2013

TRIAL

HIGHER SCHOOL CERTIFICATE

EXAMINATION

GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided
- Show all necessary working in Questions 11-14

Total marks - 70

Section 1 pages 2-3

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section 2 pages 4 - 8

- Attempt Questions 11 - 14
- Allow about 1 hour 45 minutes for this section

SECTION 1

Multiple Choice (10 marks) Circle the correct answer on this question paper.

1. Which expression is a correct factorisation of $y^3 - 125$?

   (A) $(y-5)(y^2+5y+25)$ (C) $(y-5)(y^2-5y+25)$

   (B) $(y+5)(y^2+5y+25)$ (D) $(y+5)(y^2-5y+25)$

2. The point P divides the interval from $A(-5,6)$ to $B(-2,3)$ externally in the ratio 3 : 2.

   What is the $y-$coordinate of P?

   (A) -4 (B) -3 (C) 3 (D) 4

3. A polynomial equation has roots α, β and γ where $\alpha+\beta+\gamma=-2$, $\alpha\beta+\alpha\gamma+\beta\gamma=3$ and $\alpha\beta\gamma=-4$. Which polynomial equation has the roots α, β and γ?

   (A) $x^3+2x^2+3x-4=0$ (C) $x^3-2x^2+3x-4=0$

   (B) $x^3-2x^2+3x+4=0$ (D) $x^3+2x^2+3x+4=0$

4. $\dfrac{\sin x}{1-\cos x}$ when expressed in terms of half-angle is equal to

   (A) $\tan\dfrac{x}{2}$ (B) $-\tan\dfrac{x}{2}$ (C) $\cot\dfrac{x}{2}$ (D) $-\cot\dfrac{x}{2}$

5. Find the equation of the normal to the parabola $x=6t, y=3t^2$ at the point where $t=-2$.

   (A) $x-3y+24=0$ (C) $2x+y+12=0$

   (B) $x-2y+36=0$ (D) $2x-y-12=0$

6. In the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$, the constant term is

(A) $-{}^{15}C_8$ (B) ${}^{15}C_8$ (C) $-{}^{16}C_8$ (D) ${}^{16}C_8$

7. The solutions of the equation $e^{2x} - 3e^x + 2 = 0$ are

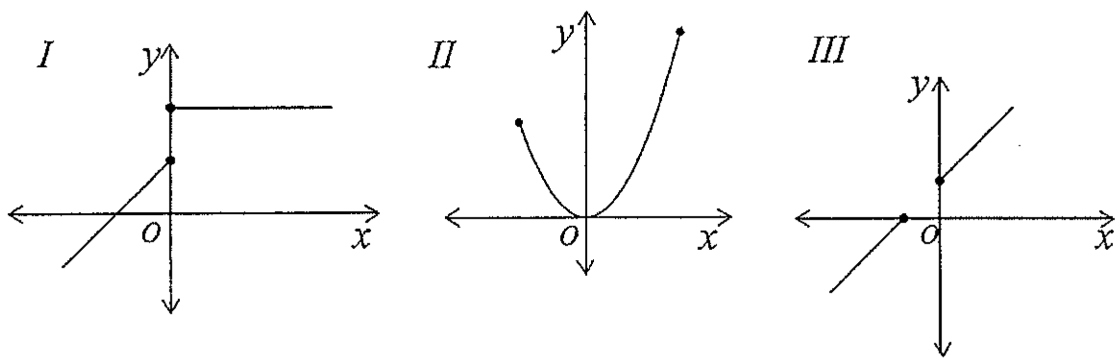
(A) $0,1$ (B) $1,2$ (C) $0, \log_e 2$ (D) $1, \log_e 2$

8. The value of k if $\int_0^1 \frac{dx}{x^2+3} = k\pi$ is

(A) $6\sqrt{3}$ (B) $\frac{1}{6\sqrt{3}}$ (C) 6 (D) $\frac{1}{6}$

9. The velocity $v\,m/s$ of a particle moving in simple harmonic motion along the x – axis is given by $v^2 = -5 + 6x - x^2$, where x is in metres. The amplitude of the oscillation is (A) $2m$ (B) $3m$ (C) $4m$ (D) $5m$

10. Which of the following graphs represent function(s) whose inverse(s) are also functions?



(A) *I* and *II* (C) *III* only

(B) *I* and *III* (D) *II* only

Question 11 (15 marks) **Marks**

(a) Evaluate: $\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx$ 2

(b) Evaluate $\int_{-1}^{0} x\sqrt{1+x}\,dx$ using the substitution $u = 1+x$. 3

(c) Solve $\frac{x}{x+4} > 2$ 3

(d) The line $y = mx$ makes an angle of $45°$ with the line $y = 3x$. Find the values of m. 3

(e) In how many ways can a jury of 7 people reach a majority decision? 1

(f) Find the value of k if the roots of the equation $x^3 - 3x^2 - 6x + k = 0$ are in arithmetic progression. 3

Question 12 (15 marks)

(a) The temperature gauge of a car is giving an overheating warning and the car is stopped. At time t minutes, the temperature T of the liquid in the radiator decreases according to the equation $\frac{dT}{dt} = -k(T-35)$, where k is a positive constant. The initial temperature of the liquid in the radiator is $95°C$ and it cools to $70°C$ after 10 minutes.

(i) Show that $T = 35 + Ae^{-kt}$ is a solution of $\frac{dT}{dt} = -k(T-35)$ 2

(ii) Find the value of A. 1

(iii) Find the value of k, correct to 3 decimal places. 1

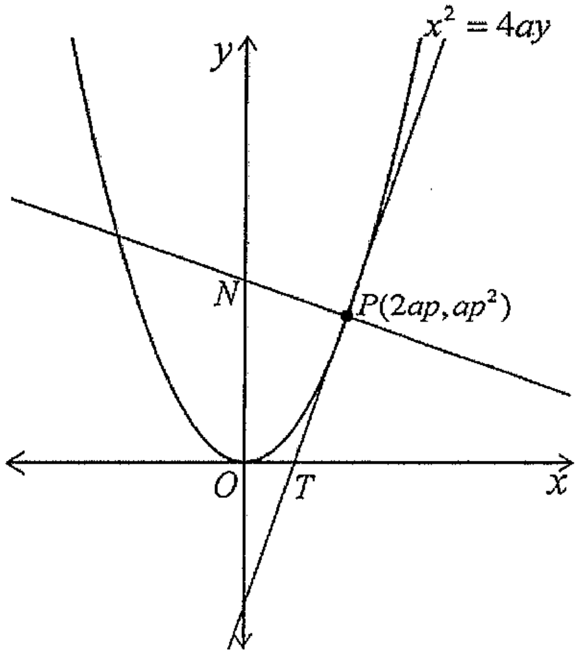
(iv) How long will it take for the temperature of the radiator to cool to $40°C$? Give your answer correct to the nearest minute. 1

(b) (i) Use mathematical induction to prove that

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n} \text{ for all integers } n \geq 2. \qquad 3$$

(ii) Hence find the exact value of $\left(1-\frac{1}{49}\right)\left(1-\frac{1}{64}\right)\left(1-\frac{1}{81}\right)\ldots\left(1-\frac{1}{144}\right)$. 2

(c) The diagram shows the graph of the parabola $x^2 = 4ay$. The tangent to the parabola at $P(2ap, ap^2)$ cuts the $x-$axis at T. The normal to the parabola at P cuts the $y-$axis at N.



(i) Find the coordinates of T, given that the equation of the tangent at P is $y = px - ap^2$ 1

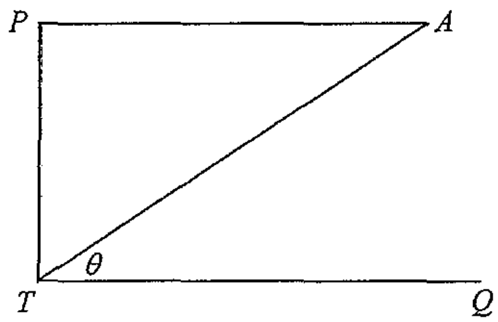
(ii) Show that the coordinates of N are $(0, a(p^2+2))$. 1

(iii) Let M be the midpoint of NT. Find the Cartesian equation of the locus of M and describe this locus. 3

Question 13 (15 marks)

(a) A person on horizontal ground is looking at an aeroplane A through a telescope T.

The aeroplane is approaching at a speed of $80m/s$ at a constant altitude of 200 meters above the telescope. When the horizontal distance of the aeroplane from the telescope is x metres, the angle of elevation of the aeroplane is θ radians.



(i) Show that $\theta = \tan^{-1}\dfrac{200}{x}$ 1

(ii) Show that $\dfrac{d\theta}{dt} = \dfrac{16000}{x^2+40000}$ 2

(iii) Find the rate at which θ is changing when $\theta = \dfrac{\pi}{4}$, (answer in degrees) 3

(b) By integrating the expansion of $(1-x)^n$ show that

$$1 - \frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} - \ldots + (-1)^n \frac{{}^nC_n}{n+1} = \frac{1}{n+1}$$ 3

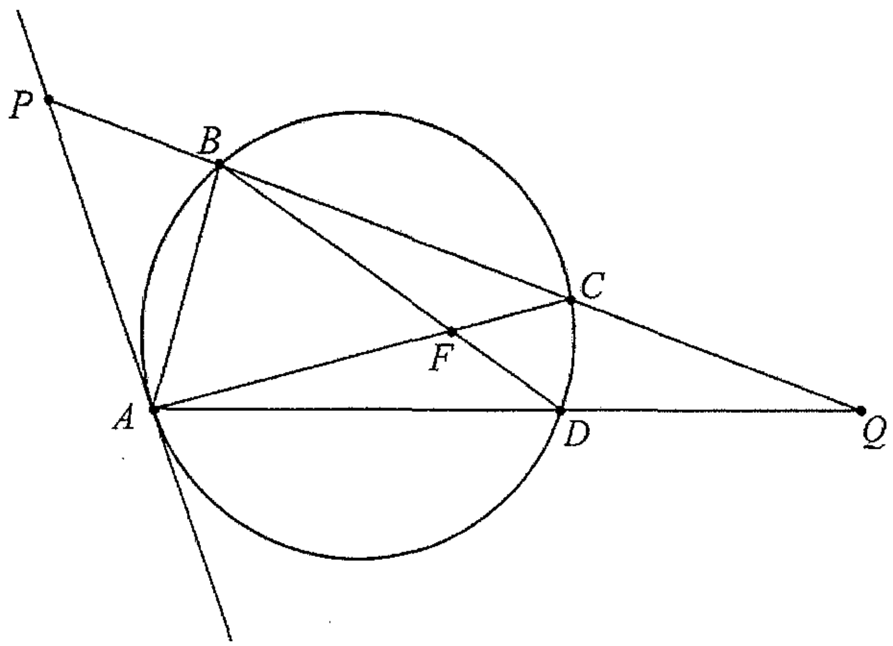
(c) **(i)** Show that the function $f(x) = x^2 - \log_e(x+3)$ has a zero between 1.1 and 1.3. 2

(ii) Use the method of halving the interval to find an approximation to this zero of $f(x)$, correct to one decimal place. 2

(d) The function $f(x)=\sin x-\frac{x}{3}$ has a zero near $x=2.2$. Use one application of Newton's method to find a second approximation to the zero. Write your answer correct to 3 significant figures. 2

Question 14 (15 marks)

(a) In the figure below, AP is a tangent to the circle at A. $PBCQ$ and ADQ are straight lines. Prove that $\angle PAB=\frac{1}{2}(\angle CFD+\angle CQD)$ 3

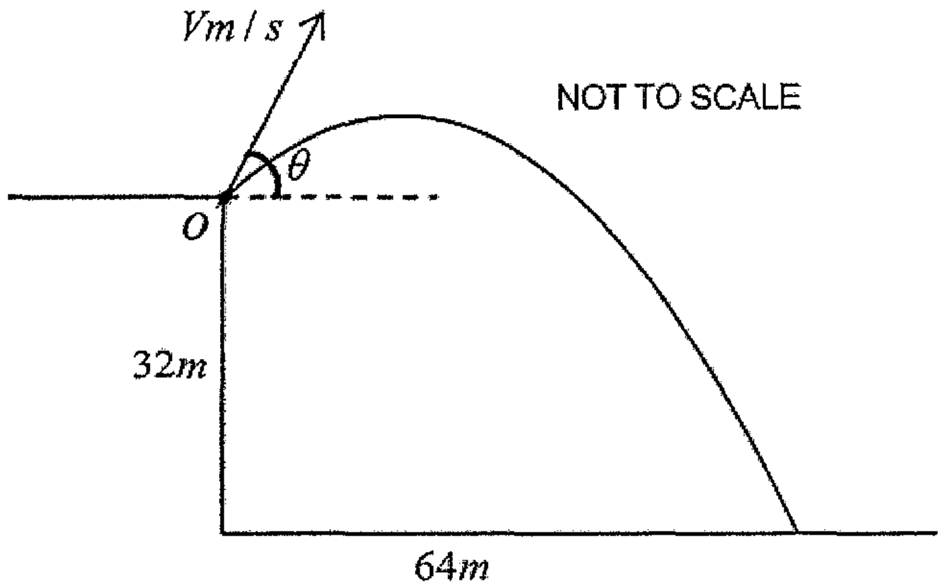


(b) The motion of a particle is given by $x=10+8\sin 2t+6\cos 2t$. Prove that the motion is simple harmonic. Write down the centre and period of the motion. 2

(c) The rise and fall of the tide at a certain port may be considered to be simple harmonic.

The interval between successive high tides is 12 hours. The port entrance has a depth of 12 m at high tide and 4m at low tide. If the low tide occurs at noon on a certain day, find the earliest time thereafter that a ship drawing 9m can pass through the entrance. 2

(d) A particle is projected with velocity Vm/s at an angle θ above the horizontal from a point O on the edge of a vertical cliff 32 metres above a horizontal beach. The particle moves in a vertical plane under gravity, and 4 seconds later it hits the beach at a point 64 metres from the foot of the cliff.(Take $g = 10m/s^2$)



(i) Use integration to show that after t seconds the horizontal displacement x metres and the vertical displacement y metres of the particle from O are given by

$x = (V\cos\theta)t$ and $y = (V\sin\theta)t - 5t^2$ respectively. 2

(ii) Write down two equations in V and θ then solve these equations to find the exact value of V and the value of θ in degrees correct to the nearest minute. 3

(iii) Find the speed of impact with the beach correct to the nearest whole number and the angle of impact with the beach correct to the nearest minute. 3

END OF TEST

Year 12 Extension 1 Trial HSC 2013

Section I (10 marks)

1. A 2. B 3. D 4. C 5. B 6. D 7. C 8. B

9. A 10. C

Question 11 (15 marks)

(a) $\displaystyle\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx$

$= \left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1$

$= \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1} 0$ (2)

$= \dfrac{\pi}{4}$

(b) $\tan 45 = \left|\dfrac{m-3}{1+3m}\right|$

$\left|\dfrac{m-3}{1+3m}\right| = 1$ (3)

$\dfrac{m-3}{1+3m} = 1$ $\dfrac{-(m-3)}{1+3m} = 1$

$m-3 = 1+3m$ $-m+3 = 1+3m$

$2m = {}^{-}4$ $4m = 2$

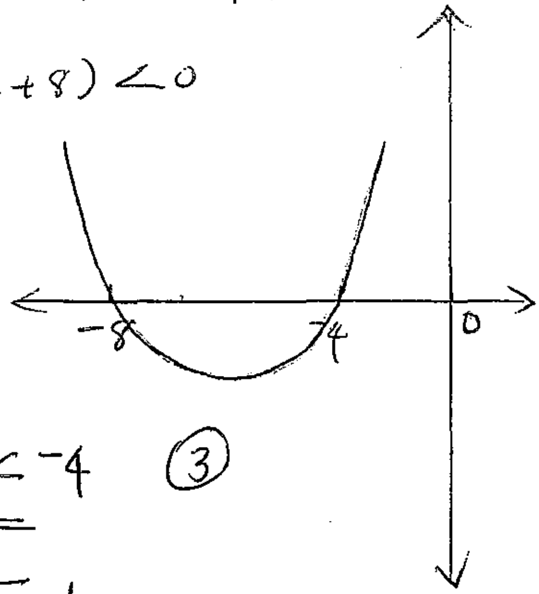
$m = -2$ $m = \dfrac{1}{2}$

(c) $\dfrac{x}{x+4} > 2$

$x(x+4) > 2(x+4)^2$

$2(x+4)^2 - x(x+4) < 0$

$(x+4)(x+8) < 0$



${}^{-}8 < x < {}^{-}4$ (3)

(d) $\displaystyle\int_{-1}^{0} x\sqrt{1+x}\,dx$

$u = 1+x$
$du = dx$
$x = u-1$
when $x = -1$, $u = 0$
when $x = 0$, $u = 1$

$= \displaystyle\int_0^1 (u-1)\sqrt{u}\,du$

$= \displaystyle\int_0^1 \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$

$= \left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$ (3)

$= \dfrac{2}{5} - \dfrac{2}{3} = \dfrac{-4}{15}$

(e)

${}^7C_7 + {}^7C_6 + {}^7C_5 + {}^7C_4$ (1)

$= 64$

(f) $x^3 - 3x^2 - 6x + k = 0$

Let the roots be $\alpha-d, \alpha, \alpha+d$

Sum of roots

$3\alpha = 3$

$\alpha = 1$

Sum of roots taken two at a time

$(\alpha-d)\alpha + (\alpha-d)(\alpha+d) + \alpha(\alpha+d)$

$3\alpha^2 - d^2 = -6$

$d^2 = 9$ (3)

Product of roots

$\alpha(\alpha-d)(\alpha+d) = \alpha(\alpha^2-d^2)$

$\alpha(\alpha^2-d^2) = -k$

$1(1-9) = -k$

$\underline{\underline{k = 8}}$

Question 12 (15 marks)

(a) (i) $T = 35 + Ae^{-kt}$

$\frac{dT}{dt} = Ae^{-kt} \times -k$

$= -kAe^{-kt}$

$= -k(T-35)$ (2)

$(\because T-35 = Ae^{-kt})$

Hence $T = 35 + Ae^{-kt}$ is a solution of $\frac{dT}{dt} = -k(T-35)$

(ii) When $t=0$, $T=95$

$95 = 35 + A$ (1)

$A = 60$

(iii) When $t=10$, $T=70$

$70 = 35 + 60e^{-10k}$

$60e^{-10k} = 35$

$e^{-10k} = \frac{35}{60} = \frac{7}{12}$

$-10k = \ln\left(\frac{7}{12}\right)$ (1)

$k = \frac{-1}{10}\ln\left(\frac{7}{12}\right) = 0.054$

(iv) $T = 40$, $t = ?$

$40 = 35 + 60e^{-kt}$

$60e^{-kt} = 5$

$e^{-kt} = \frac{5}{60} = \frac{1}{12}$

$-kt = \ln\left(\frac{1}{12}\right)$

$t = \frac{-1}{k}\ln\left(\frac{1}{12}\right)$ (1)

$= 46.02$

The temperature will fall to 40°C after 46 minutes

(b) When $n=2$, LHS $= 1-\frac{1}{4} = \frac{3}{4}$

RHS $= \frac{2+1}{2\times 2} = \frac{3}{4}$ LHS = RHS

$\therefore$ the result is true when $n=2$

Assume the result is true for $n=k$

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{k^2}\right) = \frac{k+1}{2k} \quad \text{—} \ (1)$$

Aim: To prove that it is true for $n=k+1$

ie $$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{k^2}\right)\left(1-\frac{1}{(k+1)^2}\right) = \frac{k+2}{2(k+1)} \quad \text{—} \ (2)$$

Proof:

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{k^2}\right)\left(1-\frac{1}{(k+1)^2}\right)$$

$$= \frac{k+1}{2k}\left(1-\frac{1}{(k+1)^2}\right) \quad (\text{by assumption } (1))$$

$$= \frac{(k+1)}{2k}\left(\frac{k^2+2k+1-1}{(k+1)^2}\right)$$

(3)

$$= \frac{(k+1)\,k(k+2)}{2k(k+1)^2}$$

$$= \frac{k+2}{2(k+1)} = \text{RHS of } (2)$$

Hence by the principle of mathematical Induction, the result is true for $n \geq 2$.

(ii) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{6^2}\right)\left(1-\frac{1}{7^2}\right)\cdots\left(1-\frac{1}{12^2}\right)=\frac{13}{24}$

$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{6^2}\right)=\frac{7}{12}$

$\left(1-\frac{1}{49}\right)\left(1-\frac{1}{64}\right)\cdots\left(1-\frac{1}{144}\right)=\frac{13}{24}\div\frac{7}{12}$ (2)

$=\frac{13}{14}$

(c) (i)

$y = px - ap^2$

substitute $y=0$

$px - ap^2 = 0$

$px = ap^2$ (1)

$x = ap$

$T\,(ap, 0)$

(ii) Equation of normal

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

Substitute $x=0$ (1)

$y - ap^2 = 2a$

$y = ap^2 + 2a = a(p^2+2)$

$N\,(0, a(p^2+2))$

(iii) $M = \left(\frac{ap}{2}, \frac{a(p^2+2)}{2}\right)$

$x = \frac{ap}{2} \Rightarrow p = \frac{2x}{a}$

$y = \frac{a(p^2+2)}{2}$ — (2)

substitute $p = \frac{2x}{a}$ in

$y = \frac{a\left(\frac{4x^2}{a^2}+2\right)}{2} = \frac{a\left(\frac{4x^2+2a^2}{a^2}\right)}{2}$

$y = \frac{4x^2+2a^2}{a}\times\frac{1}{2} = \frac{2x^2+a^2}{a}$

$$2x^2 + a^2 = ay$$
$$2x^2 = ay - a^2$$
$$= a(y-a)$$
$$x^2 = \frac{a}{2}(y-a) \quad (3)$$

This is a parabola with vertex $(0,a)$ and focal length $\frac{a}{8}$ units.

Question 13 (15 marks)

(a) (i)

$\angle TAP = \theta$ (alternate $\angle$s, $PA \parallel TQ$)

$$\tan\theta = \frac{200}{x}$$
$$\theta = \tan^{-1}\frac{200}{x} \quad (1)$$

(ii) $$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$$

$$= \frac{1}{1+\left(\frac{200}{x}\right)^2} \times -\frac{200}{x^2} \times -80$$

$$= \frac{x^2}{x^2+40000} \times \frac{16000}{x^2}$$

$$= \frac{16000}{x^2+40000} \quad (2)$$

(iii) when $\theta = \frac{\pi}{4}$,

$TP = AP \quad \therefore x = 200 \quad (3)$

$$\frac{d\theta}{dt} = \frac{16000}{(200)^2 + 40000} = 0.2 \text{ radians/s}$$

Hence θ is increasing at 11°/s

(b)

$(1-x)^n = 1 - {}^nC_1x + {}^nC_2x^2 - {}^nC_3x^3 + \dots + (-1)^n\,{}^nC_nx^n$

$$\int_0^1 (1-x)^n\,dx = \int_0^1 (1 - {}^nC_1x + \dots + (-1)^n\,{}^nC_nx^n)$$

$$= \left[x - {}^nC_1\frac{x^2}{2} + {}^nC_2\frac{x^3}{3} - \dots (-1)^n\,{}^nC_n\frac{x^{n+1}}{n+1}\right.$$

(b) $(1-x)^n = 1 - {}^nC_1 x + {}^nC_2 x^2 - {}^nC_3 x^3 + \dots + (-1)^n\, {}^nC_n x^n$

$$\int_0^1 (1-x)^n\,dx = \int_0^1 \left(1 - {}^nC_1 x + {}^nC_2 x^2 - {}^nC_3 x^3 + \dots + (-1)^n\, {}^nC_n x^n\right) dx$$

$$\left[\frac{(1-x)^{n+1}}{n+1} \times -1\right]_0^1 = \left[x - {}^nC_1 \frac{x^2}{2} + {}^nC_2 \frac{x^3}{3} - {}^nC_3 \frac{x^4}{4} + \dots + (-1)^n\, {}^nC_n \frac{x^{n+1}}{n+1}\right]_0^1$$

$$\frac{-1}{n+1}\left[(1-x)^{n+1}\right]_0^1 = \left[x - {}^nC_1 \frac{x^2}{2} + {}^nC_2 \frac{x^3}{3} - {}^nC_3 \frac{x^4}{4} + \dots + (-1)^n\, {}^nC_n \frac{x^{n+1}}{n+1}\right]$$

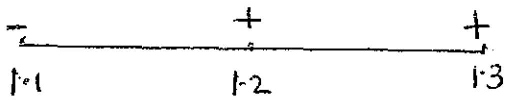
$$\frac{-1}{n+1}(0-1) = 1 - \frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} - \frac{{}^nC_3}{4} + \dots + (-1)^n \frac{{}^nC_n}{n+1}$$ 3

$$\underline{\underline{1 - \frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} - \frac{{}^nC_3}{4} + \dots + (-1)^n \frac{{}^nC_n}{n+1}}} = \frac{1}{n+1}$$

(c) (i) $f(1\cdot1) = 1\cdot1^2 - \log_e(4\cdot1) = -0\cdot201$

$f(1\cdot3) = 1\cdot3^2 - \log_e(4\cdot3) = 0\cdot231$ 2

Since $f(x)$ changes sign between $x = 1\cdot1$ and $x = 1\cdot3$, there is a zero of $f(x) = x^2 - \log_e(x+3)$ between $1\cdot1$ and $1\cdot3$.

(ii) 

$f(1\cdot2) = 1\cdot2^2 - \log_e(4\cdot2) = 0\cdot005$ 2

$\frac{1\cdot1 + 1\cdot2}{2} = 1\cdot15$

To one decimal place $\underline{\underline{x = 1\cdot2}}$ is the zero of $f(x)$.

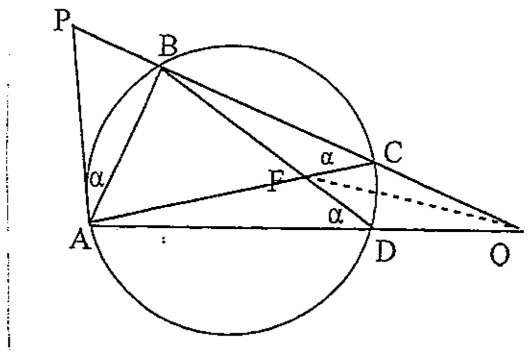
(d) $f(x) = \sin x - \frac{x}{3}$

$f'(x) = \cos x - \frac{1}{3}$

$a_1 = 2.2 - \frac{f(2.2)}{f'(2.2)}$ (2)

$= 2.28$

Question 14 (15 marks)



$\angle PAB = \angle ACB$ (angle between tangent and chord is equal to angle in the alternate segment)

$\angle ACB = \angle ADB$ (angles at the circumference standing on the same arc)

$\angle BCA = \angle CQF + \angle CFQ$ (exterior $\angle$ of ΔFQC)

$\angle ADF = \angle DQF + \angle DFQ$ (exterior $\angle$ of ΔFQD)

$\angle BCA + \angle ADF$

$= \angle CQF + \angle CFQ + \angle DQF + \angle DFQ$

$= \angle CQF + \angle DQF + \angle CFQ + \angle DFQ$

$= \angle CQD + \angle CFD$ (3)

$2\alpha = \angle CQD + \angle CFD$

$\alpha = \frac{1}{2}(\angle CQD + \angle CFD)$

(b) $x = 10 + 8\sin 2t + 6\cos 2t$

$\dot{x} = 16\cos 2t - 12\sin 2t$

$\ddot{x} = -32\sin 2t - 24\cos 2t$

$= -4(8\sin 2t + 6\cos 2t)$

$= -4(x - 10)$ (2)

Centre $x = 10$, $n = 2$

Period $T = \frac{2\pi}{n} = \pi$ seconds

(c) Period $= 12$ hrs

12.00pm

4m 8m 12m

Amplitude $= 4$m

$T = \frac{2\pi}{n}$ $n = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$

Let 12.00 pm denote $t = 0$

when $t = 0$, $x = -4$

$-4 = 4\cos\alpha$

$\cos\alpha = -1$ $\therefore \alpha = \pi$

$x = 4\cos\left(\frac{\pi}{6}t + \pi\right)$

Let at time t_1 the depth of water be 9m. Then $x=1$

$$1 = 4\cos\left(\frac{\pi}{6}t_1 + \pi\right)$$

$$\cos\left(\frac{\pi}{6}t_1 + \pi\right) = \frac{1}{4}$$

$$\frac{\pi}{6}t_1 + \pi = \cos^{-1}\left(\frac{1}{4}\right),\ 2\pi - \cos^{-1}\left(\frac{1}{4}\right)$$

$$\frac{\pi}{6}t_1 = \cos^{-1}\left(\frac{1}{4}\right) - \pi,\ \pi - \cos^{-1}\left(\frac{1}{4}\right)$$

$$t_1 = \frac{6}{\pi}\left(\cos^{-1}\left(\frac{1}{4}\right) - \pi\right),\ \frac{6}{\pi}\left(\pi - \cos^{-1}\left(\frac{1}{4}\right)\right)$$

$$= \frac{6}{\pi}\cos^{-1}\left(\frac{1}{4}\right) - 6,\ 6 - \frac{6}{\pi}\cos^{-1}\left(\frac{1}{4}\right) \quad (2)$$

$= 3.4826$ hr

$= 3$ hr 29 minutes

$t_1 = 12.00$ pm $+ 3$ hr 29 min

$= 3:29$ pm

(d) (i) $\ddot{x} = 0$ —— (1) $\qquad \ddot{y} = -g$ —— (2)

$$\frac{dx}{dt} = A \text{ —— (3)} \qquad \frac{dy}{dt} = -gt + B \text{ —— (4)}$$

where A and B are constants of integration

when $t=0$, $\frac{dx}{dt} = V\cos\theta$, $\frac{dy}{dt} = V\sin\theta$

$V\cos\theta = A \qquad V\sin\theta\ B$

$$\frac{dx}{dt} = V\cos\theta \text{ —— (5)} \qquad \frac{dy}{dt} = -gt + V\sin\theta \text{ —— (6)}$$

$$x = (V\cos\theta)t + C \text{ —— (7)} \qquad y = -\frac{gt^2}{2} + (V\sin\theta)t + D \text{ —— (8)}$$

where C and D are constants of integration.

when $t=0$, $x=0$ and $y=0$

$0 = 0 + C \quad \therefore C = 0$ $\qquad$ $0 = 0 + 0 + D \quad \therefore D = 0$ (2)

$x = (V\cos\theta)t$ —— (9) $\qquad$ $y = (V\sin\theta)t - 5t^2$ —— (10)

(ii) when $t = 4$, $x = 64$ and $y = -32$

$64 = (V\cos\theta)4$ $\qquad$ $-32 = (V\sin\theta)\times 4 - 5\times 16$

$64 = 4V\cos\theta$ $\qquad$ $-32 = 4V\sin\theta - 80$

$V\cos\theta = 16$ —— (11)

$V\sin\theta = 12$ —— (12)

Squaring and adding (11) and (12) $\qquad$ (3)

$V^2(\sin^2\theta + \cos^2\theta) = 144 + 256$
$= 400$

$V = 20$ m/s

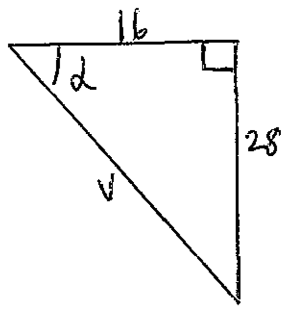
$\sin\theta = \frac{12}{20} = \frac{3}{5}$

$\theta = 36^\circ 52'$

(iii) when $t = 4$,

$\dot{x} = V\cos\theta = 16$

$\dot{y} = V\sin\theta - gt$
$= 12 - 40$
$= -28$



$V^2 = 16^2 + 28^2$
$= 1040$ $\qquad$ (3)

$V = 32.25$ m/s

$\tan\alpha = \frac{28}{16} = \frac{7}{4}$

$\alpha = 60^\circ 15'$