Student number: ______________

GIRRAWEEN HIGH SCHOOL

2020 | TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt all questions
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. The unit vector in the direction of $\underset{\sim}{u} = \underset{\sim}{i} - 2\underset{\sim}{j}$ is

A. $5\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)$

B. $\frac{1}{5}\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)$

C. $\sqrt{5}\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)$

D. $\frac{1}{\sqrt{5}}\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)$

2. A vector perpendicular to $3\underset{\sim}{i} + 4\underset{\sim}{j}$ and with magnitude 5 is

A. $-4\underset{\sim}{i} - 3\underset{\sim}{j}$

B. $5\left(4\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

C. $4\underset{\sim}{i} - 3\underset{\sim}{j}$

D. $\sqrt{5}\left(4\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

3. Which of the following is the derivative of $\tan^{-1}(3x)$?

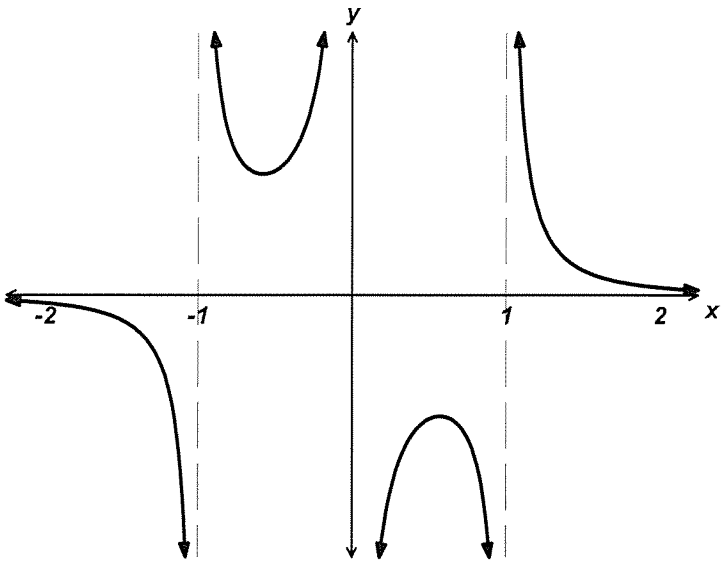
A. $3\tan^{-1}x$

B. $\frac{3}{1 + x^2}$

C. $\frac{3}{1 + 9x^2}$

D. $3\ \sec^2 3x$

4.



The graph above shows $y = \dfrac{1}{f(x)}$.

Which one of the following equations best represents $f(x)$?

A. $f(x) = x^2 - 1$

B. $f(x) = x(x^2 - 1)$

C. $f(x) = x^2(x^2 - 1)$

D. $f(x) = x^2(x^2 - 1)^2$

5. What is the value of $\sin 2x$ given that $\sin x = \dfrac{\sqrt{3}}{2}$ and x is obtuse?

A. $-\dfrac{\sqrt{3}}{4}$ B. $-\dfrac{\sqrt{3}}{2}$ C. $\dfrac{\sqrt{3}}{4}$ D. $\dfrac{\sqrt{3}}{2}$

6. Four females and four male students are to be seated around a circular table.
In how many ways can this be done if the males and females must alternate?

A. $4! \times 4!$ B. $3! \times 4!$ C. $3! \times 3!$ D. $2 \times 3! \times 3!$

7. If $\sin(\alpha + \beta) = a$ and $\sin(\alpha - \beta) = b$, then $\sin\alpha\cos\beta$ is equal to

A. $\sqrt{a^2 + b^2}$

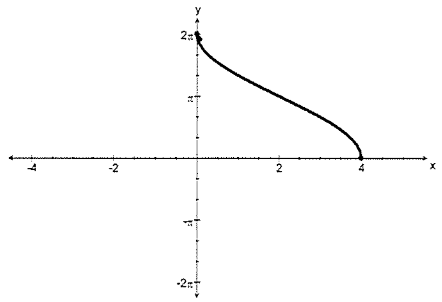
B. $\sqrt{ab}$

C. $\sqrt{a^2 - b^2}$

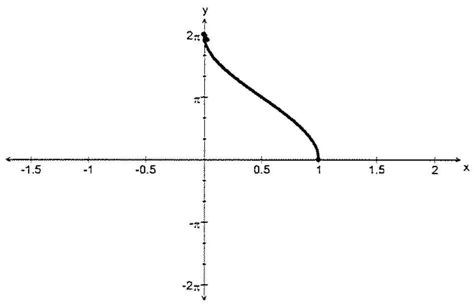
D. $\dfrac{a + b}{2}$

8. Which of the graphs below shows $y = 2\cos^{-1}\left(\dfrac{x}{2} - 1\right)$?

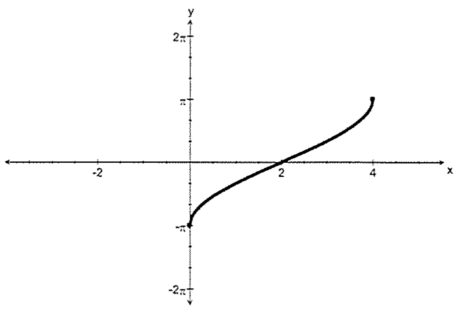
A.



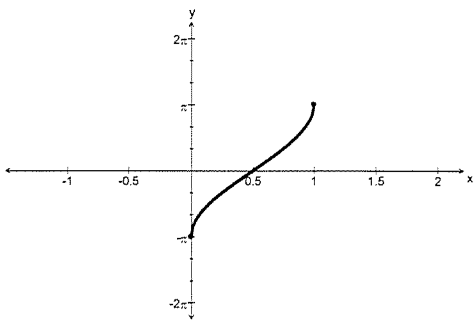
B.



C.



D.



9. $\sin 3x \sin 4y =$

A. $\frac{1}{2}[\cos(3x - 4y) - \cos(3x + 4y)]$

B. $\frac{1}{2}[\cos(3x + 4y) - \cos(3x + 4y)]$

C. $\frac{1}{2}[\cos(3x - 4y) - \cos(3x - 4y)]$

D. $\frac{1}{2}[\cos(3x - 4y) - \sin(3x + 4y)]$

10. When the polynomial $P(x)$ is divided by $(x - 2)$ and $(x + 1)$ the respective remainders are 5 and 8.
What is the remainder when $P(x)$ is divided by $(x - 2)(x + 1)$?

A. $7 - x$

B. $x - 7$

C. $x + 7$

D. $-7 - x$

Section II

60 marks
Attempt all questions
Allow about 1 hour and 45 minutes for this section

Start each question on a new page in the answer booklet provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is available on request.

Question 11 (14 marks)

a. Solve $\dfrac{3}{4x-1} \geq 2$ [3]

b. Differentiate $e^x \tan^{-1} x$ [2]

c. Consider the curve $f(x) = x^2 - 4x + 5$

(i) Find the largest possible positive domain for which $f(x)$ has an inverse function $f^{-1}(x)$. [1]

(ii) Find the point(s) of intersection of $y = f(x)$ and $y = f^{-1}(x)$ in the domain determined in part (i). [2]

(iii) State the domain of $y = f^{-1}(x)$. [1]

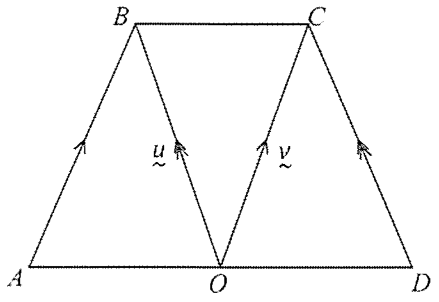
(iv) What is the equation of $y = f^{-1}(x)$? [2]

(v) For the restricted domain, sketch the function and the inverse function on the same number plane.
Clearly label the graphs and show the main features. [3]

Question 12 (11 marks)

a. Evaluate $\displaystyle\int_0^{15} \frac{x}{\sqrt{x+1}}\,dx$ using the substitution $u^2 = x + 1$. [2]

b.



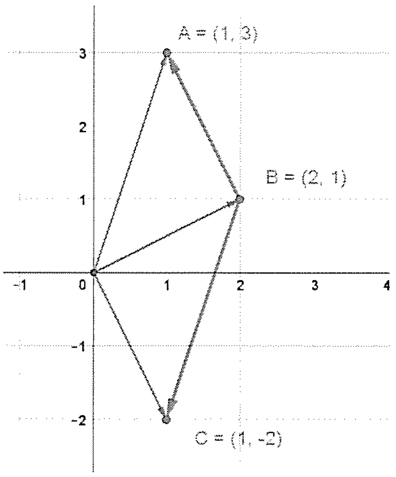
$AB \,//\, OC$; $DC \,//\, OB$; $\overrightarrow{OB} = \underset{\sim}{u}$; $\overrightarrow{OC} = \underset{\sim}{v}$

$AB = OB = OC = DC$

(i) Express $\overrightarrow{AD}$ in terms of $\underset{\sim}{u}$ and $\underset{\sim}{v}$ [2]

(ii) Express $\overrightarrow{BD}$ in terms of $\underset{\sim}{u}$ and $\underset{\sim}{v}$ [1]

c.



A, B and C are points defined by the position vectors

$\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{c} = \underset{\sim}{i} - 2\underset{\sim}{j}$ respectively.

(i) Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$ [2]

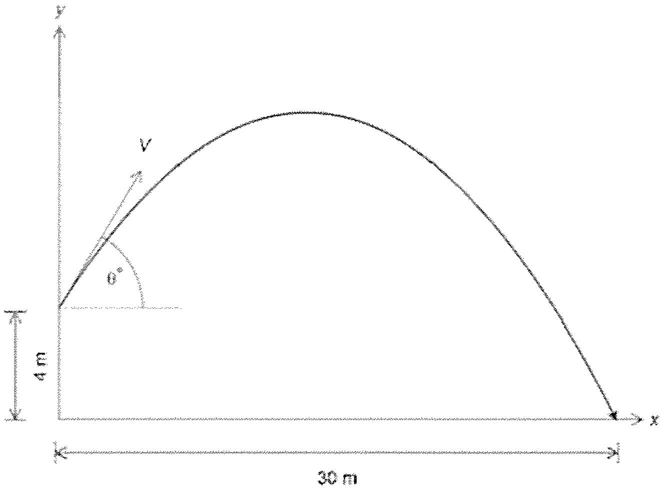
(ii) Find $|\overrightarrow{BA}|$ and $|\overrightarrow{BC}|$ [2]

(iii) Find $\angle ABC$ [2]

Question 13 (13 marks)

a. Use mathematical induction to prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7 for integers $n \geq 1$. [3]

b.



A rock is projected with a speed of $V\ ms^{-1}$ from a point 4 metres above a flat ground. The angle of projection to the horizontal is θ as shown.

(i) Taking the ground as the origin and the acceleration due to gravity as $10ms^{-2}$, show that $x = Vt\cos\theta$ and $y = Vt\sin\theta - 5t^2 + 4$ [2]

(ii) If the rock hits the ground 30 metres away, find the value of V if

$$\theta = \tan^{-1}\left(\frac{5}{12}\right)$$ [3]

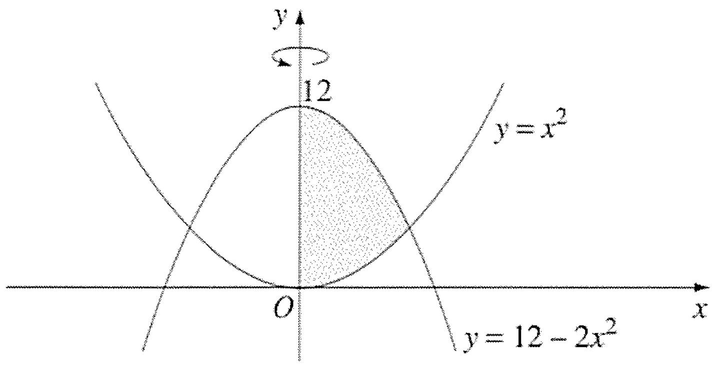
(iii) Find the maximum height reached. [2]

c. Let $P(x) = x^3 + ax^2 + bx + 5$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers.

Find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ given that $(x - 1)^2$ is a factor of $P(x)$. [3]

Question 14 (11 marks)

a.



The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.

(i) Find the points of intersection of the two curves. [1]

(ii) The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two regions, or otherwise, find the volume of the solid formed. [3]

b. A salad, which is initially at a temperature of 25°C, is placed in a refrigerator that has a constant temperature of 3°C. The cooling rate of the salad is proportional to the difference between the temperature in the refrigerator and the temperature, T, of the salad. That is T satisfies the equation

$$\frac{dT}{dt} = -k(T - 3)$$

where t is the number of minutes after the salad is placed in the refrigerator.

(i) Show that $T = 3 + Ae^{-kt}$ satisfies this equation. [1]

(ii) The temperature of the salad is 11°C after 10 minutes.
Find the temperature of the salad after 15 minutes. [3]

c. How many numbers greater than 6000 can be formed with the digits 1, 4, 5, 7, 8 if no digit is repeated? [2]

Question 15 (11 marks)

a. (i) Show that

$2\sin 2x - 3\cos 2x - 3\sin x + 3 = \sin x(6\sin x + 4\cos x - 3)$ [2]

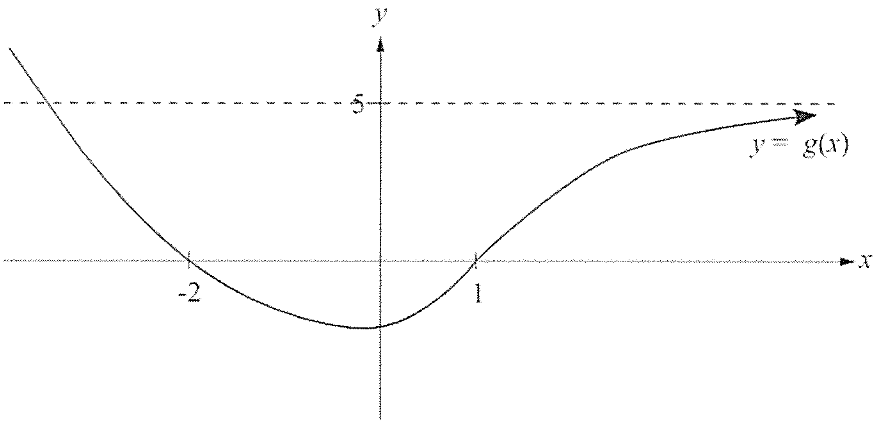
(ii) Express $6\sin x + 4\cos x$ in the form $R\sin(x + \alpha)$ where

$R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$ [2]

(iii) Hence, solve $2\sin 2x - 3\cos 2x - 3\sin x + 3 = 0$ for $0 \leq x < \pi$

Answer in radians, correct to 2 decimal places. [2]

b. The diagram shows the graph of the function $y = g(x)$



Draw a graph of $y = \sqrt{g(x)}$, showing any asymptotes and stating its domain and range. [3]

c. An equation can be expressed in the parametric form

$x = 2\cos\theta - 1$

$y = 2 + 2\sin\theta$

Express the equation in Cartesian form. [2]

End of Examination

TRIAL HSC

2020 GHS Extension 1 SOLUTIONS

Section I

MC

1. unit vector $= \dfrac{\underset{\sim}{i} - 2\underset{\sim}{j}}{\sqrt{(1)^2 + (-2)^2}}$

$= \dfrac{\underset{\sim}{i} - 2\underset{\sim}{j}}{\sqrt{5}}$ $\boxed{D}$

2. $\boxed{C}$

$(4\underset{\sim}{i} - 3\underset{\sim}{j}) \cdot (3\underset{\sim}{i} + 4\underset{\sim}{j})$

$= (4 \times 3 - 3 \times 4) = 0 \Rightarrow$ perpendicular.

$|4\underset{\sim}{i} - 3\underset{\sim}{j}| = \sqrt{4^2 + (-3)^2} = 5 \Rightarrow$ magnitude 5

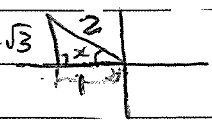
3. $y = \tan^{-1}(3x)$

$\dfrac{dy}{dx} = \dfrac{1}{1+(3x)^2} \cdot 3$

$= \dfrac{3}{1+9x^2}$ $\boxed{C}$

4. $\boxed{B}$

5. $\sin x = \dfrac{\sqrt{3}}{2}$ Q2



$\sin 2x = 2 \sin x \cos x$

$= 2 \times \dfrac{\sqrt{3}}{2} \times \dfrac{-1}{2}$

$= -\dfrac{\sqrt{3}}{2}$ $\boxed{B}$

6. $3! \times 4!$ $\boxed{B}$

7. $\sin(\alpha+\beta)$; $\sin\alpha\cos\beta + \cos\alpha\sin\beta = a$ — ①

$\sin(\alpha-\beta)$; $\sin\alpha\cos\beta - \cos\alpha\sin\beta = b$ — ②

① + ②

$$2\sin\alpha\cos\beta = a+b$$

$$\sin\alpha\cos\beta = \frac{a+b}{2}$$ $\boxed{D}$

8. $\frac{y}{2} = \cos^{-1}\left(\frac{x}{2} - 1\right)$

Domain : $-1 \leq \frac{x}{2} - 1 \leq 1$

$$0 \leq x \leq 4$$

Range : $0 \leq \frac{y}{2} \leq \pi$

$$0 \leq y \leq 2\pi$$ $\boxed{A}$

9. $\sin 3x \sin 4y = \frac{1}{2}\left[\cos(3x-4y) - \cos(3x+4y)\right]$

$\boxed{A}$ (using reference sheet)

10. $P(x) = (x-2)(x+1)\,Q(x) + Ax + b$

$P(2) = 5 \Rightarrow 2A + B = 5$ — ①

$P(-1) = 8 \quad -A + B = 8$ — ②

① − ②

$$3A = -3$$

$$A = -1$$

substitute $A = -1$ into ①

$$-2 + B = 5$$

$$B = 7$$

$\boxed{A}$

Section II

Question 11 (14 marks)

a) $\frac{3}{4x-1} \geq 2$

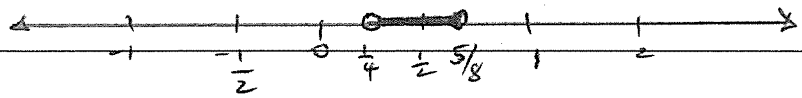
$x \neq -\frac{1}{4}$

Solve $\frac{3}{4x-1} = 2$

$8x - 2 = 3$

$8x = 5$

$x = \frac{5}{8}$



Test $x = 0$

$\frac{3}{4(0)-1} = -3 \leq 2$

×

Test $x = \frac{1}{2}$

$\frac{3}{4(\frac{1}{2})-1} = \frac{3}{1} \geq 2$

✓

Test $x = 1$

$\frac{3}{4(1)-1} = 1 \leq 2$

×

$\therefore$ solution : $\frac{1}{4} < x \leq \frac{5}{8}$

b) $y = e^{x} \tan^{-1} x$

$\frac{dy}{dx} = vu' + uv'$

$u = e^{x}$, $v = \tan^{-1} x$

$u' = e^{x}$, $v' = \frac{1}{1+x^2}$

$= e^{x} \tan^{-1} x + \frac{e^{x}}{1+x^2}$

c) i) $f(x) = x^2 - 4x + 5 \Rightarrow$ Axis of symmetry $= \frac{4}{2}$

$x = 2$

$\therefore$ largest positive domain : $x \geq 2$

11 c ii) Points of intersection are on the line $y = x$

i.e. when $x = x^2 - 4x + 5$

$$x^2 - 5x + 5 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$= \frac{5 \pm \sqrt{5}}{2}$$

In the restricted domain point of intersection is

$$\left(\frac{5+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$$

iii) For $f(x)$, when $x = 2$, $y = 1 \Rightarrow$ range is $y \geq 1$

$\therefore$ Domain of inverse function is $x \geq 1$.

iv) $f^{-1}(x)$: $x = y^2 - 4y + 5$

$$x = y^2 - 4y + 4 + 1$$

$$x - 1 = (y-2)^2$$

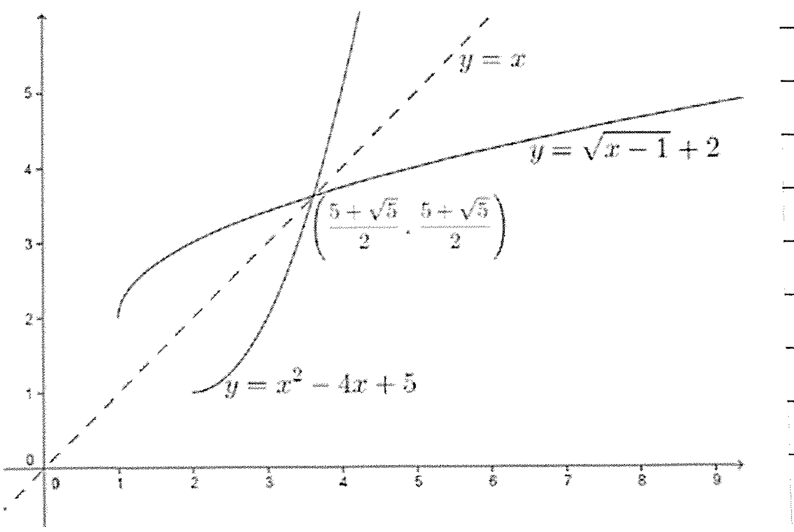
$$y - 2 = \pm\sqrt{x-1}$$

$$y = 2 \pm \sqrt{x-1}$$

Since range of inverse is $y \geq 2$

$$y = 2 + \sqrt{x-1}$$

v)



Question 12 (11 marks)

a) $$\int_0^{15} \frac{x}{\sqrt{x+1}}\,dx \quad ; \quad u^2 = x+1$$

$x = u^2 - 1$

$dx = 2u\,du$

When $x = 0$, $u = 1$

$x = 15$, $u = 4$

$$= \int_1^4 \frac{u^2-1}{u} \cdot 2u\,du$$

$$= 2\int_1^4 (u^2-1)\,du$$

$$= 2\left[\frac{u^3}{3} - u\right]_1^4$$

$$= 2\left[\left(\frac{64}{3} - 4\right) - \left(\frac{1}{3} - 1\right)\right)$$

$$= 36$$

b) i) $\vec{AO} = \vec{AB} - \vec{OB}$

$= \underset{\sim}{v} - \underset{\sim}{u}$

$\vec{AO} = \vec{OD}$ $(\Delta ABO \equiv \Delta OCD)$

$\vec{AD} = \vec{AO} + \vec{OD}$

$= (\underset{\sim}{v} - \underset{\sim}{u}) + (\underset{\sim}{v} - \underset{\sim}{u})$

$= 2\underset{\sim}{v} - 2\underset{\sim}{u}$

ii) $\vec{BD} = \vec{AD} - \vec{AB}$

$= (2\underset{\sim}{v} - 2\underset{\sim}{u}) - \underset{\sim}{v}$

$= \underset{\sim}{v} - 2\underset{\sim}{u}$

5

12 c) i) $\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$

$= \underset{\sim}{i} + 3\underset{\sim}{j} - (2\underset{\sim}{i} + \underset{\sim}{j})$

$= -\underset{\sim}{i} + 2\underset{\sim}{j}$

$\overrightarrow{BC} = \underset{\sim}{c} - \underset{\sim}{a}$

$= \underset{\sim}{i} - 2\underset{\sim}{j} - (2\underset{\sim}{i} + \underset{\sim}{j})$

$= -i - 3j$

ii) $|\overrightarrow{BA}| = \sqrt{(-1)^2 + 2^2}$
$= \sqrt{5}$

$|\overrightarrow{BC}| = \sqrt{(-1)^2 + (-3)^2}$
$= \sqrt{10}$

iii) $\cos \angle ABC = \dfrac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}| \cdot |\overrightarrow{BC}|}$

$= \dfrac{(-1 \times -1) + (2 \times -3)}{\sqrt{5} \times \sqrt{10}}$

$= \dfrac{-5}{5\sqrt{2}} = \dfrac{-1}{\sqrt{2}}$

$\angle ABC = 135°$

6

Question 13 (13 marks)

a) $3^{2n+1} + 2^{n+2}$ is divisible by 7 for $n \geq 1$

Step 1 Show true for $n=1$

$3^3 + 2^3 = 27 + 8$

$= 35$ which is divisible by 7

$\therefore$ true for $n=1$

Step 2 : Assume true for $n=k$

ie. $3^{2k+1} + 2^{k+2} = 7p$ where p is an integer

$3^{2k+1} = 7p - 2^{k+2}$

Step 3 : Prove true for $n=k+1$

ie. $3^{2k+3} + 2^{k+3} = 7q$ where q is an integer

$$
\begin{aligned}
LHS &= 3^{2k+3} + 2^{k+3} \\
&= 3^2 . 3^{2k+1} + 2^{k+3} \\
&= 3^2(7p - 2^{k+2}) + 2^{k+3} \quad \text{(from assumption)} \\
&= 9 \times 7p - 9 \times 2^{k+2} + 2 \times 2^{k+2} \\
&= 9 \times 7p - 7 \times 2^{k+2} \\
&= 7(9p - 2^{k+2}) \\
&= 7q \quad \text{where } q = 9p - 2^{k+2} \\
&= RHS
\end{aligned}
$$

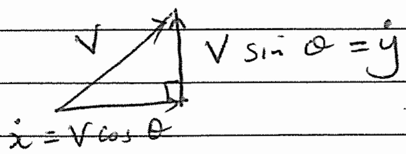
$\therefore$ If true for $n=k$ then also true for $n=k+1$

Step 4 : By the principle of Mathematical Induction the result is true for all $n \geq 1$

7

b)

i) Initially



Horizontally

$\ddot{x} = 0$

$\dot{x} = c_1$

when $t = 0$, $\dot{x} = V\cos\theta$

$\dot{x} = V\cos\theta$

$x = Vt\cos\theta + c_2$

when $t=0$, $x = 0$

$\therefore c_2 = 0$

$\therefore x = Vt\cos\theta$ — (1)

Vertically

$\ddot{y} = -10$

$\dot{y} = -10t + c_1$

when $t=0$ $\dot{y} = V\sin\theta$

$\therefore c_1 = V\sin\theta$

$\dot{y} = -10t + V\sin\theta$

$y = -5t^2 + Vt\sin\theta + c_2$

when $t=0$, $y = 4$ $\therefore c_2 = 4$

$\therefore y = Vt\sin\theta - 5t^2 + 4$ — (2)

ii) From (1)

$x = Vt\cos\theta \Rightarrow t = \dfrac{x}{V\cos\theta}$

substituting into (2)

$$y = V\sin\theta\left(\frac{x}{V\cos\theta}\right) - 5\left(\frac{x}{V\cos\theta}\right)^2 + 4$$

$$= x\tan\theta - \frac{5x^2}{V^2}\sec^2\theta + 4$$

$$= x\tan\theta - \frac{5x^2}{V^2}(1+\tan^2\theta) + 4$$

$\left(\tan\theta = \frac{5}{12}\right)$

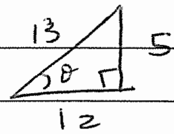
when $x = 30$, $y = 0$

$$\therefore 0 = 30 \times \frac{5}{12} - \frac{5\times 30^2}{V^2}\left(1 + \left(\frac{5}{12}\right)^2\right) + 4$$

$V^2 = 320.076\ldots$

$V = 17.87$ m/s (to 2dp)

13 b (iii) max height $\Rightarrow \dot{y} = 0$



$$-10t + V\sin\theta = 0$$

$$t = \frac{V\sin\theta}{10}$$

$$= \frac{17.89 \times \frac{5}{13}}{10}$$

$$t = 0.688 \text{ s}$$

when $t = 0.688$,

$$y = 17.89 \times 0.688 \times \frac{5}{13} - 5 \times 0.688^2 + 4$$

Max. height $= 6.37$ m (to 2dp)

c) $P(x) = x^3 + ax^2 + bx + 5$

$P(1) = 1 + a + b + 5 = 0$

$\therefore a + b = -6$ — ①

$P'(x) = 3x^2 + 2ax + b$

$P'(1) = 3 + 2a + b = 0$

$\therefore 2a + b = -3$ — ②

② − ①

$a = 3$

substitue $a = 3$ into ①

$3 + b = -6$

$b = -9$

$a = 3, \quad b = -9$

⑨

Question 14 (11 marks)

a) i) $y = x^2$ — ①

$y = 12 - 2x^2$ — ②

Solving simultaneously

$$12 - 2x^2 = x^2$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$

when $x = 2$, $y = 4$

when $x = -2$, $y = 4$

$\therefore$ Points of intersection $(2, 4)$ and $(-2, 4)$

ii) Rotation about y-axis $\Rightarrow V = \pi \int x^2 \, dy$

$$V = \pi \int_0^4 y \, dy + \pi \int_4^{12} \left(6 - \frac{y}{2}\right) dy$$

From ① $x^2 = y$

From ② $x^2 = \frac{12 - y}{2}$

$x^2 = 6 - \frac{y}{2}$

$$= \pi \left[\frac{y^2}{2}\right]_0^4 + \pi \left[6y - \frac{y^2}{4}\right]_4^{12}$$

$$= \pi \left[\frac{16}{2} - 0\right] + \pi \left[6(12) - \frac{12^2}{4}\right) - \left(6(4) - \frac{4^2}{4}\right)\right]$$

$$= 8\pi + 16\pi$$
$$= 24\pi \text{ cubic units.}$$

b) i) $T = 3 + Ae^{-kt}$ $\Rightarrow Ae^{-kt} = T - 3$

$$\frac{dT}{dt} = -k \times Ae^{-kt}$$
$$= -k(T - 3)$$

$\therefore T = 3 + Ae^{-kt}$ satisfies the equation

14 b) (ii) $T = 3 + Ae^{-kt}$

when $t = 0$, $T = 25$

$25 = 3 + Ae^{0}$

$A = 22$

$\therefore T = 3 + 22e^{-kt}$

when $t = 10$, $T = 11$

$\therefore 11 = 3 + 22e^{-10k}$

$8 = 22e^{-10k}$

$e^{-10k} = \frac{8}{22}$

$\log_e e^{-10k} = \log_e \frac{4}{11}$

$-10k = \log_e \frac{4}{11}$

$k = \frac{-1}{10}\left(\log_e \frac{4}{11}\right)$

$= 0.1011\ldots$

T when $t = 15$

$T = 3 + 22e^{-15(0.1011\ldots)}$

$= 7.8241\ldots$

Temperature after 15 mins $= 7.8^{\circ}$ (to 1dp)

c) 4 digit numbers $= 2 \times 4 \times 3 \times 2 = 48$

5 digit numbers $= 5!$

Total $= 168$

(11)

Question 15 (11 marks)

c) (i) $2\sin 2x - 3\cos 2x - 3\sin x + 3 = \sin x(6\sin x + 4\cos x - 3)$

$$\begin{aligned}
LHS &= 2\sin 2x - 3\cos 2x - 3\sin x + 3 \\
&= 2(2\sin x\cos x) - 3(1 - 2\sin^2 x) - 3\sin x + 3 \\
&= 4\sin x\cos x - 3 + 6\sin^2 x - 3\sin x + 3 \\
&= 4\sin x\cos x + 6\sin^2 x - 3\sin x \\
&= \sin x(6\sin x + 4\cos x - 3) \\
&= RHS.
\end{aligned}$$

ii) $6\sin x + 4\cos x = R\sin(x+\alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

Equating coefficients

$R\cos\alpha = 6$ — (1) $\quad R\sin\alpha = 4$ — (2)

Squaring and adding (1) & (2)

$R^2\cos^2\alpha + R^2\sin^2\alpha = 6^2 + 4^2$

$R^2 = 52$

$R = \sqrt{52} \quad R > 0$

$\cos\alpha = \frac{6}{\sqrt{52}}$; $\sin\alpha = \frac{4}{\sqrt{52}}$ $\quad \therefore \alpha < 90^\circ$

$\alpha = 0.588$

$\therefore 6\sin x + 4\cos x = \sqrt{52}\sin(x + 0.588)$

iii) $2\sin 2x - 3\cos 2x - 3\sin x + 3 = 0 \quad 0 \le x < \pi$

$\Rightarrow \sin x(6\sin x + 4\cos x - 3) = 0$

$\Rightarrow \sin x(\sqrt{52}\sin(x + 0.588) - 3) = 0$

$\therefore \sin x = 0$ or $\sqrt{52}\sin(x + 0.588) - 3 = 0$

$x = 0 \quad \sin(x + 0.588) = \frac{3}{\sqrt{52}}$

$x + 0.588 = 0.429$ or 2.7125

$x = 2.1245$ (since $0 \le x < \pi$)

$\therefore x = 0$ or $x = 2.12 \quad x = 2.12$ (to 2dp)

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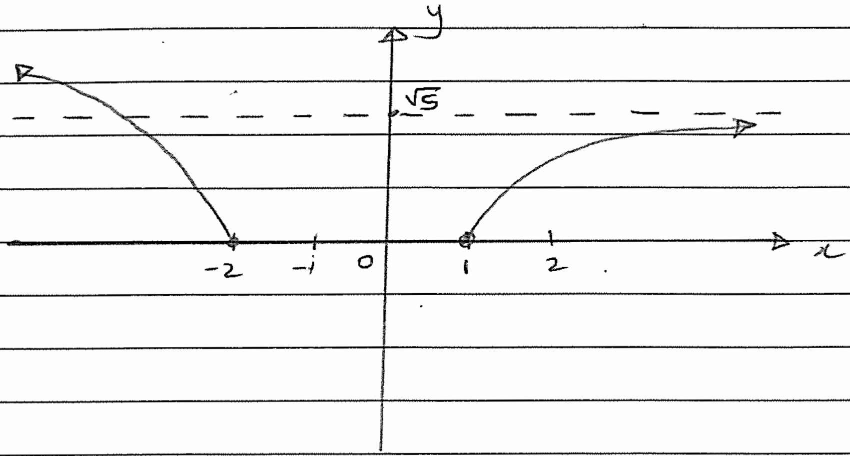
15 b)



Domain : $x \leq -2, \ x \geq 1$

Range : $y \geq 0$

a) $x = 2\cos\theta - 1 \Rightarrow \cos\theta = \dfrac{x+1}{2}$

$y = 2 + 2\sin\theta$

$\Rightarrow \sin\theta = \dfrac{y-2}{2}$

Using $\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{x+1}{2}\right)^2 + \left(\frac{y-2}{2}\right)^2 = 1$$

$$\frac{(x+1)^2}{4} + \frac{(y-2)^2}{4} = 1$$

$$(x+1)^2 + (y-2)^2 = 4$$