



**2011**  
TRIAL  
HIGHER SCHOOL CERTIFICATE

**GIRRAWEEEN HIGH SCHOOL**

# Mathematics Extension 1

## General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

**Total marks – 84**

Attempt Questions 1 – 7

All questions are of equal value

Girraween High School Trial Higher School Certificate 2011

Total marks-84

Attempt all 7 questions

All questions are of equal value

Answer each question on a separate piece of paper clearly marked Question 1, Question 2, etc.

Each piece of paper should show your name.

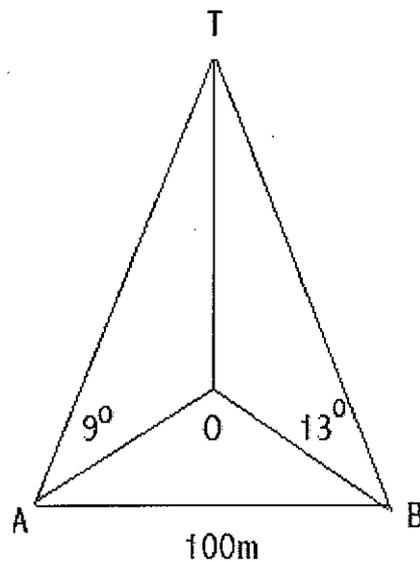
Question 1 (12 Marks) Use a separate piece of paper	Marks
(a) Factorise $x^3 + 64$	1
(b) Find $\lim_{x \rightarrow 0} \frac{\tan 3x}{2x}$	2
(c) Differentiate $y = x \tan^{-1} x$ .	2
(d) Solve the inequality $\frac{3}{x-1} \leq 2$	2
(e) Find the point $P(x, y)$ that divides the interval A (-2, 3) and B (2, -5) externally in the ratio 2 : 1	2
(f) Use the table of standard integrals to find $\int \sec 3x \tan 3x \, dx$	1
(g) Use the substitution $u = x^3 - 1$ to evaluate $\int_0^2 \frac{x^2}{(x^3 - 1)^2} \, dx$	3

Question 2 (12 Marks) Use a separate piece of paper

- (a) The population  $P$  of Happytown has been falling at a rate proportional to the current population. That is  $P$  satisfies the equation  $\frac{dP}{dt} = -kP$
- (i) Show that  $P = P_0 e^{-kt}$  satisfies this equation 1
- (ii) In 1990 the population of Happytown was 25,000 by 2000 it had fallen to 20,000. Find the values of  $P_0$  and  $k$ . 2
- (iii) Find the population of Happytown in 2010. 2
- (iv) At what rate is the population falling in 2010. (to the nearest person) 1
- (b) The polynomial  $P(x)$  has a remainder of  $-2$  when divided by  $(x - 2)$  and has a remainder of  $4$  when divided by  $(x + 1)$ . Find the remainder when  $P(x)$  is divided by  $(x - 2)(x + 1)$ . 2
- (c) A skydiver jumps from an aircraft at 4000m above the ground. The velocity  $V$  m/s at which he falls is given by  $V = 60(1 - e^{-0.16t})$
- (i) Find the acceleration 10 seconds after he jumps. (2 dec pl) 2
- (ii) Find the distance he has fallen in the first 10 seconds. (nearest metre) 2

Question 3 (12 Marks) Use a separate piece of paper.

- (a) A chess club has 7 male and 5 female members. A grade team of 4 players is selected at random.
- (i) In how many ways can the team be chosen. 1
- (ii) What is the probability that the team contains a particular boy and particular girl. 2
- (b) Using the substitution  $t = \tan \frac{\theta}{2}$  or otherwise, show that  $\operatorname{cosec} \theta + \cot \theta = \cot \frac{\theta}{2}$  2
- (c) From a point A due south of a tower OT the angle of elevation is  $9^\circ$   
 100m from A at B that is due east of the tower, the angle of elevation is  $13^\circ$ .  
 As in the diagram below. Find the height of the tower. 3



- (d) The polynomial  $x^3 - 2x^2 - 5x + 7 = 0$  has roots  $\alpha, \beta$  and  $\gamma$
- (i) Find the value of  $\alpha + \beta + \gamma$  1
- (ii) Find the value of  $\alpha\beta\gamma$  1
- (iii) Find the cubic equation with roots  $\alpha + 2, \beta + 2, \gamma + 2$  2

Question 4 (12 Marks) Use a separate piece of paper

- (a) Two points  $P(2ap, ap^2)$  and  $Q(2aq, aq^2)$  lie on the parabola  $x^2 = 4ay$
- (i) Show that the equation of the tangent at  $P$  is  $y = px - ap^2$  1
- (ii) Show that the point of intersection of the tangents from  $P$  and  $Q$  is  $\{a(p+q), apq\}$  2
- (iii) The tangents from  $P$  and  $Q$  intersect at  $45^\circ$ . Show that  $p - q = 1 + pq$  1
- (b) The curve  $y = \tan x + \sec x$  is rotated about the X axis between  $x = \frac{\pi}{6}$  and  $\frac{\pi}{3}$ .
- Find the volume of the solid formed. 3
- (c) The 3<sup>rd</sup> and 5<sup>th</sup> term of a series are 10 and 4 respectively
- (i) If the series is an A.P. find  $a$  the first term and  $d$  the common difference 2
- (ii) Find the least number of terms of the A.P. for the sum to be negative. 1
- (iii) If the series is a G.P. find  $a$  the first term and  $r$  the common ratio/s 2

Question 5 (12 Marks) Use a separate piece of paper

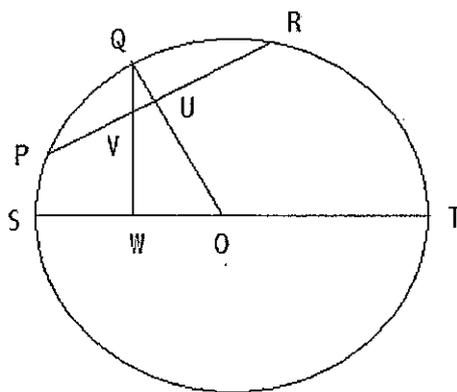
- (a) (i) Express  $2 \sin x - 3 \cos x$  in the form  $R \sin(x - \alpha)$  2
- (ii) Hence or otherwise solve  $2 \sin x - 3 \cos x = 2.5$ ,  $0 \leq x \leq 360^\circ$  (nearest minute) 2
- (b) (i) Two fair six sided dice are rolled. Show that the probability of the total being 7 is  $\frac{1}{6}$  1
- (ii) If the dice are rolled seven times find the probability that the total is 7 exactly twice. 2
- (c) Find the greatest coefficient in the expansion of  $(2x+3)^7$  2
- (leave answer in unexpanded form)
- (d) (i) Use the double angle expansion to express  $\cos 2\theta$  in terms of  $\cos \theta$  1
- (ii) Using part (i) or otherwise find the exact value of  $\cos 36^\circ$  given  $\cos 18^\circ = \frac{\sqrt{5}+1}{4}$  2

Question 6. (12 Marks) Use a separate piece of paper

(a) Use Mathematical Induction to show that for all integers  $n \geq 1$  that

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \dots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!} \quad 3$$

(b) In the circle below  $Q$  is the mid point of the arc  $PR$ . The radius  $OQ$  meets the chord  $PR$  at  $U$ . From  $Q$  a perpendicular is dropped to  $W$  on the diameter  $ST$  meeting  $PR$  at  $V$ .



Prove that  $OUVW$  is a cyclic quadrilateral.

3

(c) For the function defined by  $f(x) = \frac{2x^2}{x^2 + 1}$

- (i) Show that  $f(x)$  is an even function 1
- (ii) Show that  $f(x)$  is monotonic increasing for  $x \geq 0$  2
- (iii) Find the inverse function  $f^{-1}(x)$   $x \geq 0$  2
- (iv) State the domain and range of  $f^{-1}(x)$  1

Question 7 (12 Marks) Use a separate piece of paper

(a) The velocity  $v$  m/s of a particle moving along the  $x$ -axis is given by  $v^2 = 24 + 8x - 2x^2$

where  $x$  is the displacement from the origin.

- |       |   |   |
|-------|---|---|
| (i)   | Show that the motion is Simple Harmonic | 2 |
| (ii)  | Find the centre of the motion           | 1 |
| (iii) | Find the amplitude of the motion.       | 1 |
| (iv)  | Find the period of the motion.          | 1 |
| (v)   | Find the greatest velocity.             | 1 |

(b) A cricketer at the centre of an oval strikes a cricket ball from the ground level producing a velocity of 30m/s. 60 meters from the batsman and in the same plane as the motion of the ball is a fence 1.2m high. The path of the ball is given by the parametric equations

$$x = vt \cos \theta \text{ and } y = vt \sin \theta - \frac{1}{2}gt^2 \text{ where } g = 10ms^{-2} \text{ (there is no need to prove this)}$$

(i) Show that the Cartesian equation of the path of the ball is given by

$$y = (\tan \theta)x - \frac{\sec^2 \theta}{180}x^2 \quad 2$$

(ii) Show that the angle of projection can be found by solving

$$20 \tan^2 \theta - 60 \tan \theta + 21.2 = 0 \quad 2$$

(iii) Find the range of angles for which the ball will clear the fence. 1

(iv) Find the maximum distance the ball can land beyond the fence. 1

END OF PAPER

H.S.C TRIAL EXTENSION 1

Question 1

a)  $x^3 + 64 = (x+4)(x^2 - 4x + 16)$  (1)

b)  $\lim_{x \rightarrow 0} \frac{\tan 3x}{2x} = \lim_{x \rightarrow 0} \frac{\tan 3x}{3x} \cdot \frac{3}{2}$   
 $= 1 \cdot \frac{3}{2}$   
 $= \frac{3}{2}$  (1)

c)  $y = x \tan^{-1} x$   
 let  $u = x$   $v = \tan^{-1} x$   
 $\frac{du}{dx} = 1$   $\frac{dv}{dx} = \frac{1}{1+x^2}$   
 $\frac{d(u \cdot v)}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$   
 $= \tan^{-1} x + \frac{x}{1+x^2}$  (2)

d)  $\frac{3}{x-1} \leq 2$   
 $3(x-1) \leq 2(x-1)^2$   
 $0 \leq 2(x-1)^2 - 3(x-1)$   
 $0 \leq (x-1) \{ 2x-2-3 \}$   
 $0 \leq (x-1)(2x-5)$   
 $x < 1$   $x > \frac{5}{2}$  (2)

e) A(-2, 3) B(2, -5)  
 $2: -1$   
 $(\frac{2 \times 2 + (-2) \times (-1)}{2-1}, \frac{-5 \times 2 + 3 \times (-1)}{2-1})$   
 $(6, -13)$  (2)

(f)  $\int \sec 3x \tan 3x dx$   
 $= \frac{1}{3} \sec 3x + C$  (1)

(g)  $I = \int_0^2 \frac{x^2}{(x^3+1)^2} dx$   
 let  $u = x^3 + 1$   
 $\frac{du}{dx} = 3x^2$   
 $du = 3x^2 dx$   
 when  $x = 0$   $u = 1$   
 $x = 2$   $u = 9$

$= \frac{1}{3} \int_1^9 \frac{3x^2 dx}{(x^3+1)^2}$   
 $= \frac{1}{3} \int_1^9 \frac{du}{u^2}$  (3)  
 $= \frac{1}{3} \left[ -\frac{1}{u} \right]_1^9$   
 $= \frac{1}{3} \left[ -\frac{1}{9} + \frac{1}{1} \right]$   
 $= \frac{8}{27}$

Question 2

(a) (i)  $P = P_0 e^{-kt}$   
 $\frac{dP}{dt} = -k(P_0 e^{-kt})$   
 $= -kP$  (1)

(ii)  $t = 0$  (1990)  $P = 25,000$   
 $25000 = P_0 e^{-k(0)}$   
 $P_0 = 25000$  (1)  
 $t = 10$  (2000)  $P = 20,000$   
 $20000 = 25000 e^{-10k}$   
 $0.8 = e^{-10k}$   
 $\ln 0.8 = -10k \ln e$   
 $k = 0.022314$  (1)

(iii)  $t = 20$  (2010)  
 $P = 25000 e^{-20(0.022314)}$   
 $P = 16000$  (2)

(iv)  $\frac{dP}{dt} = -kP$  (1)  
 $= -0.022314 \times 16000$   
 $= -357 \text{ people/year}$

(b)  $P(x) = Q(x)(x-2)(x+1) + ax + b$   
 $P(2) = -2$   
 $-2 = Q(2)(0)(3) + 2a + b$   
 $-2 = 2a + b$  (α)  
 $P(-1) = 4$   
 $4 = Q(-1)(-3)(0) - a + b$   
 $4 = -a + b$  (β)  
 $(\alpha) - (\beta)$   
 $-6 = 3a$   $a = -2$

$\therefore b = 2$

$\therefore \text{Remainder} = -2x + 2$  (2)

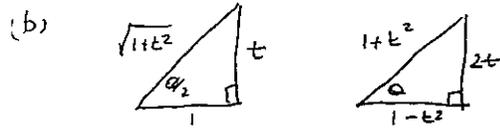
(c) (i)  $V = 60(1 - e^{-0.16t})$   
 $a = \frac{dV}{dt} = 60(0.16 e^{-0.16t})$   
 $a_{(t=10)} = 60(0.16 e^{-1.6})$   
 $= 1.9382 \text{ m/s}$  (2)  
 $= 1.94 \text{ m/s}$  2 D.P.

(ii)  $x = \int v dt$   
 $x = 60 \int_0^{10} (1 - e^{-0.16t}) dt$   
 $= 60 \left[ t + \frac{e^{-0.16t}}{0.16} \right]_0^{10}$   
 $= 60 \left[ (10 + 1.26185) - (0 + 6.25) \right]$   
 $= 300.711 \text{ m}$   
 $= 301 \text{ m}$  (2)

### Question 3.

(a) (i)  ${}^{12}C_4 = 495$  ways (1)

(ii)  $P(B+G+***)) = \frac{{}^{10}C_2}{{}^{12}C_4}$   
 $= \frac{45}{495}$   
 $= \frac{1}{11}$  (2)



$\therefore \operatorname{cosec} \theta + \cot \theta = \frac{1+t^2}{2t} + \frac{1-t^2}{2t}$   
 $= \frac{2}{2t}$   
 $= \frac{1}{t}$   
 $= \frac{1}{\tan \theta/2}$   
 $= \cot \theta/2$  (2)

(c)  $\tan \theta = \frac{h}{AO}$      $\tan 13 = \frac{h}{OB}$

$AO = \frac{h}{\tan \theta}$      $OB = \frac{h}{\tan 13}$

BY PYTHAGORAS IN  $\triangle AOB$

$AO^2 + OB^2 = AB^2$

$\frac{h^2}{\tan^2 \theta} + \frac{h^2}{\tan^2 13} = 10000$

$h^2 \left( \frac{1}{\tan^2 \theta} + \frac{1}{\tan^2 13} \right) = 10000$

$h^2 = \frac{10000}{58.6251414}$

$h^2 = 170.57528$

$h = 13.06$  m. (3)

(d)  $x^3 - 2x^2 - 5x + 7 = 0$

$\alpha, \beta, \gamma$

(i)  $\alpha + \beta + \gamma = -\frac{b}{a} = 2$  (1)

(ii)  $\alpha\beta\gamma = -\frac{d}{a} = -7$  (1)

(iii) WE REQUIRE  $Y = X + 2$

$\therefore X = Y - 2$

$(Y-2)^3 - 2(Y-2)^2 - 5(Y-2) + 7 = 0$

$Y^3 - 6Y^2 + 12Y - 8 - 2Y^2 + 8Y - 8 - 5Y + 7 = 0$

$Y^3 - 8Y^2 + 15Y - 8 = 0$  (2)

CHECK WHEN USING SUM OF ROOT =  $-\frac{b}{a} = 8$ .

WHICH IS EQUAL TO  $\alpha + \beta + \gamma + 6$ .

### Question 4.

(a) (i)  $x = 2ap$      $y = ap^2$

$\frac{dx}{dp} = 2a$      $\frac{dy}{dp} = 2ap$

$\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = p$

EQN

$Y - ap^2 = P(X - 2ap)$

$Y - ap^2 = pX - 2ap^2$

$Y = pX - ap^2$  (1)

(ii) EQN OF TANG FROM Q

$Y = qX - aq^2$

$\therefore pX - ap^2 = qX - aq^2$

$(p - q)X = a(p^2 - q^2)$

$X = \frac{a(p - q)(p + q)}{(p - q)}$

$X = a(p + q)$

$\therefore Y = qa(p + q) - aq^2$

$Y = apq + aq^2 - aq^2$

$Y = apq$

$P_t (a(p + q), apq)$

(iii) grad from P, p  
grad from Q, q

$\therefore \tan \theta = \frac{p - q}{1 + pq}$

But  $\theta = 45$ ,  $\tan \theta = 1$

$1 = \frac{p - q}{1 + pq}$

$1 + pq = p - q$  (1)

(b)  $V = \pi \int_{\pi/6}^{\pi/3} (\tan x + \sec x)^2 dx$

$= \pi \int_{\pi/6}^{\pi/3} \tan^2 x + 2 \tan x \sec x + \sec^2 x dx$

$= \pi \int_{\pi/6}^{\pi/3} 2 \tan x \sec x + 2 \sec^2 x - \tan^2 x dx$

$= \pi [2 \sec x + 2 \tan x - x]$

$= \pi \left[ (4 + 2\sqrt{3} - \pi/3) - \left( \frac{4}{\sqrt{3}} + \frac{2}{3} - \pi/6 \right) \right]$

$= \pi \left[ 4 - \pi/6 \right] u^3$  (3)

(c) (i)  $T_3 = 10 = a + 2d$  (A)

$T_5 = 4 = a + 4d$  (B)

(B) - (A)  $2d = -6$      $d = -3$

$\therefore 10 = a - 6$      $a = 16$

(ii)  $S_n = \frac{n}{2} (2a + (n-1)d)$

$0 = \frac{n}{2} (32 + (n-1)(-3))$

$0 = 16n - \frac{3n^2}{2} + \frac{3n}{2}$

Question 4 Cont.

$$-3n^2 - 35n = 0$$

$$n(3n - 35) = 0$$

$$n = 0 \text{ REJECT}$$

OR  $n = 35/3 \therefore n = 12^{th} \text{ term}$

$$(iii) T_3 = ar^2 = 10 \quad (A)$$

$$T_5 = ar^4 = 4 \quad (B)$$

B/A  $r^2 = 2/5$

$$r = \pm \sqrt{2/5}$$

$$T_3 = a \left(\frac{2}{5}\right)^2 = 10 \quad (2)$$

$$a = 25$$

Question 5.

a)  $R \sin(x - \alpha) = R(\sin x \cos \alpha - \cos x \sin \alpha)$

$$R \sin x \cos \alpha - R \cos x \sin \alpha = 2 \sin x - 3 \cos x$$

equating  $\sin x, \cos x$

$$R \sin x \cos \alpha = 2 \sin x \quad (A)$$

$$R \cos \alpha = 2$$

$$R \cos x \sin \alpha = 3 \cos x \quad (B)$$

$$R \sin \alpha = 3$$

$$\tan \alpha = 1.5$$

$$\alpha = 56.31^\circ$$

$$R^2 (\sin^2 \alpha + \cos^2 \alpha) = 2^2 + 3^2$$

$$R = \sqrt{13} \quad (2)$$

$$2 \sin x - 3 \cos x = 2.5$$

$$\sqrt{13} \sin(x - 56.31) = 2.5$$

$$\sin(x - 56.31) = \frac{2.5}{\sqrt{13}}$$

$$x - 56.31 = 43.8979, 136.1021$$

$$x = 100.2078, 192.4121 \quad (2)$$

$$x = 100^\circ 12', 192^\circ 25'$$

(b) (i)  $n(7) = 6$  (WITH TWO DICE)

$$n(5) = 36 \text{ (36 OUTCOMES)}$$

$$\therefore P(7) = \frac{6}{36} = \frac{1}{6} \quad (1)$$

$$(ii) \binom{7}{7} = \binom{7}{0} \left(\frac{5}{6}\right)^7 + \binom{7}{1} \left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right)^1 + \binom{7}{2} \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)^2$$

$$= P(\text{EXACTLY 2 '7's}) = 21 \cdot \frac{5^5}{6^7}$$

$$P(7's) = 0.2344 \quad (2)$$

(c) Term  $n = \binom{7}{n-1} 2^{8-n} 3^{n-1}$

$$\text{Term } n+1 = \binom{7}{n} 2^{7-n} 3^n$$

Require  $\frac{T_{n+1}}{T_n} \geq 1$

$$\frac{7!}{(7-n)! n!} 2^{7-n} 3^n \geq 1$$

$$\frac{7!}{(8-n)! (n-1)!} 2^{8-n} 3^{n-1}$$

$$\frac{(8-n) \cdot 3}{n} \geq 1$$

$$24 - 3n \geq 2n$$

$$24 \geq 5n$$

$$n \leq 4.8$$

Let  $n = 4$

$\therefore n+1^{th}$  term is the 5<sup>th</sup> term

$$= \binom{7}{4} 2^3 3^4 \quad (2)$$

(d) (i)  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$   
 $= 2 \cos^2 \theta - 1$

$$\therefore (ii) \cos 36 = 2(\cos 18)^2 - 1$$

$$= 2 \left(\frac{\sqrt{5}+1}{4}\right)^2 - 1$$

$$= \frac{6+2\sqrt{5}}{8} - 1$$

$$= \frac{6+2\sqrt{5}-8}{8}$$

$$= \frac{2\sqrt{5}-1}{8}$$

$$= \frac{\sqrt{5}-1}{4} \quad (2)$$

Question 6

$$(a) \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} \dots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

S.1 Prove true for  $n=1$

$$\text{LHS} = \frac{1}{2!} = \frac{1}{2}$$

$$\text{RHS} = 1 - \frac{1}{2!} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{LHS} = \text{RHS} \text{ TRUE FOR } n=1$$



S.2. Assume true for  $n=k$

$$\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$$

PROVE TRUE FOR  $n=k+1$

$$\text{RHS} = 1 - \frac{1}{(k+2)!}$$

$$\text{LHS} \frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k}{(k+1)!} + \frac{k+1}{(k+2)!}$$

$$= 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!}$$

$$= 1 - \frac{k+2}{(k+2)!} + \frac{k+1}{(k+2)!}$$

$$= 1 - \frac{1}{(k+2)!} = \text{RHS} \quad (3)$$

S.3. Step 1 implies true for  $n=1$  Step 2 implies if true for  $n=1$  true for  $n=2, 3, 4, \dots$

$\therefore$  By the Principle of Mathematical Induction true for all  $n \geq 1$ .

(b) MUST PROVE  $\triangle OPQ \equiv \triangle ORQ$

CONNECT O TO P and O TO R.

IN  $\triangle OPQ, \triangle ORQ$

$$OP = OR \text{ (RADII)}$$

$$\angle POQ = \angle ROQ \text{ (STAND ON EQUAL ARCS)}$$

OQ IS COMMON

$$\therefore \triangle OPQ \equiv \triangle ORQ \text{ (SAS)}$$

$$\therefore PQ = RQ \text{ (MATCHING SIDES IN } \equiv \text{ } \Delta\text{'s)}$$

$\therefore$  OQ BISECTS CHORD PR

$\therefore$  OQ  $\perp$  PR

IN: QUADRILATERAL OUVW

$$\angle OUV = 90^\circ \text{ (above)}$$

$$\angle VWO = 90^\circ \text{ (data)} \quad (3)$$

\(\therefore\) OUVW IS CYCLIC QUAD

OPPOSITE ANGLES SUPPLEMENTARY.

$$(c) (i) f(x) = \frac{2x^2}{x^2+1}$$

$$f(a) = \frac{2a^2}{a^2+1}$$

$$f(-a) = \frac{2(-a)^2}{(-a)^2+1} = \frac{2a^2}{a^2+1}$$

$$f(a) = f(-a) \therefore \text{EVEN} \quad (1)$$

$$(ii) f'(x) = \frac{(x^2+1)(4x) - (2x^2)2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

$$f'(x) \geq 0 \quad x \geq 0 \quad (2)$$

\(\therefore\) MONOTONIC INCREASING

$$(iii) \text{ let } y = \frac{2x^2}{x^2+1}$$

$$\therefore \text{ INVERSE } x = \frac{2y^2}{y^2+1}$$

$$y^2x + x = 2y^2$$

$$x = 2y^2 - y^2x$$

$$y^2(2-x) = x$$

$$y = \sqrt{\frac{x}{2-x}} \quad (2)$$

$$(iv) \text{ Domain } 0 \leq x < 2$$

$$\text{Range } y \geq 0 \quad (1)$$

Question 7

$$(a) v^2 = 24 + 8x - 2x^2$$

$$(i) a = \frac{d}{dt}v^2 = \frac{d}{dt}(24 + 8x - 2x^2) = 4 - 2x$$

$$a = -2(x-2)$$

$$\therefore \ddot{x} = -x^2(x-2)$$

\(\therefore\) SIMPLE HARMONIC \quad (2)

$$(ii) x = 2 \quad (1)$$

$$(iii) \text{ FOR MAX AMPLITUDE } v=0$$

$$0 = 24 + 8x - 2x^2$$

$$0 = x^2 - 4x - 12$$

$$0 = (x-6)(x+2)$$

$$x = 6 \quad x = -2$$

$$\text{AMP} = 4. \quad (1)$$

$$(iv) \text{ PERIOD } T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{\sqrt{2}}$$

$$= \sqrt{2}\pi \text{ sec} \quad (1)$$

$$(v) \text{ MAX VEL } x = 2$$

$$v^2 = 24 + 8(2) - 2(2)^2$$

$$v^2 = 32$$

$$v = 4\sqrt{2} \text{ m/s} \quad (1)$$

$$(b) (i) x = vt \cos \theta \quad 1.$$

$$y = vt \sin \theta - \frac{1}{2}gt^2 \quad 2.$$

$$\text{From 1. } t = \frac{x}{v \cos \theta}$$

$$\text{SUBST 2. } y = \frac{v x \sin \theta}{v \cos \theta} - \frac{g}{2} \frac{x^2}{v^2 \cos^2 \theta}$$

$$y = \tan \theta x - \frac{g}{2} \frac{x^2 \sec^2 \theta}{v^2}$$

$$y = (\tan \theta)x - \frac{\sec^2 \theta}{180} x^2 \quad (2)$$

$$(ii) \text{ when } x = 60 \quad y = 1.2$$

$$1.2 = 60 \tan \theta - \frac{(\tan^2 \theta + 1) \cdot 3600}{180}$$

$$1.2 = 60 \tan \theta - 20 \tan^2 \theta - 20$$

$$20 \tan^2 \theta - 60 \tan \theta + 21.2 = 0 \quad (2)$$

$$(iii) \tan \theta = \frac{60 \pm \sqrt{3600 - 4(20)(21.2)}}{40}$$

$$\tan \theta = \frac{60 \pm \sqrt{1904}}{40}$$

$$22^\circ 15' < \theta < 68^\circ 54' \quad (1)$$

$$(iv) x_{\text{max}} = \frac{v^2}{g}$$

$$= \frac{360^2}{10}$$

$$= 90 \text{ m}$$

\(\therefore\) 30 metres beyond fence.

END OF SOLUTIONS

$$x_{\text{max}} \text{ when } \theta = 45$$

$$R = v^2 \frac{g}{g} \cdot 2\theta \quad *$$