Girraween High School

2016 Year 12 Trial Higher School Certificate
Mathematics Extension 1

General Instructions

- Reading Time - 5 minutes
- Working Time - 2 hours
- Calculators and ruler may be used
- All necessary working out must be shown
- Write on both sides of the paper

Total Marks - 70

- Attempt all questions
- Marks may be deducted for careless or badly arranged work

Section I

10 marks

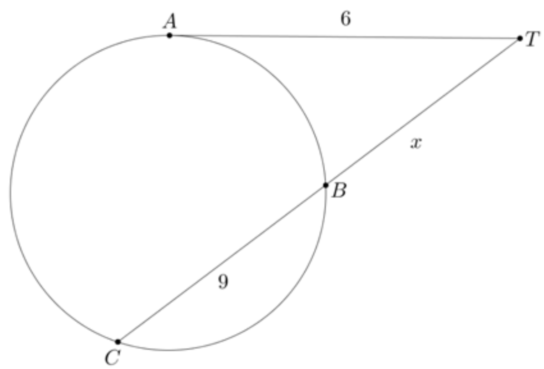
Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Question 1-10

Question 1 (1 mark)

Line AT is a tangent to the circle at A. TB is a secant cutting the circle at B and C. If $AT = 6$, $TB = x$ and $BC = 9$, what is the value of x?



A. 2

B. 3

C. 4

D. 12

Question 2 (1 mark)

What is the derivative of $\cos^{-1}(3x)$?

A. $\dfrac{1}{3\sqrt{1-9x^2}}$

B. $\dfrac{-1}{3\sqrt{1-9x^2}}$

C. $\dfrac{3}{\sqrt{1-9x^2}}$

D. $\dfrac{-3}{\sqrt{1-9x^2}}$

Question 3 (1 mark)

What is the value of $\lim\limits_{x \to 0} \dfrac{2 \sin 3x}{x}$?

A. $\dfrac{2}{3}$

B. 2

C. 6

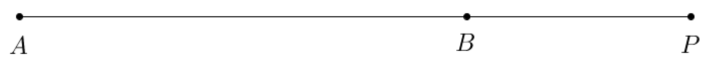
D. undefined

Question 4 (1 mark)

The point P divides the interval AB in the ratio $-1 : 2$. Which of the following diagrams is correct?

A.

B.



C.

D.

Question 5 (1 mark)

The degrees of two polynomials $P(x)$ and $Q(x)$ are m and n respectively, where $m > n$. What is the degree of $P(x) + Q(x)$?

A. $m + n$

B. mn

C. m

D. n

Question 6 (1 mark)

What is the term independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^9$?

A. ${}^9C_3\,(-2)^3$

B. ${}^9C_3\,(2)^3$

C. ${}^9C_6\,(-2)^6$

D. ${}^9C_6\,(2)^6$

Question 7 (1 mark)

A hotel has 3 different rooms.

How many different ways can 4 people be accommodated?

A. 3^4

B. 4^3

C. ${}^4C_3$

D. ${}^4P_3$

Question 8 (1 mark)

The position function of a particle is given by $x = 2\sin \pi t + 1$. Which of the following is true?

A. The maximum velocity of the particle occurs at $t = \dfrac{1}{2}$ at $x = 3$

B. The maximum velocity of the particle occurs at $t = \dfrac{3}{2}$ at $x = -1$

C. The maximum velocity of the particle occurs at $t = 2$ at $x = 1$

D. The maximum velocity of the particle occurs at $t = 6$ at $x = -1$

Question 9 (1 mark)

For which of the following is true?

A. If $f(x) = \sin x$ for $0 \leq x \leq \pi$ then $f^{-1}(x)$ exists

B. If $f(x) = x^2$ for all real x then $f^{-1}(x)$ exists

C. If $f(x) = mx$ for all real x then $f^{-1}(x)$ exists for any real value of m

D. $f^{-1}(x)$ does not exist for any of the above

Question 10 (1 mark)

Projectiles A and B are launched at same time at velocity V and angle α. However projectile A is launched from a higher position. The two projectiles land in the same horizontal plane. Which of the following is always true?

A. A and B will reach the ground at the same time

B. A and B will have the same range

C. A will reach its maximum height earlier than B

D. The maximum speed of A is greater than the maximum speed of B

Question 11 on the next page

Section II

60 marks

Attempt Questions 11-14

Allow about 1 hour and 45 minutes for this section

Write your answers on the paper provided.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Solve $\dfrac{3}{x+2} < 4$. [3]

(b) Find the size of the acute angle between the lines $2x + y = 5$ and $3x - y = 1$. [3]

(c) The point P divides the interval joining $A(-1, -2)$ to $B(9, 3)$ internally in the ratio $4 : 1$. Find the coordinates of P. [2]

(d) If $\cos\theta = \dfrac{4}{5}$ and $\dfrac{3\pi}{2} < \theta < 2\pi$ find the exact value of $\sin 2\theta$. [2]

(e) Using the substitution $u = \sqrt{x}$ find $\displaystyle\int \frac{1}{\sqrt{x}(1+x)}\,dx$. [3]

(f) Find the value of a if $P(x) = x^3 + ax^2 + ax + 5$ gives the same remainders when it is divided by $x + 2$ or $x - 4$. [2]

The exam continues on the next page

Question 12 (15 marks)

(a) Prove by mathematical induction that for all integers $n \geq 1$, [3]

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \cdots + n \times 2^{n-1} = 1 + (n-1)2^n$$

(b) i. Find the coefficient of x in the expansion of $\left(3x - \dfrac{1}{2x}\right)^7$. [2]

ii. Hence state the constant term in the expansion of $\left(1 + \dfrac{1}{x}\right)^2 \left(3x - \dfrac{1}{2x}\right)^7$. [1]

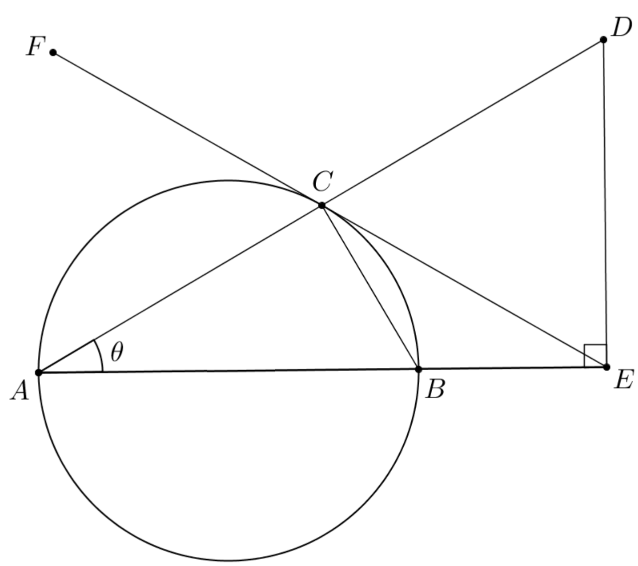
(c) Consider the function $f(x) = x^3 + x$.

i. State the domain of $f(x)$. [1]

ii. Show that $f(x)$ does not have any stationary points. [2]

iii. By giving reasons, state whether $f^{-1}(x)$ exists. [1]

(d) The diagram shows a circle with diameter AB. EF is a tangent to the circle at C. AC is extended to D such that DE is perpendicular to AE. Let $\angle BAC = \theta$.



i. Copy the diagram and prove that $BCDE$ is a cyclic quadrilateral. [2]

ii. Prove that $EC = ED$. [3]

The exam continues on the next page

Question 13 (15 marks)

(a) i. A fair coin is tossed 4 times, what is probability of getting more heads than tails? [2]

ii. A person decides to flip a two dollar coin 4 times each day, and if he gets more heads than tails he will contribute the coin towards his savings. He does this for 7 days. What is the probability that he will contribute four dollars towards his savings by the end of the 7 days? Give your answer to one decimal place. [1]

(b) A particle moves along a straight line with displacement $x\,m$ and velocity $v\,ms^{-1}$. Initially the particle is at the origin at with velocity $-1\,ms^{-1}$. The acceleration of a particle is given by

$$\ddot{x} = 4x + 2$$

i. Show that $v^2 = 4x^2 + 4x + 1$ [2]

ii. Show that $x = \frac{1}{2}(e^{-2t} - 1)$ [3]

iii. What happens to x as $t \to \infty$? [1]

(c) A particle moves in a straight line and its position at time t is given by

$$x = 5 + \sqrt{3}\sin 3t - \cos 3t$$

i. Express $\sqrt{3}\sin 3t - \cos 3t$ in the form $R\sin(3t - \alpha)$, where α is in radians. [2]

ii. Prove that the particle is undergoing simple harmonic motion and find its period. [2]

iii. State the particle's maximum displacement. [1]

iv. When does the particle first reach its minimum acceleration? [1]

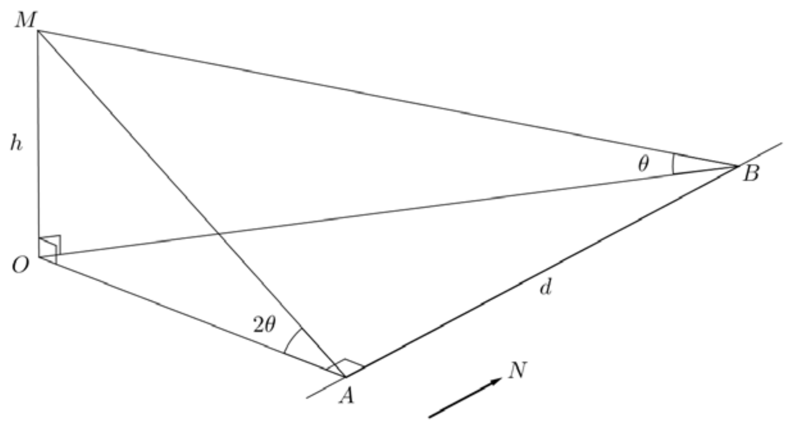
The exam continues on the next page

Question 14 (15 marks)

(a) A person walks a length of d metres due north along a road from point A to point B. The point A is due east of a mountain OM, where M is the top of the mountain. The point O is directly below point M and is on the same horizontal plane as the road. The height of the mountain above point O is h metres.

From point A, the angle of elevation to the top of the mountain is 2θ.

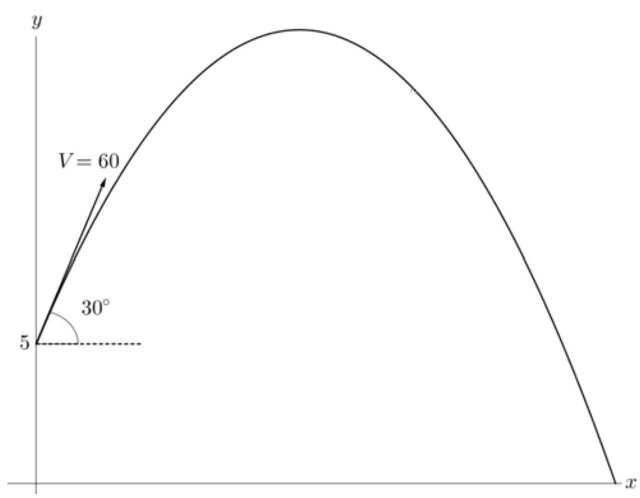
From point B, the angle of elevation to the top of the mountain is θ.



i. Find the expressions for OA and OB in terms of h and θ. [1]

ii. Show that $d^2 = \dfrac{h^2 \operatorname{cosec}^2 \theta}{4}(3 - \tan^2 \theta)$ [3]

(b) A projectile is launched from a height of $5\,m$ above the ground at $V = 60\,ms^{-1}$ at $30°$ to the horizontal. You may assume $g = 10\,ms^{-2}$.



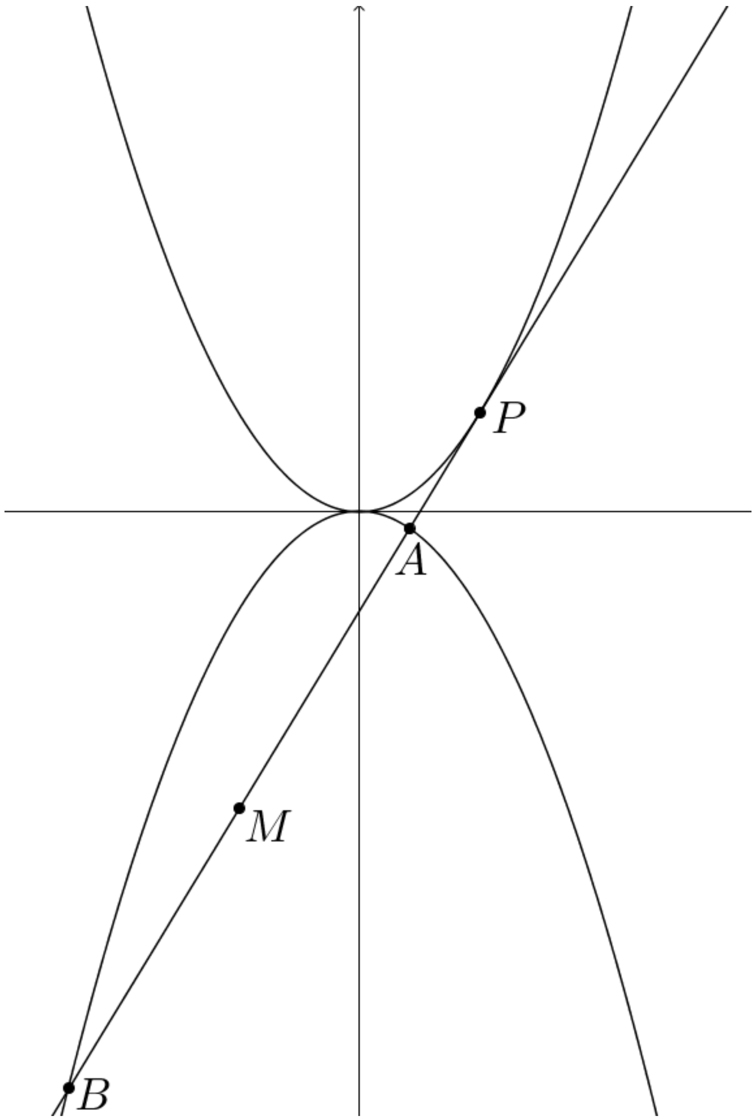
i. Derive the equations for x and y. [2]

ii. Find the maximum height of the projectile. [1]

iii. Find the speed and angle (to nearest degree) at which the projectile hits the ground. [3]

The exam continues on the next page

(c) The diagram shows a point $P(2at, at^2)$ on the parabola $x^2 = 4ay$. The tangent to the parabola at P cuts the reflection of the parabola in the x axis at points A and B. The point M is the midpoint of the interval AB.



i. Show that the equation of the tangent is $y = tx - at^2$. [1]

ii. Show that the coordinates of M are $(-2at, -3at^2)$ [3]

iii. Show that the locus of M is $x^2 = -\frac{4}{3}ay$ [1]

End of exam

Yr12 Ext1 TRIAL 2016

MC: BDCAC DACDD
(or C)

Q1

$6^2 = x(x+9)$

$36 = x^2 + 9x$

$x^2 + 9x - 36 = 0$

$(x+12)(x-3) = 0$

$\therefore x = 3 \quad \therefore$ (B)

Q2 $y = \cos^{-1}(3x)$

$y' = -\frac{1}{\sqrt{1-(3x)^2}} \times 3$

$= -\frac{3}{\sqrt{1-9x^2}} \quad \therefore$ (D)

Q3 $\lim_{x \to 0} \frac{2\sin 3x}{x} = 2 \lim_{x \to 0} \frac{\sin 3x}{x}$

$= 2 \times 3 \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= 6 \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= 6 \quad \therefore$ (C)

Q4

$AP : PB = -1 : 2$

$\therefore$ P is external to AB and P is closer to A

$\therefore$ (A)

Q5

$P(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_0$

$Q(x) = b_n x^n + b_{n-1} x^{n-1} + \dots + b_0$

$\therefore \deg[P(x) + Q(x)] = m \quad \therefore$ (C)

Q6

Constant term occurs when

$\binom{9}{3} (x^2)^3 \left(-\frac{2}{x}\right)^6$

$= \binom{9}{3} (-2)^6 \frac{x^6}{x^6}$

$= \binom{9}{3} 2^6 \quad \therefore$ (D) or (C)

Q7

Each person can choose one of the 3 rooms $\therefore 3 \times 3 \times 3 \times 3$

$= 3^4 \quad \therefore$ (A)

Q8

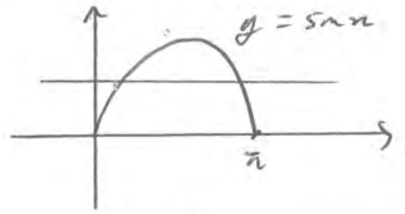
$\therefore$ max V occurs at $x = 1$

$T = \frac{2\pi}{\pi} = 2$

$\therefore$ max V occurs when $t = 0$ as it is at the centre initially

$\therefore$ it will return to this position at $t = 2 \quad \therefore$ (C)

Q9



clearly not 1:1

∴ A is false.

$f(x) = x^2$ is clearly even as

$f(-x) = (-x)^2 = x^2 = f(x)$.

As even functions are symmetrical about the y-axis they are not 1:1

∴ B is false

If $m = 0$ then $f(x) = mx$ is a horizontal line which is clearly not 1:1 ∴ C is false

∴ (D)

Q10

Since $\dot{y} = V\sin\alpha - gt$ for both projectiles they will reach the highest points at the same time

∴ C is false

A clearly has a longer flight time as it needs to fall a greater vertical distance ∴ A is false

With longer flight time A must travel a longer horizontal distance ∴ B is false.

∴ (D) is true as A will gain a more negative vertical velocity, resulting in greater maximum speed.

Q11

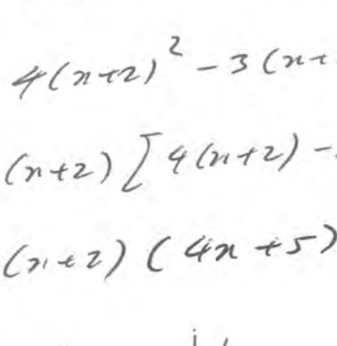
(a) $\frac{3}{x+2} < 4$

$3(x+2) < 4(x+2)^2$

$4(x+2)^2 - 3(x+2) > 0$

$(x+2)[4(x+2) - 3] > 0$

$(x+2)(4x+5) > 0$



∴ $x < -2$ & $x > -\frac{5}{4}$.

(b) $y = -2x+5$ ∴ $m_1 = -2$

$y = 3x+1$ ∴ $m_2 = 3$

$\tan\theta = \left|\frac{-2-3}{1-6}\right| = 1$

∴ $\theta = 45°$

Q11

(c)

$A(-1,-2) \quad B(9,3)$

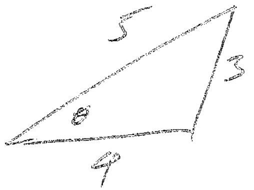
$4 : 1$

$x = \frac{4\times 9 - 1}{4+1} \qquad y = \frac{3\times 4 - 2}{5}$

$x = 7 \qquad y = 2$

$\therefore P = (7, 2)$

(d) $\cos\theta = \frac{4}{5}$



Since $\frac{3\pi}{2} < \theta < 2\pi \quad \therefore \sin\theta = -\frac{3}{5}$

$\therefore \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\times -\frac{3}{5} \times \frac{4}{5} = -\frac{24}{25}$

(e) $I = \int \frac{1}{\sqrt{x}(1+x)}\,dx$

let $u = \sqrt{x}$

$u = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\therefore I = 2\int \frac{1}{2\sqrt{x}(1+x)}\,dx$

$= 2\int \frac{1}{1+u^2}\,du$

$= 2\tan^{-1}u + C$

$= 2\tan^{-1}\sqrt{x} + C.$

(f) $P(-2) = P(4)$

$\therefore (-2)^3 + 4a - 2a + 5 = 4^3 + 16a + 4a + 5$

$-8 + 2a = 64 + 20a$

$\therefore 18a = -72$

$\therefore a = -4$

Q12

(a)

- Prove true for $n=1$.

When $n=1$: $LHS = 1\times 2^0 = 1$

$RHS = 1 + 0\times 2^1 = 1$

$\therefore LHS = RHS = 1$

$\therefore$ true for $n=1$

- Assume true for $n=k$, ie

$1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \dots + k\times 2^{k-1}$

$= 1 + (k-1)2^k.$

- Prove true for $n=k+1$, ie.

$1\times 2^0 + 2\times 2^1 + \dots + k\times 2^{k-1} \times (k+1)\times 2^k$

$= 1 + k\,2^{k+1}$

$LHS = 1\times 2^0 + 2\times 2^1 + \dots + k\times 2^{k-1} \times (k+1)\times 2^k$

$= 1 + (k-1)2^k + (k+1)\times 2^k$ by assumption.

$= 1 + 2^k(k-1+k+1)$

$= 1 + 2^k(2k)$

$= 1 + k2^{k+1} = RHS.$

Q12

(a) ∴ true for n=k+1

∴ By the principle of induction it is true for n≥1.

(b)

(i) $T_{k+1} = \binom{7}{k}(3x)^{7-k}\left(-\frac{1}{2x}\right)^{k}$

$= \binom{7}{k} 3^{7-k} x^{7-k} (-2)^{-k} x^{-k}$

$= \binom{7}{k} 3^{7-k} (-2)^{-k} x^{7-2k}$

$\therefore 7-2k=1 \therefore 2k=6 \therefore k=3$

$\therefore$ Coefficient is $\binom{7}{3} 3^{4} (-2)^{-3}$

$= -\frac{2835}{8}$

(ii)

$\left(1+\frac{2}{x}+\frac{1}{x^2}\right)\left(3x-\frac{1}{2x}\right)^7$

Only way to get constant is multiplying $\frac{2}{x}$ with x term from $\left(3x-\frac{1}{2x}\right)^7$

$\therefore$ constant is $2 \times -\frac{2835}{8} = -\frac{2835}{4}$.

(c) (i) D: all real x.

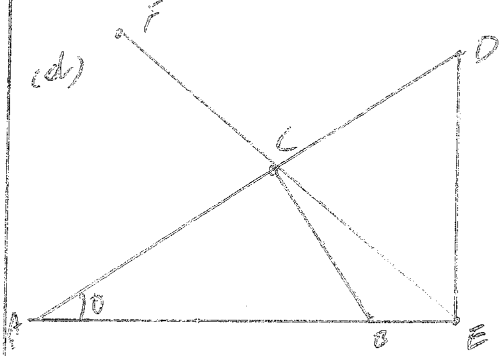
(ii) $f'(x) = 3x^2+1 \neq 0$ for all x.

(iii) $f'(x) \geq 1$ for all x

∴ f is a monotonically increasing function, so it is 1:1

∴ f^{-1} exists.

(d)



(i) $\angle ACB = 90$ ($\angle$ in a semi circle with diameter AB)

$\therefore \angle CBE = 90+\theta$ (exterior $\angle$ of $\triangle ABC$).

$\angle ADE = 90-\theta$ ($\angle$ sum of $\triangle ADE$)

$\therefore \angle CBE + \angle ADE = 180$

∴ BCDE is a cyclic quadrilateral (opposite $\angle$'s are supplementary).

(ii)

$\angle ABC = 90-\theta$ ($\angle$ sum of $\triangle ABC$).

$\therefore \angle ACF = 90-\theta$ ($\angle$ between tangent EF and chord AC equals the $\angle$ in the alternate segment).

$\therefore \angle DCE = 90-\theta$ (vertically opposite $\angle$'s)

$\therefore \angle DCE = \angle ADE = 90-\theta$

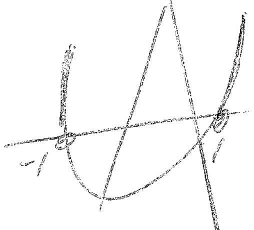
$\therefore CE = ED$ (Equal sides opposite equal $\angle$'s of $\triangle ECD$)

Q12

(d)

(i) $\frac{x+1}{x-1} > 0$

$(x+1)(x-1) > 0$



$\therefore$ D: $x < -1$ & $x > 1$

(ii) $f(x) = \ln\sqrt{\frac{x+1}{x-1}}$

$f(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$f(x) = \frac{1}{2}\left[\ln(x+1) - \ln(x-1)\right]$

$f'(x) = \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right]$

$= \frac{1}{2}\left[\frac{x-1-(x+1)}{(x+1)(x-1)}\right]$

$f'(x) = \frac{1}{2} \times \frac{-2}{(x+1)(x-1)} = -\frac{1}{(x+1)(x-1)}$

$\neq 0$ for all x

$\therefore$ no stationary points.

(iii) Since $f'(x) < 0$ for all x

$\therefore f$ is a monotonically decreasing function, so it is 1:1

$\therefore f^{-1}$ exists.

Q13

(a)

(i)

$= P(3H) + P(4H)$

$= \binom{4}{3}\left(\frac{1}{2}\right)^4 + \binom{4}{4}\left(\frac{1}{2}\right)^4$

$= \frac{5}{16}$

(ii) $\binom{7}{2}\left(\frac{5}{16}\right)^2\left(\frac{11}{16}\right)^5$

$= 31.5\%$ (1dp).

(b)

(i) $\frac{d}{dx}\frac{1}{2}v^2 = 4x+2$

$\therefore \frac{1}{2}v^2 = \int 4x+2\, dx$

$\frac{1}{2}v^2 = 2x^2 + 2x + C$

but $v = -1$ when $x = 0$

$\therefore \frac{1}{2} = C$

$\therefore \frac{1}{2}v^2 = 2x^2 + 2x + \frac{1}{2}$

$\therefore v^2 = 4x^2 + 4x + 1$

(ii) $v^2 = (2x+1)^2$

$\therefore v = \pm(2x+1)$

but $v = -1$ when $x = 0$

$\therefore v = -(2x+1)$

$\therefore \frac{dx}{dt} = -(2x+1)$

$\therefore \frac{1}{2x+1}\, dx = -dt$

Q13

(b)

(ii) $\int \frac{1}{2x+1}\,dx = -\int dt$

$\frac{1}{2}\int \frac{2}{2x+1}\,dx = -t + c$

$\frac{1}{2}\ln(2x+1) = -t + c$

$t=0 \quad x=0$

$\therefore \frac{1}{2}\ln 1 = c \quad \therefore c = 0.$

$\therefore \ln(2x+1) = -2t$

$\therefore 2x+1 = e^{-2t}$

$\therefore x = \frac{1}{2}(e^{-2t} - 1)$

(iii) As $t \to \infty \quad x \to -\frac{1}{2}$.

(c)

(i)

$\sqrt{3}\sin 3t - \cos 3t = R\sin(3t - \alpha)$

$\sqrt{3}\sin 3t - \cos 3t = R[\sin 3t \cos\alpha - \cos 3t \sin\alpha]$

$\sqrt{3}\sin 3t - \cos 3t = R\cos\alpha \sin 3t - R\sin\alpha \cos 3t$

$\therefore R\cos\alpha = \sqrt{3} \;\&\; R\sin\alpha = 1$

$\therefore R^2\cos^2\alpha + R^2\sin^2\alpha = 3+1$

$\therefore R^2(\sin^2\alpha + \cos^2\alpha) = 4$

$\therefore R^2 = 4 \quad \therefore R = 2$

$\therefore \cos\alpha = \frac{\sqrt{3}}{2} \;\&\; \sin\alpha = \frac{1}{2}$

$\therefore \alpha = \frac{\pi}{6}$

$\therefore \sqrt{3}\sin 3t - \cos 3t = 2\sin(3t - \frac{\pi}{6})$.

(ii) $x = 5 + 2\sin(3t - \frac{\pi}{6})$

$\dot{x} = 6\cos(3t - \frac{\pi}{6})$

$\ddot{x} = -18\sin(3t - \frac{\pi}{6})$

$\ddot{x} = -3^2 \times 2\sin(3t - \frac{\pi}{6})$

$\ddot{x} = -3^2(x-5)$

$\therefore$ SHM with $T = \frac{2\pi}{3}$

(iii) $x_{max} = 5+2 = 7$

(iv) maximum acceleration when $x = 7$

$\therefore 7 = 5 + 2\sin(3t - \frac{\pi}{6})$

$\therefore \sin(3t - \frac{\pi}{6}) = 1$

$\therefore 3t - \frac{\pi}{6} = \frac{\pi}{2}$

$\therefore t = \frac{1}{3} \times (\frac{\pi}{2} + \frac{\pi}{6})$

$t = \frac{1}{3} \times \frac{4\pi}{6} = \frac{2\pi}{9}$

Q14

(a)

(i) $\tan 2\theta = \frac{h}{OA}$

$\therefore OA = \frac{h}{\tan 2\theta}$.

$\tan\theta = \frac{h}{OB}$

$\therefore OB = \frac{h}{\tan\theta}$.

Q14

(a)

(ii) $d^2 = OB^2 - OA^2$

$$= \frac{h^2}{\tan^2\theta} - \frac{h^2}{\tan^2 2\theta}$$

$$= h^2\left(\frac{1}{\tan^2\theta} - \frac{1}{\tan^2 2\theta}\right).$$

$$= h^2\left(\frac{1}{\tan^2\theta} - \frac{(1-\tan^2\theta)^2}{4\tan^2\theta}\right).$$

$$= h^2\left(\frac{4 - 1 + 2\tan^2\theta - \tan^4\theta}{4\tan^2\theta}\right).$$

$$= h^2\left(\frac{3 + 2\tan^2\theta - \tan^4\theta}{4\tan^2\theta}\right)$$

$$= h^2\,\frac{(3-\tan^2\theta)(1+\tan^2\theta)}{4\tan^2\theta}.$$

$$= \frac{h^2(3-\tan^2\theta)}{4\tan^2\theta \times \cos^2\theta}$$

$$= \frac{h^2\operatorname{cosec}^2\theta}{4}(3-\tan^2\theta).$$

(c) (14(b) on next page).

(i) $y = \frac{1}{4a}x^2$

$y' = \frac{1}{2a}x$

$y'(2at) = t$

$\therefore y - at^2 = t(x - 2at)$

$y - at^2 = tx - 2at^2$

$y = tx - at^2$

Q14

(c)

(ii) The reflection of the parabola is $x^2 = -4ay$.

So its intersection with the tangent is given by:

$$\frac{-x^2}{4a} = tx - at^2.$$

$-x^2 = 4atx - 4a^2t^2.$

$x^2 + 4atx - 4a^2t^2 = 0$

x coordinate of M is average of roots

$\therefore$ the x coordinate of M is given by $x = \frac{-4at}{2(1)} = -2at.$

$\therefore y = t(-2at) - at^2$

$= -2at^2 - at^2 = -3at^2.$

$\therefore M = (-2at, -3at^2).$

(iii) $\therefore t = -\frac{x}{2a}$

$\therefore y = -3a\left(-\frac{x}{2a}\right)^2$

$y = -3a \times \frac{x^2}{4a^2}$

$y = -\frac{3x^2}{4a}$

$\therefore x^2 = -\frac{4}{3}ay$

$x^2 = -4\left(\frac{a}{3}\right)y$

$\therefore$ focal length $= \frac{1}{3}$ of original focal length.

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(b) $\ddot{x} = 0$

(i)

$\dot{x} = \int 0 \, dt$

$\dot{x} = c_1$

but $t=0$ $\dot{x} = 60\cos 30$

$\dot{x} = 30\sqrt{3}$

$\therefore c_1 = 30\sqrt{3}$

$\therefore \dot{x} = 30\sqrt{3}$

$x = \int 30\sqrt{3} \, dt$

$x = 30\sqrt{3}t + c_2$

but $t=0$ $x=0$ $\therefore c_2 = 0$.

$\therefore x = 30\sqrt{3}t$.

$\ddot{y} = -10$

$\dot{y} = \int -10 \, dt$

$\therefore \dot{y} = -10t + c_3$

$t=0$ $\dot{y} = 60\sin 30$

$\dot{y} = 30$

$\therefore c_3 = 30$

$\therefore \dot{y} = -10t + 30$

$y = \int -10t + 30 \, dt$

$y = -5t^2 + 30t + c_4$

$t=0$ $y=5$ $\therefore c_4 = 5$

$\therefore y = -5t^2 + 30t + 5$.

(ii)

$\dot{y} = 0$ when $-10t + 30 = 0$.

$\therefore 10t = 30$

$t = 3$

$y(3) = -5(3)^2 + 30(3) + 5$

$= 50$

$\therefore$ max height is 50 m.

(iii) $y = 0$ when:

$-5t^2 + 30t + 5 = 0$.

$t^2 - 6t - 1 = 0$.

$t = \dfrac{6 \pm \sqrt{36 - 4(1)(-1)}}{2}$

$t = \dfrac{6 + \sqrt{40}}{2} = 3 + \sqrt{10}$

$\dot{y}(3+\sqrt{10}) = -30 - 10\sqrt{10} + 30$.

$= -10\sqrt{10}$.

$v^2 = (30\sqrt{3})^2 + (-10\sqrt{10})^2$

$v^2 = 3700$

$\therefore v = 10\sqrt{37}$ ms^{-1}

$\tan\theta = \dfrac{10\sqrt{10}}{30\sqrt{3}} = \dfrac{\sqrt{10}}{3\sqrt{3}}$

$\therefore \theta = 31°$ (nearest deg).

$\therefore v = 10\sqrt{37}$ ms^{-1} at 31° below the horizontal.