GIRRAWEEN HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1999

MATHEMATICS

3 UNIT (ADDITIONAL)

AND

3/4 UNIT (COMMON)

Time allowed - Two hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on the last page.
- Board-approved calculators may be used.
- Each question attempted is to be returned on a *separate* piece of paper clearly marked Question 1 , Question 2 ; etc. Each piece of paper must show your name.
- You may ask for extra pieces of paper if you need them.

Question 1. (14 marks)

(a) Solve $\cos\theta\sin\theta = \sin\theta$ for $0 \le \theta \le 2\pi$ 2

(b) (i) Find the derivative of $\ln\cos 2x$ 2

(ii) Hence, or otherwise find $\int_0^{\frac{\pi}{8}} \tan 2x\,dx$ 3

(c) Evaluate $\int_0^9 \frac{x^3}{1+x^8}\,dx$ using the substitution $u = x^4$ 4

(d) Find the gradient of the curve $y = \sin^{-1} x$ at $x = \frac{1}{2}$ 3

Question 2. (12 marks)

(a) Find the coefficient of x^4 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^8$ 2

(b) (i) How many different arrangements of BALLOON are possible ? 2

(ii) How many of these arrangements start with L ? 2

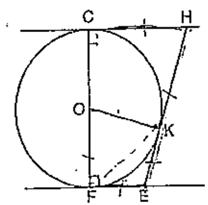
(c) CF is the diameter of the circle, centre O. 6

The lines CH, HE and EF are tangents to the circle.

(i) Copy the figure.

(ii) Prove that CHEF is a trapezium.

(iii) Prove that 2 x Area CHEF = CF.HE



Question 3. (14 marks)

(a) A body moves in Simply Harmonic Motion according to $x = a\cos nt$.
The period of the motion is 8 seconds and the amplitude a is 6 cm.
What is the speed of the body when the displacement x is 3 cm ? **4**

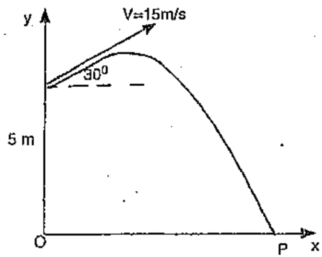
(b) Find the volume when the area enclosed between the cuves $y = x^3$ and $y = x^2$ is rotated about the x axis. **3**

(c) Use the method of mathematical induction to prove that $n^3 + 5n$ is divisible by 3 for all positive integers n. **4**

(d) The surface area of a cube is increasing at a rate of 10 cm^2/s. Find the rate at which the volume of the cube is increasing when the edge of the cube is 12 cm **3**

Question 4 (17 marks)

(a) The diagram shows a 5 metre high water spray set at an angle of 30 degrees to the horizontal. The water leaves the nozzle at 15 metres/second and strikes the ground at a point P.



Given that the displacement equations are:
$x = Vt\cos\theta$ and $y = -5t^2 + Vt\sin\theta + 5$, where V is the velocity of projection and θ is the angle of projection.

(i) Find how long it would take for a water droplet to reach the ground after leaving the nozzle. (Neglect air resistance and $g = 10 m/s^2$) **3**

(ii) Find the horizontal distance OP covered by the spray. **2**

(b) The acceleration of a particle moving in a straight line is given by $\ddot{x} = \frac{-900}{x^3}$
where x metres is the displacement of the particle from the origin after t seconds. Initially the particle is 10 m to the right of the origin moving with a velocity of 3 m/s
Find the equation for the speed of the particle. **4**

(c) Col invests \$P for 3 years at 6% per annum. Interest is compounded monthly and at the end of 3 years his investment was worth \$2991.70
Find the value of P, correct to the nearest dollar. **3**

(d) (i) Show that the normal to the parabola $x^2 = 4ay$ at $T(2at, at^2)$ has the equation $x + ty = 2at + at^3$. **3**

(ii) Hence show there is only one normal to the parabola that passes through its focus. **2**

Question 5. (12 marks)

(a) Find the value of k if $x+1$ is a factor of $x^3 + kx^2 + 3x + 2$. **2**

(b) (i) Use Newton's method of approximation once to find the root of the equation $2x^3 - 6x - 3 = 0$ near $x = 2$. **3**

(ii) Briefly explain why using Newton's method of finding a root near $x = 1$ would not work. **2**

(c) Prove $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$ **2**

(d) Find the general solution to the equation $\cos\left(4x + \frac{\pi}{4}\right) = 0$ **3**

Question 6. (12 marks)

(a) Consider the function $f(x) = 2\cos^{-1}\left(\frac{x}{3}\right)$

(i) Evaluate $f(3)$ **1**

(ii) State the domain and range of $y = f(x)$ **3**

(iii) Sketch $y = f(x)$ 2

(b) Find the exact value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ 2

(c) It is known that 5% of men are colour blind. A random sample of 20 men is chosen. Find the probability, correct to two decimal places, that the sample contains:

(i) at most one colour blind man. 2

(ii) at least two colour blind men. 2

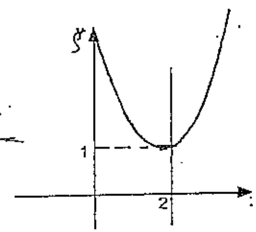
Question 7. (13 marks)

(a) (i) Write out the binomial expansion for $(1+x)^{2n}$ and hence show that

$$\sum_{r=0}^{2n}\binom{2n}{r} = 2^{2n}$$ 2

(ii) By determining the coefficient of x^r on both sides of the identity $(1+x)(1+x)^n = (1+x)^{n+1}$, show that $\binom{n}{r-1}+\binom{n}{r}=\binom{n+1}{r}$ 3

(b) This is a graph of $y=(x-2)^2+1$.



(i) Find the largest positive domain such that the graph of $y=(x-2)^2+1$ defines a function $f(x)$ which has an inverse function. 2

(ii) Find the inverse function $y=f^{-1}(x)$ and state its domain and range. 3

(iii) Sketch the graphs of $y=f(x)$ and $y=f^{-1}(x)$ on the same axes. 3

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Question 1.

(a) $\cos\theta\sin\theta = \sin\theta$

$\sin\theta(\cos\theta - 1) = 0$

$\sin\theta = 0 \quad \cos\theta = 1$

$\therefore \theta = 0, \pi, 2\pi \quad \theta = 0$

$\therefore$ Solution $\theta = 0, \pi, 2\pi$ (2)

(b) i) $y = \ln\cos 2x$

$\frac{dy}{dx} = \frac{1}{\cos 2x}\cdot -\sin 2x \cdot 2$

$= -2\tan 2x$ (2)

(ii) $\int_0^{\frac{\pi}{8}} \tan 2x\,dx = -\frac{1}{2}\int_0^{\frac{\pi}{8}} -2\tan 2x\,dx$

$= -\frac{1}{2}\left[\ln\cos 2x\right]_0^{\frac{\pi}{8}}$

$= -\frac{1}{2}\left[\ln\cos\frac{\pi}{4} - \ln\cos 0\right]$

$= -\frac{1}{2}\left[\ln\frac{1}{\sqrt{2}} - \ln 1\right]$

$= -\frac{1}{2}\ln\frac{1}{\sqrt{2}}$ or $-\frac{1}{2}\ln 2^{-\frac{1}{2}}$

or $\frac{\ln 2}{4}$ (3)

(c) $\int_0^9 \frac{x^3}{1+x^8}\,dx \qquad u = x^4$

$\frac{du}{dx} = 4x^3$

$\frac{du}{4} = x^3\,dx$

when $x=0 \quad u=0$

$x=9 \quad u=6561$

$\therefore \int_0^{6561}\frac{1}{1+u^2}\cdot\frac{du}{4}$

$= \frac{1}{4}\left[\tan^{-1}u\right]_0^{6561}$

$= \frac{1}{4}\left(\tan^{-1}6561 - \tan^{-1}0\right)$

$= .392$

(3)

$=$

(d) $y = \sin^{-1}x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

at $x = \frac{1}{2} \qquad \frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{1}{4}}}$

$= \frac{1}{\sqrt{\frac{3}{4}}}$

$\therefore$ gradient is $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

(3)

Question 2

(a) Term in $x^4 = {}^8C_4\,(3x^2)^4\cdot(-$

$= {}^8C_4\,3^4 x^8\cdot\frac{2^4}{x^4}$

$\therefore T_5 = {}^8C_4\,3^4\cdot 2^4$

$= 90\,720$ (2)

(b) i) Arrangements $= \frac{7!}{2!\,2!}$

$= 1260$

(ii) Start with L $\frac{6!}{2!} = 360$

(2)