qquad = \frac{1}{a}\tan^{-1}\frac{x}{a}, \ a \neq 0$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx \qquad = \sin^{-1}\frac{x}{a}, \ a > 0, \ -a < x < a$

$\int \frac{1}{\sqrt{x^2 - a^2}} dx \qquad = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$

$\int \frac{1}{\sqrt{x^2 + a^2}} dx \qquad = \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE: $\ln x = \log_e x, \ \ x > 0$

Mathematics Trial 2015

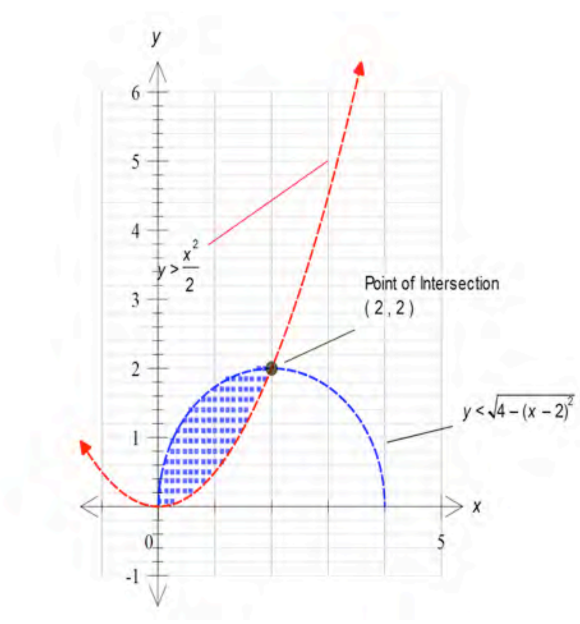
1.	$\frac{x^3-1}{x^2-1}\times\frac{x^2-4x-5}{4x^2+4x+4}=\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}\times\frac{(x+1)(x-5)}{4(x^2+x+1)}$ $=\frac{(x-5)}{4}$	1 Mark: A
2.	$\frac{6}{x}=2x+4$ $x\times\left(\frac{6}{x}\right)=(2x+4)\times x \qquad x\neq 0$ $6=2x^2+4x$ $2x^2+4x-6=0$ $2(x+3)(x-1)=0$ $\therefore\ x=1 \text{ or } x=-3$	1 Mark: D
3.	$\alpha+\beta=-\frac{b}{a}=-\frac{-7}{3}=\frac{7}{3}$	1 Mark: B
4.	$\int_0^{\frac{\pi}{4}}(\sec^2 x-x)dx=\left[\tan x-\frac{x^2}{2}\right]_0^{\frac{\pi}{4}}$ $=\left(\tan\frac{\pi}{4}-\frac{\left(\frac{\pi}{4}\right)^2}{2}\right)-\left(\tan 0-\frac{0^2}{2}\right)$ $=1-\frac{\pi^2}{32}$	1 Mark: A
5.	$A=\pi r^2=\pi\times 6^2=36\pi \text{ cm}^2$ $A=\int\frac{\pi t}{32}dt$ $=\frac{1}{64}\pi t^2+c$ $A=\frac{1}{64}\pi t^2+36\pi$ Hence $=\frac{1}{64}\pi\times 8^2+36\pi=37\pi$	1 Mark: D

6.	$T_3 = ar^2 = 1.5$ and $T_7 = ar^6 = 20$ Divide the two equations $\frac{ar^6}{ar^2} = \frac{20}{1.25}$ $r^4 = 16$ $r = \pm 2$ $T_7 = a \times (\pm 2)^6 = 20$ $a = \frac{20}{64} = \frac{5}{16}$	1 Mark: C
7.	$x^2 = 6y + 2x + 11$ $x^2 - 2x = 6y + 11$ $(x-1)^2 - 1 = 6y + 11$ $(x-1)^2 = 6(y+2)$ $(x-1)^2 = 4 \times \frac{3}{2} \times (y+2)$ Vertex is $(1, -2)$ and focal length is $\frac{3}{2}$. Focus is $\left(1, -\frac{1}{2}\right)$	1 Mark: D
8.	$a = 3$ and $S = 1.8$ $S = \frac{a}{1-r}$ $1.8 = \frac{3}{1-r}$ $1.8 - 1.8r = 3$ $1.8r = -1.2$ $r = -0.6$	1 Mark: B
9.	$\cos^2\left(\frac{\pi}{2} - \theta\right)\cot\theta$ $= \sin^2\theta \cot\theta$ $= \sin^2\theta \times \frac{\cos\theta}{\sin\theta}$ $= \sin\theta\cos\theta$	1 Mark: B

10.	$\log_5 200 - 3\log_5 2$ $= \log_5 200 - \log_5 2^3$ $= \log_5\left(\frac{200}{8}\right)$ $= \log_5 25$ $= 2$	1 Mark: B

Q11

(a)	$\frac{y}{y^2-4} - \frac{2}{y-2} = \frac{y}{(y+2)(y-2)} - \frac{2}{(y-2)}$ $= \frac{y-2(y+2)}{(y+2)(y-2)}$ $= \frac{-y-4}{y^2-4}$	2 Marks: Correct answer. 1 Mark: Finds a common denominator or shows some understanding.	
(b)	$\int \frac{3x}{x^2+1}\,dx$ $= \frac{3}{2}\int \frac{2x}{x^2+1}\,dx$ $= \frac{3}{2}\ln\left(x^2+1\right) + C$	1	1 for correct answer.
(c)	$\frac{2}{\sqrt{7}+3} - \frac{3\sqrt{7}}{\sqrt{7}-3} = \frac{2\sqrt{7}-6-21-9\sqrt{7}}{7-9}$ $= \frac{-7\sqrt{7}-27}{-2}$ $= \frac{7\sqrt{7}+27}{2}$	2	1 (rational denominator) 1 for simplification
(d)	$\frac{x}{x+4.2} = \frac{5.6}{8.2}$ $8.2x = 5.6(x+4.2)$ $8.2x = 5.6x + 23.52$ $2.6x = 23.52$ $x = 9.046$ $x = 9.0$ (nearest mm)	2	1 for correct ratio 1 for solving equation

(e) (i)	$\frac{d}{dx}(\tan 5x) = \sec^2 5x \times \frac{d}{dx}(5x)$ $= 5\sec^2 5x$	1 Mark: Correct answer.	
(ii)	$\frac{d}{dx}\left(\frac{\log_e x}{x}\right) = \frac{x \times \frac{1}{x} - \log_e x \times 1}{x^2}$ $= \frac{1 - \log_e x}{x^2}$	1 Mark: Correct answer.	
(iii)	$\frac{d}{dx}(x\cos x) = -x\sin x + \cos x$	1 Mark: Correct answer.	
(f)		2	1 for correct functions 1 for correct shading of intersection. Point of intersection not required for marks.

(g) (i)	$\angle BAC = 180 \ -30 \ -50 \ = 100$ $\frac{AB}{\sin \angle ACB} = \frac{BC}{\sin \angle BAC}$ $\frac{AB}{\sin 30} = \frac{10}{\sin 100}$ $AB = \frac{10\sin 30}{\sin 100}$	2 Marks: Correct answer. 1 Mark: Finds angle *BAC* or uses the Sine Rule with two correct values.

(ii)	$\sin 50 = \dfrac{AD}{AB}$ $AD = \dfrac{10\sin 30 \ \sin 50}{\sin 100}$ $= 3.8893095... \approx 3.89$ cm	1 Mark: Correct answer.

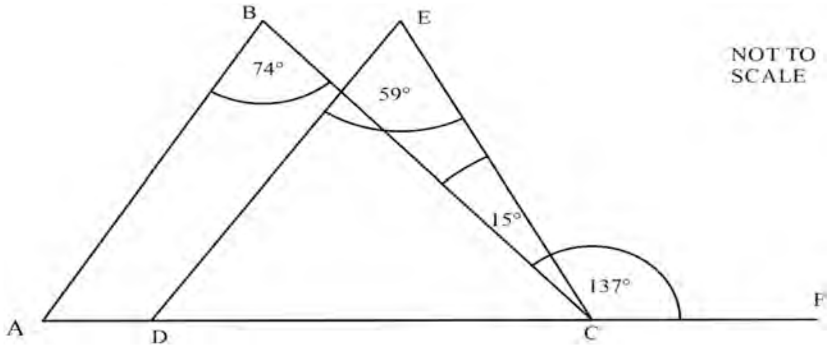
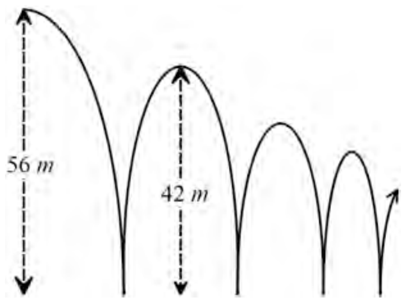
Q12

(a) (i)	Gradient of *OA*: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-1}{0--3} = -\dfrac{1}{3}$	1 Mark: Correct answer.
(ii)	Gradient of *OB*: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-3}{0-1} = 3$ Perpendicular lines occur when $m_1 m_2 = -1$ $m_1 m_2 = -1$ $-\dfrac{1}{3} \times 3 = -1$ True	1 Mark: Correct answer.
(iii)	If *BC* is parallel to *OA* then it has the same gradient or $m = -\dfrac{1}{3}$ $y - y_1 = m(x - x_1)$ $y - 3 = -\dfrac{1}{3}(x-1)$ $3y - 9 = -x + 1$ or $x + 3y - 10 = 0$	2 Marks: Correct answer. 1 Mark: Uses the gradient intercept form with at least 1 correct value.
(iv)	The point *C* lies on the line $x = -2$ Substitute –2 for *x* into $x + 3y - 10 = 0$ $-2 + 3y - 10 = 0$ $3y = 12$ or $y = 4$ Coordinates of *C* are $(-2, 4)$.	1 Mark: Correct answer.
(v)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $BC = \sqrt{(-2-1)^2 + (4-3)^2} = \sqrt{10}$	1 Mark: Correct answer.
(vi)	Quadrilateral *OACD* is a square (Rectangle with all sides equal) $A = s^2 = \left(\sqrt{10}\right)^2 = 10$ square units	1 Mark: Correct answer

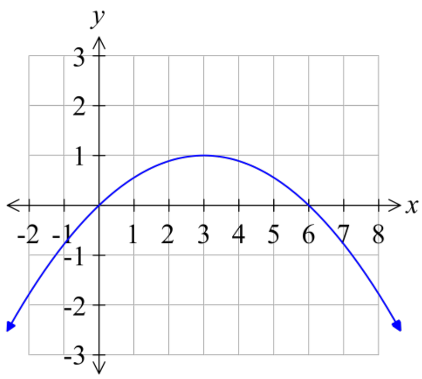
(b)	$\int_1^3 f(x)dx \approx \frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2y_2\right]$ $\approx \frac{0.5}{3}\left[4 + 8 + 4(1.5 + 2.5) + 2\times 2\right] \approx \frac{16}{3}$	2 Marks: Correct answer. 1 Mark: Uses Simpson's rule.	
(c)	$\sqrt{3}\ \cos x = \sin x$ $\sqrt{3} = \frac{\sin x}{\cos x}$ $\tan x = \sqrt{3}$ $x = \frac{\pi}{3}$ tan positive 1st, 3rd quadrant $\therefore\ x = \frac{\pi}{3}, \frac{4\pi}{3}$	2	1 for determining equation 1 for all solutions
(d)	$3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$ *RHS* $= A\left(x^2 + 2x - x - 2\right) + Bx + B + C$ $= Ax^2 + Ax - 2A + Bx + B + C$ $= x^2 A + x(A+B) + (-2A + B + C)$	2	1 for expansion and determining coefficients 1 for solving to find the values of A,

	Equating coefficients $A = 3$ $A + B = 1$ ① $-2A + B + C = 1$ ② From ① $A + B = 1$ $3 + B = 1$ $\therefore \quad B = -2$ From ② $-2A + B + C = 1$ $-6 - 2 + C = 1$ $C = 9$		B C.
(e)	$y = x \cos x$ $u = x \qquad v = \cos x$ $u' = 1 \qquad v' = -\sin x$ $y' = \cos x - x \sin x$ when $x = \frac{\pi}{2}$ $y' = \cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}$ $= -\frac{\pi}{2}$ Equation of tangent $y - 0 = -\frac{\pi}{2}\left(x - \frac{\pi}{2}\right)$ $y = \frac{-\pi x}{2} + \frac{\pi^2}{4}$	2	1 for gradient of tangent 1 for equation of tangent.

Q13

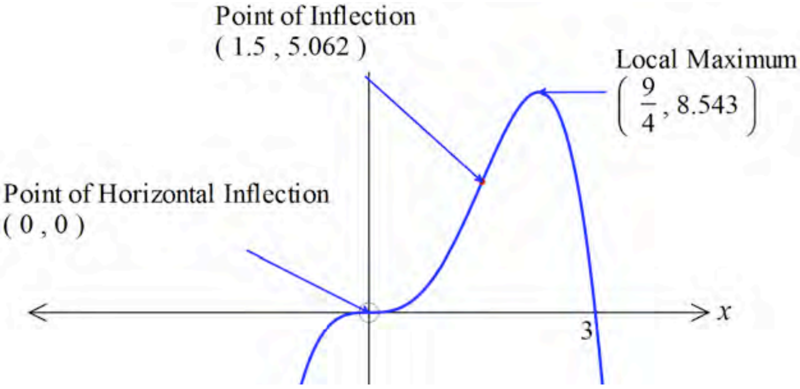
(a)	 Prove AB||DE $\angle BAD + 74° = 137°$ (exterior$\angle$ Δ) $\therefore \angle BAD = 63°$ $\angle EDC + 59° = 137° - 15°$ (exterior$\angle$ Δ) $\angle EDC = 63°$ $\angle BAD = \angle EDC$ (both $= 63°$) $\therefore$ AB | | DE (equal corresponding $\angle$'s)	2	1 for showing angles are 63^o 1 for stating lines parallel with reason
(b)	(i) Taking 42 as the first term as it is the first completed rebound.  $T_n = ar^{n-1}$ For the rise on the 5th bounce $a = 42 \quad r = \frac{3}{4} \quad n = 5$ $T_5 = 42\left(\frac{3}{4}\right)^{5-1}$ $T_5 = 42\left(\frac{3}{4}\right)^{4}$ $\approx 13.29m$ Note if take 56 as first term, need to find the 6^{th} term.	2	1 for determining the series 1 for finding correct term (only 1 mark if found T_5 after using a = 56.)
	(ii)	1	1 for correct answer

	$\|r\| < 1 \quad r = \frac{3}{4}$ Consider one bounce up and down as a term, so $t_1 = 84$ $\therefore s_\infty = \frac{a}{1-r}$ $= \frac{84}{1-\frac{3}{4}}$ $= \frac{84}{\frac{1}{4}}$ $= 336m$ Total distance travelled will be $336 + 56 = 392m$ Alternately take 42 as first term and double result from $S\,\infty$	
(c) (i)	In ΔABE and ΔDEC $\angle AEB = \angle DEC$ (vertically opposite angles are equal) $\frac{AE}{EC} = \frac{BE}{ED}$ ($\frac{AE}{EC} = \frac{3}{6} = \frac{1}{2}$ and $\frac{BE}{ED} = \frac{4}{8} = \frac{1}{2}$) $\Delta ABE \,\|\|\|\, \Delta DEC$ (two pairs of corresponding sides are in proportion and the include angles are equal)	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: One relevant statement
(ii)	$\angle BAE = \angle DCE$ (matching angles in similar triangles are equal) Therefore $\angle BAE$ and $\angle DCE$ are alternate angles and equal. $\therefore AB \quad CD$ (alternate angles are only equal if the lines are parallel) Therefore *ABCD* is a trapezium (one pair of opposite sides parallel)	2 Marks: Correct answer. 1 Mark: Shows some understanding.
(d)	$d = \left\| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\|$ $= \left\| \frac{3 \times 0 - 1 \times 5 + 1}{\sqrt{3^2 + (-1)^2}} \right\|$ $= \left\| \frac{-4}{\sqrt{10}} \right\| = \frac{4}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$ $= \frac{2\sqrt{10}}{5}$	2 Marks: Correct answer. 1 Mark: Uses the perpendicular distance formula with one correct value.
(e (i))	The parabola is symmetrical about the vertex of (3, 1). If the parabola passes through the origin it is concave down and the other *x*-intercept is	1 Mark: Correct answer.

	(6, 0). 	
(ii)	The points (0, 0), (3, 1) and (6, 0) satisfy $y = ax^2 + bx + c$ Sub (0, 0) into $y = ax^2 + bx + c$ results in $c = 0$ Sub (6, 0) into $y = ax^2 + bx + c$ results in $0 = 36a + 6b$ (1) Sub (3, 1) into $y = ax^2 + bx + c$ results in $1 = 9a + 3b$ (2) Multiply eqn (2) by 2 $2 = 18a + 6b$ (3) Eqn (1) – (3) $-2 = 18a$ or $a = -\frac{1}{9}$ Sub $a = -\frac{1}{9}$ into eqn (2) $1 = 9 \times -\frac{1}{9} + 3b$ $3b = 2$ or $b = \frac{2}{3}$ Therefore $a = -\frac{1}{9}$, $b = \frac{2}{3}$ and $c = 0$	2 Marks: Correct answer. 1 Mark: Finds one correct value or shows some understanding.

Q14

(a)	The object comes to rest when $x = 0$ $x = 3e^{-2t} + 10e^{-t} + 4t$ $x = -6e^{-2t} - 10e^{-t} + 4$ $= -2(3e^{-2t} + 5e^{-t} - 2)$ Let $m = e^{-t}$ $-2(3m^2 + 5m - 2) = 0$ $-2(3m - 1)(m + 2) = 0$	3 Marks: Correct answer. 2 Marks: Finds and factorises a a quadratic equation. 1 Mark: Correctly differentiates *x*.

	Hence $3m-1=0$ or $m+2=0$ $m=\frac{1}{3}$ $\quad m=-2$ $e^{-t}=\frac{1}{3}$ $\quad e^{-t}=-2$ (no solution) Therefore $t=-\log_e \frac{1}{3}$ $=\log_e 3$ $\approx 1.0986...$		
(b) (i)	$y = x^3(3-x) = 3x^3 - x^4$ $y' = 9x^2 - 4x^3$ Stationary points where $y' = 0$ $9x^2 - 4x^3 = 0$ $x^2(9-4x) = 0$ $x = 0$ or $x = \frac{9}{4}$ $y = 0$ or $y = 8.543$ $y'' = 18x - 12x^2$ $x = 0, y'' = 0$ so possible inflexion test $x = -1$, $y'' = , -30$ $x = 1$ $y'' = 6$ so change of concavity **so (0,0) is horizontal inflexion** $x = \frac{9}{4}$, $y'' = -20\frac{1}{4}$ ∴ concave down **so $\left(\frac{9}{4}, 8.543\right)$ is a local maximum.**	3	1 for the two x values of stationary pts 1 for second derivative used to determine possible nature. 1 for checking inflexion and naming the two points and their nature.
(ii)	Use second derivative to check for other turning points. $y'' = 18x - 12x^2$ $y'' = 0$ when $18x - 12x^2 = 0$ $6x(3-2x) = 0$ $x = 0$ or $x = \frac{3}{2}$ $x = 0$ is horizontal inflexion found in part i) $x = \frac{3}{2}$, $y = 5\frac{1}{16}$ $x = 2$, $y'' = -12$ $x = 1$, $y'' = 6$ ∴ change of concavity so inflexion at $\left(\frac{3}{2}, 5\frac{1}{16}\right)$ Intercepts on x axis $x^3(3-x) = 0$ (ii) $x = 0$ or $x = 3$  	3	1 for determining other inflexion 1 for general shape of sketch

			1 for showing all features
(c)	(i) $P = \$650000 \quad r = 5.4 \div 100 \div 12 = 0.0045$ $A = P(1+r)^n - M$ $A_1 = 650000(1.0045)^1 - M$ $A_2 = A_1(1.0045)^1 - M$ $A_2 = [650000(1.0045)^1 - M](1.0045) - M$ $A_2 = 650000(1.0045)^2 - M(1.0045) - M$ $A_2 = 650000(1.0045)^2 - M[1 + 1.0045]$ $A_3 = (650000(1.0045)^2 - M[1 + 1.0045])(1.0045) - M$ $A_3 = 650000(1.0045)^3 - M[1 + 1.0045 + 1.0045^2]$ . . . $A_n = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$	1	1 for following pattern to establish required formula
	(ii) Months $= 30 \times 12 = 360$ repayments $A_{360} = 0$ (loan repaid) $A_n = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$ $0 = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$ $M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right] = 650000(1.0045)^n$ $M = \dfrac{650000(1.0045)^n}{1 + 1.0045 + ... + 1.0045^{n-1}}$ The denominator is a geometric series with $a = 1$, $r = 1.0045$ and $n = 360$ $S_n = \dfrac{a(r^n - 1)}{r}$ $S_{360} = \dfrac{1((1.0045)^{360} - 1)}{0.0045}$ $S_{360} = \dfrac{(1.0045)^{360} - 1}{0.0045}$ $\therefore M = \dfrac{(650000(1.0045)^{360}) \times 0.0045}{(1.0045)^{360} - 1}$ $M = \$3649.95$	2	1 for expression for M 1 for substituting into sum of series and finding M (can use rounded answer for S_n)

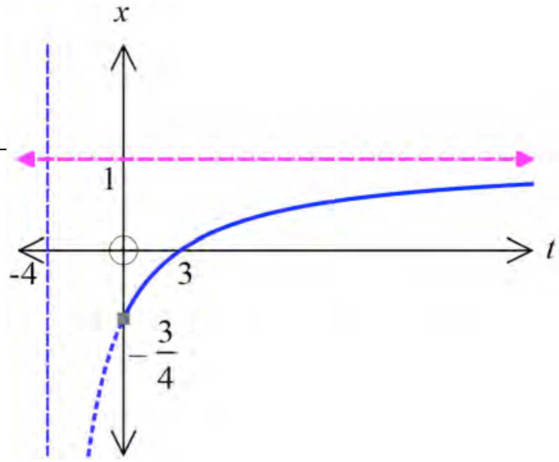
	(iii) $A_n = 650000(1.0045)^n - 5000S_n$ A_n = \$0 paid off $5000S_n = 650000(1.0045)^n$ $5000\left[\dfrac{(1.0045)^n - 1}{0.0045}\right] = 650000(1.0045)^n$ $5000(1.0045)^n - 5000 = 2925(1.0045)^n$ $5000(1.0045)^n - 2925(1.0045)^n = 5000$ $(1.0045)^n[5000 - 2925] = 5000$ $(1.0045)^n = \dfrac{5000}{2075}$ $\ln(1.0045^n) = \ln\left[\dfrac{5000}{2075}\right]$ $n\ln(1.0045) = \ln\left[\dfrac{5000}{2075}\right]$ $n = \dfrac{\ln\left[\dfrac{5000}{2075}\right]}{\ln 1.0045}$ $n = 195.88$ = 196 months	2	1 for using sum to establish equation 1 for solving to find n
	(iv) Total of loan over 30 years 360 × \$3 649.95 = \$1 313 982 Total of loan by paying \$5000/month 196 × \$5 000 = \$980 000 Interest Saving" \$1 313 982 - \$980 000 = \$333 982	1	1 for answer

Q15

(a)	(i) $3x + 3x + 4y = 300$ $4y = 300 - 6x$ $y = 75 - \dfrac{3x}{2}$	1	1 for correct expression

	(ii) $A = 3x \times y$ $A = 3x\left[75 - \frac{3x}{2}\right]$ $A = 225x - \frac{9x^2}{2}$ Maximum Area find A' $A' = 225 - \frac{18x}{2}$ $= 225 - 9x$ $A' = 0$ $0 = 225 - 9x$ $9x = 225$ $x = 25m$ *When* $x = 25m \quad y = 37.5m$ Test maximum point $A'' = -9$ < 0 ∴ Maximum Area ∴ $x = 25m$ will produce the maximum area	3	1 for A' 1 for *x* 1 for test that it is maximum
	(iii) $A = 25 \times 37.5$ $= 937.5m^2$	1	1 for area
	(iv) $3 \times 937.5 = 2812.5m^2$ $1Ha = 10000m^2$ $10000 - 2812.5 = 7187.5m^2$ So Greg and his wife will have 7187.5 m^2 left.	1	1 for answer
(b)	(i) $\ddot{x} = -\frac{14}{(t+4)^3}$ $= -14(t+4)^{-3}$ $\dot{x} = \int -14(t+4)^{-3}\,dt$ $= 7(t+4)^{-2} + c_1$ when $t = 0 \quad v = \frac{7}{16}$ $\frac{7}{16} = 7(0+4)^{-2} + c_1$ $c_1 = 0$	2	1 for velocity

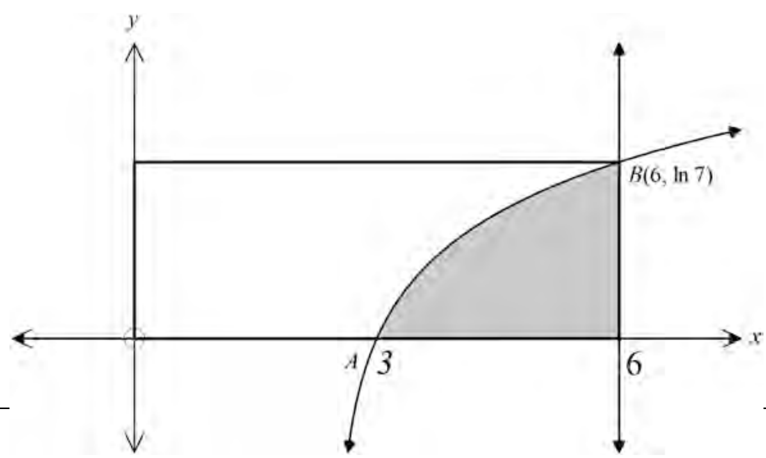
	$\therefore \quad \dot{x} = \dfrac{7}{(t+4)^2}$ $x = \displaystyle\int \frac{7}{(t+4)^2}\,dt$ $= \dfrac{-7}{t+4} + c_2$ when $t = 0$ $x = -\dfrac{3}{4}m$ $-\dfrac{3}{4} = \dfrac{-7}{0+4} + c_2$ $c_2 = 1$ $\therefore \quad x = \dfrac{-7}{t+4} + 1$		1 for displacement
	(ii) When $x = 0$ the particle is at the origin $0 = \dfrac{-7}{t+4} + 1$ $\dfrac{7}{t+4} = 1$ $7 = t + 4$ $t = 3$ When $t = 3$ $\dot{x} = \dfrac{7}{(t+4)^2}$ $= \dfrac{7}{(3+4)^2}$ $= \dfrac{1}{7}$ m/s	2	1 for value of t 1 for velocity
	(iii) $x = 1 - \dfrac{7}{t+4} = \dfrac{t+4-7}{t+4} = \dfrac{t-3}{t+4}$ $t \neq -4$, so $t = -4$ would be a vertical asmptote. $t = 0 \Rightarrow x = -\dfrac{3}{4}$ $x = 0, \Rightarrow \dfrac{t-3}{t+4} = 0 \Rightarrow t = 3$ As $t \to +\infty, \dfrac{t-3}{t+4} \to 1$ from below. e.g. $t = 1000,\ x = \dfrac{1000-3}{1000+4} = \dfrac{997}{1004} \approx 0.993$ So $x = 1$ is an horizontal asymptote.	2	1 for intercepts

			1 for sketch with asymptote
(c)	$\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$ LHS $= \frac{(10^3)^n \times (5^2)^{n+2}}{(2^3)^n}$ $= \frac{(1000)^n \times (5^2)^{n+2}}{(2^3)^n}$ $= \frac{(2^3 \times 5^3)^n \times 5^{2n+4}}{(2^{3n})}$ $= \frac{(2^{3n} \times 5^{3n}) \times (5^2)^{n+2}}{(2^{3n})}$ $= 5^{3n} \times 5^{2n+4}$ $\therefore 5^{5n+4} = 1$ $5^0 = 1$ $\therefore 5n + 4 = 1$ $n = -\frac{4}{5}$	3	1 for expanding the terms 1 for collecting powers of 2 and of 5 1 for solving for n

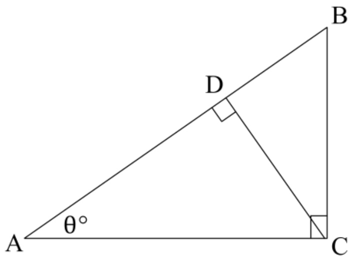
Q16

(a) (i)	Initially $t = 0$ and $R = 8000$ $R = R_0 e^{-kt}$ $8000 = R_0 e^{-k\times 0}$ $R_0 = 8000$ Also $t = 1$ and $R = 7000$ $7000 = 8000e^{-k\times 1}$ $e^{-k} = \frac{7000}{8000}$	2 Marks: Correct answer. 1 Mark: Finds the correct value for R_0 or k.

	$-k = \log_e \frac{7}{8}$ $k = -\log_e \frac{7}{8} = 0.13353139...$		
(ii)	We need to find *R* when $t = 10$ $R = 8000e^{\log_e \frac{7}{8} \times 10}$ $= 2104.604609... \approx 2105 \text{ bq}$	1 Mark: Correct answer.	
(iii)	We need to find *t* when $R = 50$. $50 = 8000e^{-k \times t}$ $e^{-kt} = \frac{1}{160}$ $-kt = \log_e \frac{1}{160}$ $t = -\frac{1}{k}\log_e \frac{1}{160}$ $= \log_e \frac{1}{160} \div \log_e \frac{7}{8}$ $= 38.0073458..$ $\therefore \textit{the } 39\textit{th year}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.	
(b)	(i) A is on the x axis so $y = 0$ $\ln(2x - 5) = 0$ $2x - 5 = e^0 = 1$ $2x = 6$ $x = 3$ A is the point (3, 0) For B, $x = 6$ so $y = \ln(2 \times 6 - 5)$ $y = \ln 7$ B is the point (6, ln 7)	1	1 for use of logs to show both values



	(ii) Given $\ln(2x-5)$ change subject to x. $2x - 5 = e^y$ $2x = e^y + 5$ $x = \dfrac{e^y + 5}{2}$	1	1 for changing the subject.
	(iii) Can't integrate $\ln(2x-5)$ so use the area between the curve and the y axis and subtract from the rectangle shown. Area to y axis $= \displaystyle\int_0^{\ln 7} \frac{e^y + 5}{2}\,dy$ $= \left[\dfrac{e^y + 5y}{2}\right]_0^{\ln 7}$ $= \dfrac{\left(e^{\ln 7} + 5(\ln 7)\right)}{2} - \dfrac{e^0 + 5 \times 0}{2}$ $= \dfrac{(7 + 5(\ln 7) - 1)}{2}$ $= \dfrac{(6 + 5\ln 7)}{2}$ Area Rectangle $= 6 \times \ln 7 = 6\ln 7$ Shaded area $= 6\ln 7 - \dfrac{(6 + 5\ln 7)}{2}$ $= \dfrac{(12\ln 7 - (6 + 5\ln 7))}{2}$ $= \dfrac{7\ln 7 - 6}{2}$ square units		1 for correct integral 1 for finding area to y axis 1 for shaded area

(c) (i)	 $\cos\theta = \dfrac{AD}{AC}$ $\tan\theta = \dfrac{BC}{AC}$ $\cos\theta = \dfrac{AC}{AB}$ $AD = AC\cos\theta$ $BC = AC\tan\theta$ $AB = AC\sec\theta$ Now $8AD + 2BC = 7AB$ $8AC\cos\theta + 2AC\tan\theta = 7AC\sec\theta$ $8\cos\theta + 2\tan\theta = 7\sec\theta$	2 Marks: Correct answer. 1 Mark: Draws the diagram and makes some progress towards the solution.

(ii)	$8\cos\theta + 2\tan\theta = 7\sec\theta$ $8\cos\theta + \dfrac{2\sin\theta}{\cos\theta} = \dfrac{7}{\cos\theta}$ $8\cos^2\theta + 2\sin\theta = 7$ $8(1-\sin^2\theta) + 2\sin\theta = 7$ $8\sin^2\theta - 2\sin\theta - 1 = 0$ $(2\sin\theta - 1)(4\sin\theta + 1) = 0$ $2\sin\theta - 1 = 0$ or $4\sin\theta + 1 = 0$ $\sin\theta = \dfrac{1}{2}$ $\sin\theta = -\dfrac{1}{4}$ $\theta = 30$ $\theta = 165\ 31'$ Now $0 \le \theta \le 90$ as θ is in a right-angled triangle. $\therefore \theta = 30$	2 Marks: Correct answer. 1 Mark: Finds the quadratic equation in $\sin\theta$ or shows some understanding of the problem.