Girraween High School

2018 Year 12 Trial Higher School Certificate

Mathematics Extension 1

Time allowed: Two (2) Hours

(Plus 5 minutes reading time)

Instructions

- Attempt all questions.
- For Questions 1 -10, shade the circle for the letter corresponding to the correct answer on your answer sheet.
- For Questions 11 – 15, start each question on a new page. Each question should be clearly labelled.
- All necessary working must be shown for Questions 11– 15.
- Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- A Mathematics reference sheet is provided.
- All diagrams are NOT TO SCALE.

Section 1

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Question 1

The point C which divides the interval between $A(14, -20)$ and $B(-4,1)$ in the ratio $4: -1$ is:

(A) $(-10,8)$ **(B)** $(20, -27)$ **(C)** $(2, -6)$ **(D)** (8,-13)

Question 2

$\lim_{x \to 0} \dfrac{2x}{sin3x} =$

(A) $\frac{3}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{1}{3}$ **(D)** 3

Question 3

The number of different ways of arranging the letters of the word MOORABOOL in a circle is

(A) 362880 **(B)** 40 320 **(C)** 15 120 **(D)** 1680

Question 4

The remainder when $ax^3 + x^2 + x + 11$ is divided by $(2x + 3)$ is 5. The value of a is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Question 5

The co-efficient of x^2 in the expansion of $\left[2x + \frac{3}{x^2}\right]^{11}$ is

(A) $\binom{11}{2} \times 2^9 \times 3^2$ **(B)** $\binom{11}{3} \times 2^8 \times 3^3$ **(C)** $\binom{11}{8} \times 2^3 \times 3^8$ **(D)** $\binom{11}{9} \times 2^2 \times 3^9$

Question 6

$cos5xcos2x - sin5xsin2x =$

(A) $cos3x$ **(B)** $sin3x$ **(C)** $cos7x$ **(D)** $sin7x$

Examination continues on the following page

Question 7

$\int sin^2 3x.dx =$

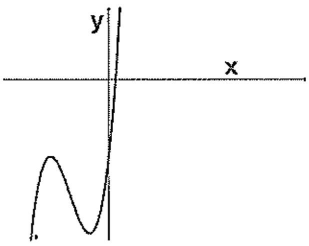
(A) $\frac{1}{3}sin^3 3x + C$ **(B)** $\frac{1}{2}x - \frac{1}{2}cos6x + C$ **(C)** $\frac{1}{2}x - \frac{1}{12}cos6x + C$

(D) $\frac{1}{2}x - \frac{1}{12}sin6x + C$

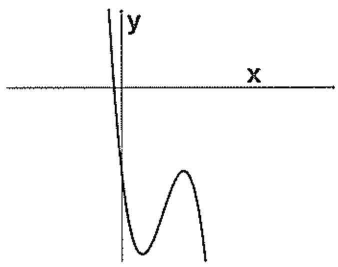
Question 8

Which of the graphs below is of $y = x^3 - 6x^2 + 9x - 4$?

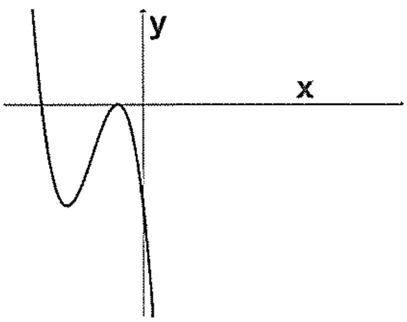
(A)



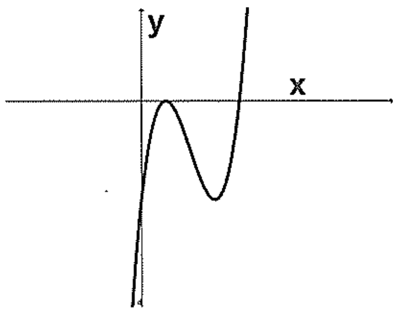
(B)



(C)



(D)



Question 9

The general solution of $sin2x = -\frac{1}{2}$ is

(A) $x = n\pi + (-1)^n \times \frac{\pi}{6}$ **(B)** $x = \frac{n\pi}{2} + (-1)^n \times \frac{\pi}{12}$ **(C)** $x = n\pi + (-1)^{n+1} \times \frac{\pi}{6}$

(D) $x = \frac{n\pi}{2} + (-1)^{n+1} \times \frac{\pi}{12}$

Question 10

If $y = sin^{-1}(5x), \frac{dy}{dx} =$

(A) $\frac{5}{\sqrt{1-25x^2}}$ **(B)** $\frac{1}{\sqrt{1-25x^2}}$ **(C)** $\frac{5}{\sqrt{25-x^2}}$ **(D)** $\frac{1}{\sqrt{25-x^2}}$

Examination continues on the following page

Section II

69 marks

Attempt Questions 11-15

Allow about 1 hour and 45 minutes for this section

Start all answers on a separate page in your answer booklet.

In Questions 11-15 your responses should include all relevant mathematical reasoning and/ or calculations.

Question 11 ***(16 Marks)*** **Marks**

(a) Solve for x: $\frac{5}{3x-2} \geq -\frac{1}{4}$ **3**

(b) Find the acute angle between the lines $y = 2x - 1$ and $x + 3y = 2$ **3**

(c) Find the *exact* value of $\cos 75^o$. **2**

(d) If α, β and γ are the roots of the polynomial equation

$2x^3 - x^2 + 3x - 4 = 0$, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$ **2**

(e) Use the substitution $u = cosx$ to find $\int_0^{\frac{\pi}{4}} \frac{\sin x}{1+\cos x}.dx$ **3**

(f) Use the substitution $x = u^2$ to find $\int \frac{1}{1+\sqrt{x}}.dx$ **3**

Question 12 ***(12 Marks)***

(a) Use the method of mathematical induction to prove

$2 + 6 + 20 + \ldots + (n^2 + n) = \frac{n}{3}(n + 1)(n + 2)$ for all positive integers n. **4**

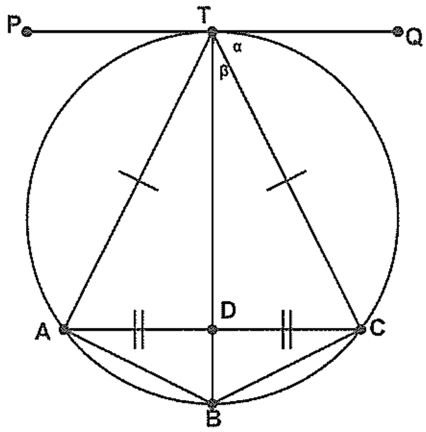
Question 12 continues on the following page

Question 12 (continued) **Marks**

(b) In the diagram below, line PTQ is a tangent to the circle $TABC$ at T.

BT bisects AC at D and $AT = CT$. $\angle QTC = \alpha$ and $\angle CTB = \beta$.

(see diagram)



(i) Copy the diagram in to your answer booklet and state why $\angle TAC = \alpha$. **1**

(ii) Prove $\angle ATP = \alpha$. **2**

(iii) Prove $\angle ATB = \beta$. **2**

(iv) Prove TB is a diameter of the circle $TABC$. **3**

Question 13 ***(14 Marks)*** **Marks**

(a) Sketch the graph of $y = 2cos^{-1}\left(\frac{x}{3}\right)$, stating its domain and range. **3**

(b) The probability that it will rain on a certain day in August in Sydney is $\frac{2}{9}$. Assuming that the probability that it will rain on one day is *independent* of the probability that it will rain on the next day:

(i) Find the probability that it will rain on 3 days in the next week. **2**

(ii) Find the probability that it will rain on at least 2 days in the next week. **2**

Question 13 continues on the following page

Question 13 (continued) **Marks**

(c) A particle is moving so that its position at time t is given by

$x = 2sin3t + 2\sqrt{3}cos3t.$

(i) By showing that $\ddot{x} = -n^2x$, n a real number, prove that the particle is moving with simple harmonic motion. **1**

(ii) By expressing the particle's position at time t in the form $x = Rcos(3t - \alpha)$, find the period and amplitude of the motion. **3**

(iii) Find the first time that the particle reaches $x = 0$ and find its velocity and acceleration then. **3**

Question 14 *(14 Marks)*

(a) A particle is moving with simple harmonic motion about $x = 0$ with acceleration $\ddot{x} = -n^2x$ and amplitude a. *(Note that in this question a refers to the amplitude of the motion).*

(i) Show that $v^2 = n^2(a^2 - x^2)$. **2**

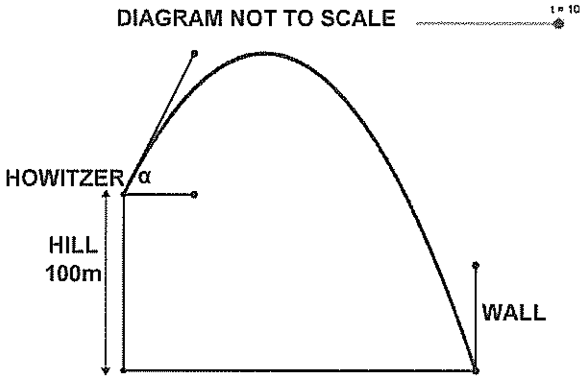
(ii) For this particle, $v = 10$ when $x = 5$ and $v = 5$ when $x = 7$. **3**

Find the period of the motion.

Question 14 continues on the following page

Question 14 (continued) **Marks**

(b) A howitzer with a muzzle velocity of $200m/s$ is located $100m$ up the side of a hill. It is firing shells at an angle of α to the horizontal. *(see diagram)*



(i) Given that $\ddot{x} = 0$ and $\ddot{y} = -g$, show that $x = 200t\cos\alpha$ and $y = -\frac{1}{2}gt^2 + 200t\sin\alpha + 100.$ **4**

(ii) Show that $y = -\frac{gx^2}{80\,000}\sec^2\alpha + x\tan\alpha + 100.$ **2**

(iii) The shell is being aimed at the base of a wall $3000m$ away horizontally. At which two angles must the shell be fired in order to hit this target? *(Let $g = 10m/s^2$).* **3**

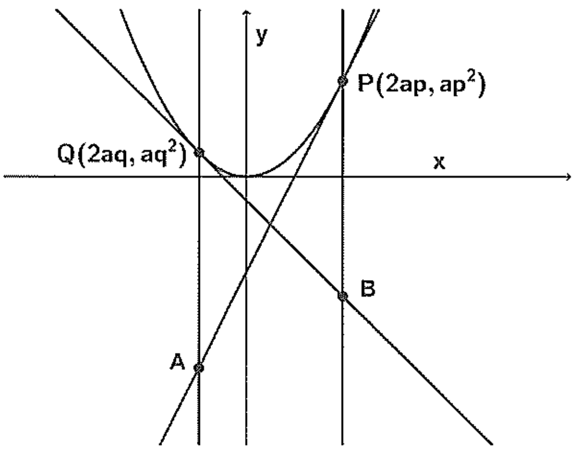
Examination continues on the following page

Question 15 *(13 Marks)* **Marks**

(a) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are variable points on the parabola $x^2 = 4ay$. A is the intersection of the tangent to the parabola at P and the line through Q parallel to the axis of the parabola, while B is the intersection of the tangent to the parabola at Q and the line through P parallel to the axis of the parabola. *(see diagram)*

The equation of the tangent to the parabola at P is $y = px - ap^2$

(Do NOT prove this!).



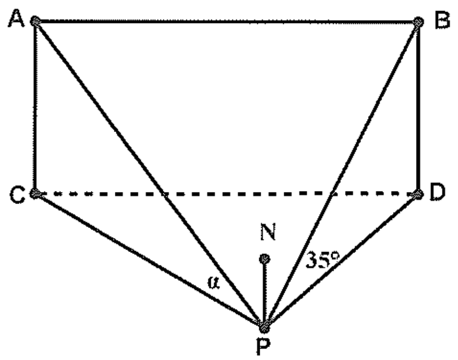
(i) Prove that $PQAB$ is a parallelogram. **3**

(ii) If $p > q$, find the area of the parallelogram in terms of p and q. **2**

Question 15 continues on the following page

Question 15 (continued) **Marks**

(b) An aeroplane flying due East at $900km/h$ at a constant altitude is seen by an observer on the ground on a bearing of 290^oT at an angle of elevation of α. One minute later it is sighted by the observer at an angle of elevation of 35^o and on a bearing of 050^oT. If the aeroplane flies from A to B, the observer is at P and C and D are on the ground directly beneath A and B *(see diagram)*



(i) Show that $AB = 15000m$ and $\angle CPD = 120^O$. **2**

(ii) Show that the height of the aeroplane above the ground is given by **3**

$$BD = \frac{15000sin20^o tan35^o}{sin60^o}.$$

(iii) Find the angle of elevation (α) when the aeroplane is first seen **3**

(answer to the nearest minute).

END OF EXAMINATION

Maths Ext 1 GHS 2018 Trial
Solutions

(1) A (2) B (3) D (4) B (5) B (6) C (7) D (8) D (9) D (10) A

Q.(1) 14   −4   −20   1

4   −1   4   −1

$= \left(\frac{-14-16}{4-1}, \quad \frac{20+4}{4-1} \right)$

$= (-10, \quad 8)$. A

Q.(2) $= \frac{2}{3} \lim_{x \to 0} \frac{3x}{\sin 3x} = \frac{2}{3} \times 1 = \frac{2}{3}$. B

1680 ways

Q.(3) $= \frac{8!}{4!}$ D

$= 1680$ ways.

Q.(4) $a\left(\frac{-3}{2}\right)^3 + \left(\frac{-3}{2}\right)^2 + \left(\frac{-3}{2}\right) + 11 = 5$ by remainder theorem.

$\frac{-27a}{8} + \frac{47}{4} = 5$. B

$a = 2$.

Q.(5) $x^{11-k} \times (x^{-2})^k = x^2$

$11 - 3k = 2$.

$k = 3$.

Co-efficient $= T_4$ co-eff

$= {}^{11}C_3 \times 2^8 \times 3^3$ B

Q.(6) $= \cos(5x + 2x)$

$= \cos 7x$ C

Q.(7) $\int \sin^2 3x \, dx$

$= \frac{1}{2} \int 1 - \cos 6x \, dx$

$= \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C$

$= \frac{1}{2}x - \frac{1}{12}\sin 6x + C$. D

Q.(8) $y = x^3 - 6x^2 + 9x - 4$

8&C out as $-x^3$.

$y' = 3x^2 - 12x + 9$.

$y' = 0$: $x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1$ or 3 → must be D

Q.(9) $\sin 2x = -\frac{1}{2}$

$$2x = n\pi + (-1)^n \times -\frac{\pi}{6}$$

$$= n\pi + (-1)^{n+1} \times \frac{\pi}{6}$$

$$\therefore x = \frac{n\pi}{2} + (-1)^{n+1} \times \frac{\pi}{12} \quad \text{D}$$

19 min

Q.(10) $y = \sin^{-1}(5x)$

$$y' = \frac{1}{\sqrt{\frac{1}{25} - x^2}} \times 5 \quad \times 5$$

$$= \frac{5}{\sqrt{1-25x^2}} \quad \text{A}$$

Q.(11)(a) By critical points:

Equalities:

$$\frac{5}{3x-2} = -\frac{1}{4}$$

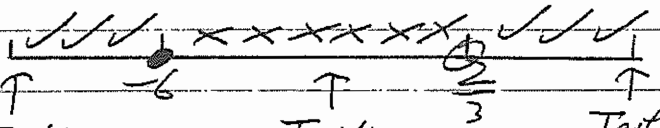
$\times 4(3x-2)$

$$20 = -3x+2$$

$$\underline{-6 = x}$$

Discontinuity:

$$x \neq \frac{2}{3}$$



Test: $x=-7$

$$\frac{5}{3\times -7-2} = \frac{-5}{23} \geq -\frac{1}{4} \checkmark$$

Test: $x=0$

$$\frac{5}{3\times 0-2} = \frac{-5}{2} \not\geq \frac{-1}{4} \times$$

Test: $x=1$

$$\frac{5}{3\times 1-2} = 5 \geq -\frac{1}{4} \checkmark$$

$$\therefore x \leq -6 \ \& \ x > \frac{2}{3}$$

By "New York":

$$\frac{5}{3x-2} \geq -\frac{1}{4}$$

$\times 4(3x-2)^2$

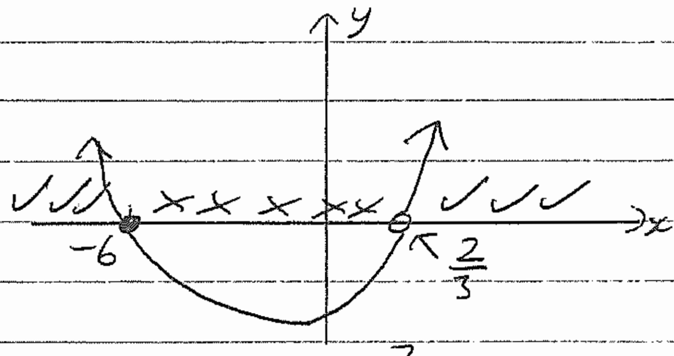
$$20(3x-2) \geq -(3x-2)^2$$

$$60x-40 \geq -9x^2+12x-4$$

$$9x^2+48x-36 \geq 0$$

$$3x^2+16x-12 \geq 0$$

$$(3x-2)(x+6) \geq 0$$



$$\underline{x \leq -6 \text{ or } x > \frac{2}{3}}$$

(b) $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$$= \frac{2 + \frac{1}{3}}{1 + 2 \times -\frac{1}{3}}$$

$$= 7$$

$$\underline{\theta = 81^\circ 52'}$$

(c) $\cos 75^\circ$

$$= \cos(45^\circ + 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

Note: Equivalents such as $\frac{\sqrt{2-\sqrt{3}}}{2}$ are also OK.

Q (11) [continued]:

(d) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\alpha\gamma} + \dfrac{1}{\beta\gamma}$

$= \dfrac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$

$= \dfrac{(\frac{1}{2})}{(\frac{4}{2})}$

$= \dfrac{1}{4}.$

(e) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x}{1+\cos x} . dx$

$\displaystyle = -\int_0^{\frac{\pi}{4}} \frac{1}{1+\cos x} . -\sin x . dx$

Let $u = \cos x$. $du = -\sin x . dx$

$\displaystyle = -\int_1^{\frac{1}{\sqrt{2}}} \frac{1}{1+u} . du$

$= -\left[\ln(1+u)\right]_1^{\frac{1}{\sqrt{2}}}$

$= \ln\left[1+u\right]_{\frac{1}{\sqrt{2}}}^{1}$

$= \ln 2 - \ln\left(1+\frac{1}{\sqrt{2}}\right)$

$= 0.1583$ [4DP].

(f) $\displaystyle\int \frac{1}{1+\sqrt{x}} . dx$ Let $x = u^2$. $dx = 2u . du$ [$u = \sqrt{x}$].

$\displaystyle = \int \frac{1}{1+u} . 2u . du$

$\displaystyle = \int \frac{2u+2}{1+u} - \frac{2}{1+u} . du$

$\displaystyle = \int 2 - \frac{2}{1+u} . du$

$= 2u - 2\ln(1+u) + C$

$= 2\sqrt{x} - 2\ln(1+\sqrt{x}) + C$

Q. (12)(a) Step 1: Show true for $n=1$.

LHS $= 2$

RHS $= \frac{1}{3}(1+1)(1+2)$
$= 2$

LHS $=$ RHS. True for $n=1$.

Step 2: Assume true for $n=k$

i.e. $2+6+20+\ldots+(k^2+k) = \frac{k}{3}(k+1)(k+2)$

Step 3: Prove true for $n=k+1$

i.e. $2+6+20+\ldots+(k^2+k)+(k+1)^2+(k+1) = \frac{(k+1)}{3}(k+2)(k+3)$

LHS:

$2+6+20+\ldots+(k^2+k)+(k+1)^2+(k+1)$

$= \frac{k}{3}(k+1)(k+2) + (k+1)^2 + (k+1)$ [Using step 2 or By assumption].

$= \frac{(k+1)}{3}\left[k(k+2) + 3(k+1) + 3\right]$

$= \frac{(k+1)}{3}\left[k^2+5k+6\right]$

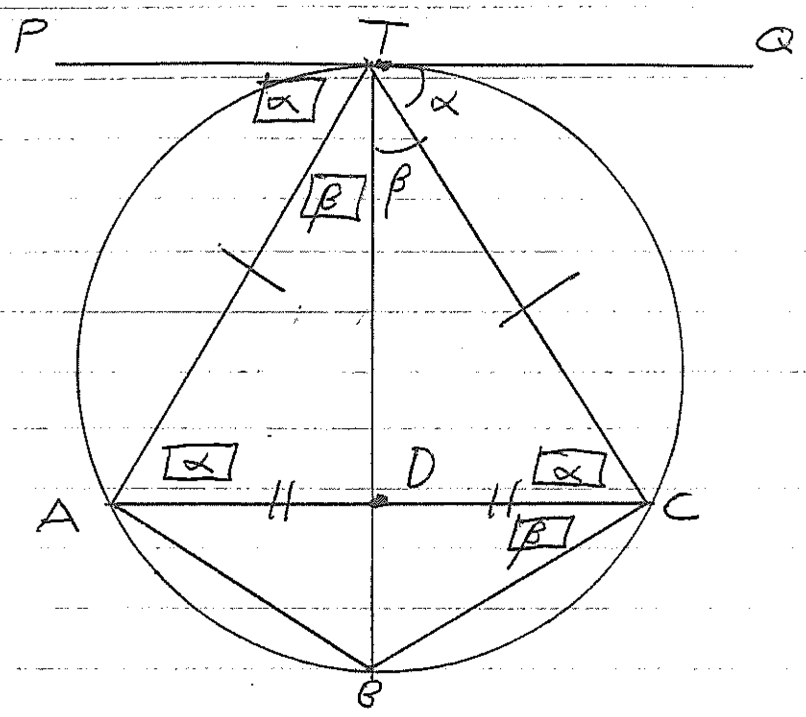
$= \frac{(k+1)}{3}(k+2)(k+3)$

$=$ RHS.

$\therefore$ If it is true for $n=k$ it will be true for $n=k+1$.

$\therefore$ As it is true for $n=1$ it will be true for $n=1+1=2$ & so on for all positive integers n.

Q.(12).(b)
(i)



$\angle TAC = \alpha$ [$\angle$ between tangent & chord = $\angle$ in alternate segment].

(ii) $\angle ACT = \alpha$ [$\angle$'s opposite = sides in isosceles $\triangle TAC$].
$\angle ATP = \alpha$ [$\angle$ between tangent & chord = $\angle$ in alternate segment].

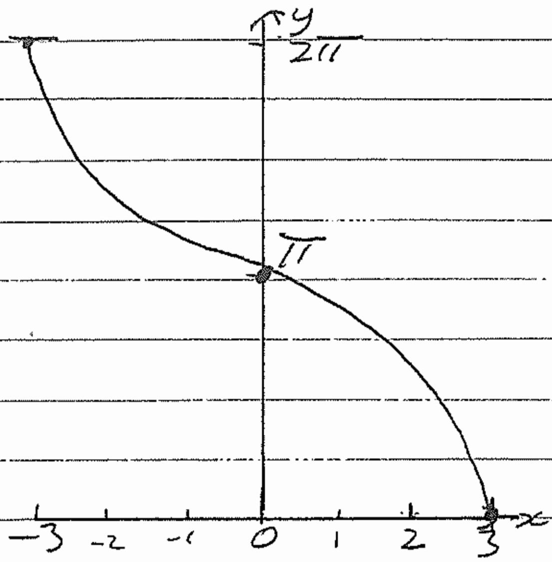
(iii) In $\triangle ATD$ & $\triangle CTD$
$AT = CT$ [data]
$AD = CD$ [data]
$\angle TAD = \angle TCD = \alpha$ [proven in (ii)]
$\triangle ATD \equiv \triangle CTD$ [SAS].
$\therefore \angle ATD = \angle CTD = \beta$ [matching $\angle$'s, $\triangle ATD \equiv \triangle CTD$].

(iv) $2\alpha + 2\beta = 180^\circ$ [$\angle$'s in straight line PTQ].
$\therefore \alpha + \beta = 90^\circ$.
$\angle ACB = \angle ATB = \beta$ [$\angle$'s at circumference subtended by arc AB].
$\therefore \angle TCB = \alpha + \beta = 90^\circ$ [adjacent $\angle$'s].
TB is a diameter of circle TACB [$\angle$ in semicircle $= 90^\circ$].

Q. (13)(a) $y = 2\cos^{-1}\left(\frac{x}{3}\right)$

Key points:

$\frac{x}{3} = -1$	$\frac{x}{3} = 0$	$\frac{x}{3} = 1$	Domain: $-3 \leq x \leq 3$.
$x = -3$.	$x = 0$	$x = 3$.	Range: $0 \leq y \leq 2\pi$.
$y = 2\cos^{-1}(-1)$	$y = 2\cos^{-1}(0)$	$y = 2\cos^{-1}(1)$	
$= 2\pi$.	$= \pi$.	$= 0$.	



(b)(i) $Pr = {}^7C_3 \times \left(\frac{2}{9}\right)^3 \times \left(\frac{7}{9}\right)^4$

$= 0.1405570473\ldots$

$\doteq 0.1406$ (4DP).

(ii) $Pr = 1 - \left(P(0) \text{ days} + P(1 \text{ day})\right)$

$= 1 - \left[{}^7C_0 \times \left(\frac{2}{9}\right)^0 \times \left(\frac{7}{9}\right)^7 + {}^7C_1 \times \left(\frac{2}{9}\right)^1 \times \left(\frac{7}{9}\right)^6\right]$

$= 0.4834528511\ldots$

$= 0.4835$ (4DP).

(c)(i) $x = 2\sin 3t + 2\sqrt{3}\cos 3t$

$\dot{x} = 6\cos 3t - 6\sqrt{3}\sin 3t$

$\ddot{x} = -18\sin 3t - 18\sqrt{3}\cos 3t$

$= -9(2\sin 3t + 2\sqrt{3}\cos 3t)$

$\ddot{x} = -9x$

$\therefore$ Particle is moving with SHM.

Q. (13)(c)(ii) Let $x = 2\sin 3t + 2\sqrt{3}\cos 3t \equiv R\cos(3t - \alpha)$

$\therefore 2\sin 3t + 2\sqrt{3}\cos 3t \equiv R\cos 3t\cos\alpha + R\sin 3t\sin\alpha$

Equating parts,

$2\sin 3t = R\sin 3t\sin\alpha$ | $2\sqrt{3}\cos 3t = R\cos 3t\cos\alpha$

$2 = R\sin\alpha \quad (1)$ | $2\sqrt{3} = R\cos\alpha. \quad (2)$

Squaring & adding (1) & (2):

$R^2(\sin^2\alpha + \cos^2\alpha) = 2^2 + (2\sqrt{3})^2.$

$R = 4.$

Sub. $R = 4$ in (1):

$2 = 4\sin\alpha$

$\frac{1}{2} = \sin\alpha.$

Sub. $R = 4$ in (2):

$2\sqrt{3} = 4\cos\alpha$

$\frac{\sqrt{3}}{2} = \cos\alpha.$

$\alpha = \frac{\pi}{6}.$

$\therefore \underline{x = 4\cos(3t - \frac{\pi}{6})}.$

Period of motion $= \frac{2\pi}{3}$ seconds. Amplitude $= 4$m.

(iii) Particle reaches $x = 0$:

$4\cos(3t - \frac{\pi}{6}) = 0$

$\cos(3t - \frac{\pi}{6}) = 0$

$3t - \frac{\pi}{6} = \frac{\pi}{2}.$

$t = \frac{2\pi}{9}$ seconds.

When $t = \frac{2\pi}{9}$ seconds,

$\dot{x} = -12\sin(3t - \frac{\pi}{6})$

$= -12\sin(3\times\frac{2\pi}{9} - \frac{\pi}{6})$

$= -12\sin\frac{\pi}{2}$

$= -12$ m/s.

$\ddot{x} = 36\cos(3t - \frac{\pi}{6})$

$= 36\cos\frac{\pi}{2}$

$= 0$ m/s^2.

[Could also use
$\ddot{x} = -9x$
$= -9\times 0$
$= 0$ m/s^2.]

(14)(a) $\ddot{x} = -n^2 x$.

(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -n^2 x$.

$$\frac{d}{dx}\left(v^2\right) = -2n^2 x.$$

$$v^2 = \int -2n^2 x \cdot dx$$

$$= -n^2 x^2 + C.$$

As $v = 0$ when $x = a$, $\underline{2}$

$$0^2 = -n^2 a^2 + C$$

$$n^2 a^2 = C$$

$$v^2 = n^2 a^2 - n^2 x^2$$

$$v^2 = n^2 (a^2 - x^2).$$

(ii) $v = 10$ when $x = 5$. $10^2 = n^2(a^2 - 5^2)$

$$100 = n^2 a^2 - 25n^2 \quad (1)$$

$v = 5$ when $x = 7$. $5^2 = n^2(a^2 - 7^2)$

$$25 = n^2 a^2 - 49n^2 \quad (2)$$

(1) − (2) $\quad 75 = 24n^2$. $\underline{3}$

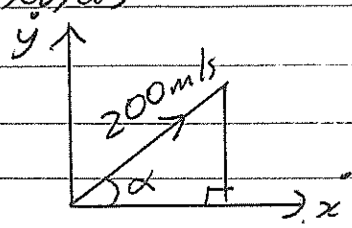
$$n = \frac{5\sqrt{2}}{4}.$$

Period of motion $= \frac{2\pi}{n}$

$$= \frac{2\pi}{\left(\frac{5\sqrt{2}}{4}\right)}$$

$$= \frac{4\pi\sqrt{2}}{5} \text{ time units (seconds).}$$

Q(14)(b)(i)



Initial $\dot{x} = 200\cos\alpha$

Initial $\dot{y} = 200\sin\alpha$.

$\ddot{x} = 0$

$\dot{x} = \int 0 . dt$

$= C$

$= 200\cos\alpha$.

$x = \int 200\cos\alpha . dt$

$= 200t\cos\alpha + C$

As $x = 0$ when $t = 0$,

$0 = 200 \times 0 \times \cos\alpha + C$

$0 = C$

$x = 200t\cos\alpha$ (1)

$\ddot{y} = -g$

$\dot{y} = \int -g . dt$

$= -gt + C$

As $\dot{y} = 200\sin\alpha$ when $t = 0$

$200\sin\alpha = -g \times 0 + C$.

$\dot{y} = -gt + 200\sin\alpha$

$y = \int -gt + 200\sin\alpha . dt$

$= -\frac{1}{2}gt^2 + 200t\sin\alpha + C'$.

As $y = 100$ when $t = 0$

$100 = -\frac{1}{2} \times g \times 0^2 + 200 \times 0 \times \sin\alpha + C$

$y = -\frac{1}{2}gt^2 + 200t\sin\alpha + 100$. (2)

(ii) $x = 200t\cos\alpha$

$\frac{x}{200\cos\alpha} = t$ (3).

Sub. (3) in (2):

$y = -\frac{1}{2} \times g \times \left(\frac{x}{200\cos\alpha}\right)^2 + 200\sin\alpha \times \frac{x}{200\cos\alpha} + 100$

$y = \frac{-gx^2\sec^2\alpha}{80\,000} + x\tan\alpha + 100$

(iii) $y = 0$ when $x = 3000$ [$g = 10$].

$0 = -\frac{10 \times 3000^2}{80\,000}(1 + \tan^2\alpha) + 3000\tan\alpha + 100$

$0 = -1125(1 + \tan^2\alpha) + 3000\tan\alpha + 100 \quad \div -25.$

$0 = 45\tan^2\alpha - 120\tan\alpha + 41.$

$\tan\alpha = \frac{120 \pm \sqrt{(-120)^2 - 4 \times 45 \times 41}}{2 \times 45}.$

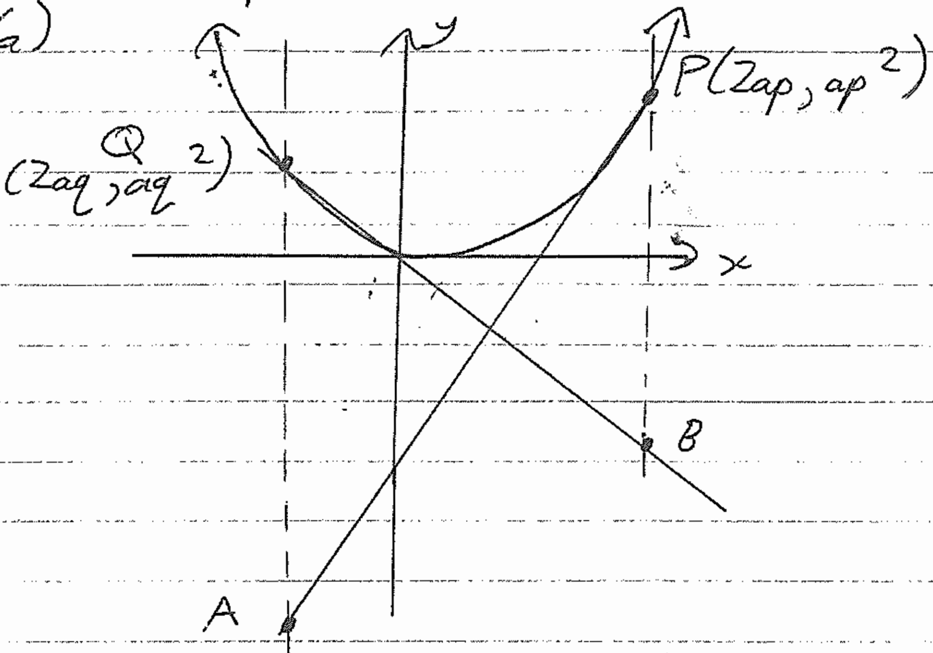
$\tan\alpha = \frac{120 \pm \sqrt{7020}}{90}$

$\alpha = 66°10'$ or

$\alpha = 21°55'$.

Shell needs to be fired at $21°55'$ or $66°10'$ to hit target.

Q.(15)(a)



(i) Line QA is $x = 2aq$.

Finding y co-ordinate of A:

$y = px - ap^2$

$= p \times 2aq - ap^2$

$= 2apq - ap^2$

$\therefore A = (2aq, 2apq - ap^2)$

Similarly, $B = (2ap, 2apq - aq^2)$.

To prove parallelogram

Either

Distance $QA = aq^2 - (2apq - ap^2)$

$= a(q^2 - 2pq + p^2)$

$= a(q-p)^2$.

Similarly, distance $PB = ap^2 - (2apq - aq^2)$

$= a(p-q)^2$

$= a(q-p)^2$.

$\therefore$ PQAB is a parallelogram
[1 pair of opposite sides = & ∥].

Note also that finding midpoint PA & midpoint QB & finding they coincide [∥ as diagonals bisect each other].
or showing $PQ = BA$ as well as $QA = PB$ [both pairs of opposite sides =] are also acceptable.

Or

$m\,PQ = \frac{ap^2 - aq^2}{2ap - 2aq}$

$= \frac{a(p-q)(p+q)}{2a(p-q)}$

$= \frac{p+q}{2},\ p \neq q$

$m\,BA = \frac{2apq - aq^2 - (2apq - ap^2)}{2ap - 2aq}$

$= \frac{a(\ \ -\ \ )}{2a(p-q)}$

$= \frac{a(\ \ -\ \ )(\ \ +\ \ )}{2a(p-q)}$

$= \frac{p+q}{2}$

$= m\,PQ$

$\therefore$ PQAB is a parallelogram
[both pairs of opposite sides ∥].

Q.(15)(a)(continued):

(ii) $A = Lh$

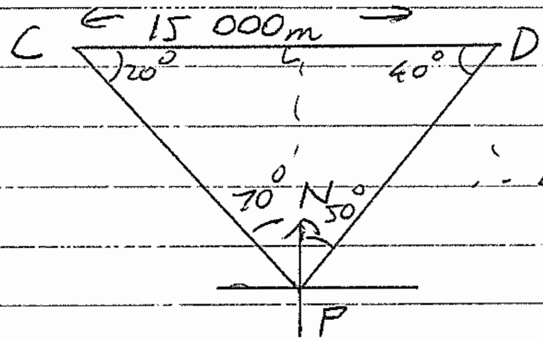
$= a(q-p)^2 \times (2ap - 2aq)$

$= a(p-q)^2 \times 2a(p-q)$ [Noting that $(p-q)^2 = (q-p)^2$].

$= 2a^2(p-q)^3$.

(b)(i) In 1 minute,

plane flies $\frac{900\,000}{60} = 15000$m.



$\angle CPN = 70°$ [bearing $PC = 290°$].

$\angle DPN = 50°$ [bearing $PD = 050°$].

$\therefore \angle CPD = 120°$ [adjacent $\angle$'s].

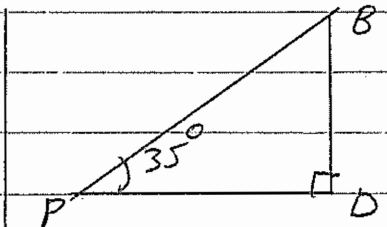
(ii) Finding PD: Using $\triangle CPD$

$\frac{PD}{\sin 20°} = \frac{15000}{\sin 120°}$

$\frac{PD}{\sin 20°} = \frac{15000}{\sin 60°}$ [as $\sin 60° = \sin 120°$].

$\times \sin 20°$

$PD = \frac{15000 \sin 20°}{\sin 60°}$.



$\tan 35° = \frac{BD}{PD}$

$\therefore PD \tan 35° = BD$

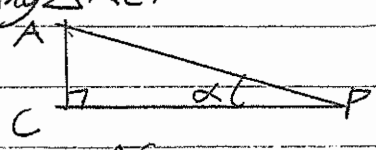
$\frac{15000 \sin 20° \tan 35°}{\sin 60°} = BD = $ Height.

(iii) Using $\triangle CPD$

$\frac{CP}{\sin 40°} = \frac{15000}{\sin 60°}$

$CP = \frac{15000 \sin 40°}{\sin 60°}$.

Using $\triangle ACP$



$\tan\alpha = \frac{AC}{CP}$

$= \frac{15000 \sin 20° \tan 35°}{\sin 60°} \times \frac{\sin 60°}{15000 \sin 40°}$

$\tan\alpha = \frac{\sin 20° \tan 35°}{\sin 40°}$

$\alpha = 20°26'$.

END OF SOLUTIONS!