GOSFORD HS 2004 Ext 1

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $x(3-2x)>0$ **2**

(b) Find $\frac{d}{dx}\{e^{-x}\cos^{-1}x\}$ **2**

(c) The remainder when x^3+ax^2-3x+5 is divided by $(x+2)$ is 11. **2**

Find the value of a.

(d) Find the general solution of $2\cos x+\sqrt{3}=0$ **2**

(e) Solve $\frac{x^2-9}{x}\geq 0$ **2**

(f) Find $\int_0^2 (4+x^2)^{-1}\,dx$ **2**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Use the substitution $x=\ln u$ to find $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$ **3**

(b) Use one application of Newton's method to find an approximation to the root of the equation $\cos x = x$ near $x=0.5$. Give your answer correct to two decimal places. **3**

(c) The curves $y=e^{2x}$ and $y=1+4x-x^2$ intersect at the point $(0,1)$. **3**

Find the angle between the two curves at this point of intersection.

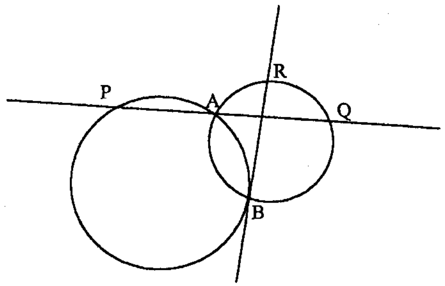
(d) (i) In how many ways can a committee of 2 Englishmen, 2 Frenchmen and 1 American be chosen from 6 Englishmen, 7 Frenchmen and 3 Americans. **2**

(ii) In how many of these ways do a particular Englishman and a particular Frenchman belong to the committee? **1**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the term independent of x in the expansion of $\left(3x-\frac{5}{x^3}\right)^8$ **3**

(b) Two circles cut at A and B. A line through A meets one circle at P and the other at Q. BR is a tangent to circle ABP and R lies on circle ABQ. Prove that PB || QR. **3**



(c) The area bounded by the curve $y=\sin^{-1}x$ the y axis and the abscissa at $y=\frac{\pi}{2}$ is rotated about the y axis.

(i) Show that the volume of the solid so formed is given by $\pi\int_0^{\frac{\pi}{2}}\sin^2 y\,dy$ **1**

(ii) Hence find the volume of this solid. **2**

(d) A particle moves in a straight line from a position of rest at a fixed origin O. its velocity is v when its displacement from O is x. **3**

If its acceleration is $\frac{1}{(x+3)^2}$, find v in terms of x.

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Expand $\cos(\alpha+\beta)$ **1**

(ii) Show that $\cos 2\alpha = 1-2\sin^2\alpha$ **1**

(iii) Evaluate $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2}$ **1**

(b) If $\alpha = tan^{-1}\left(\frac{5}{12}\right)$ and $\beta=\cos^{-1}\left(\frac{4}{5}\right)$, calculate the exact value of $\tan(\alpha-\beta)$. **2**

(c) A and B are the points $(-1,7)$ and $(5,-2)$; P divides AB in the ratio $k:1$.

(i) Write down the coordinates of P in terms of k. **2**

(ii) If P lies on the line $5x-4y=1$, find the ratio of AP:PB **1**

(d) A biased coin has a probability of coming up heads equal to $p=0.6$.Find the probability of getting exactly four heads in ten tosses of the coin. **1**

(e) Use mathematical induction to prove that **3**

$$1\times 1!+2\times 2!+3\times 3!+\ldots+n\times n! = (n+1)!-1$$

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $\cos x - \sqrt{3}\sin x = 1$, where $0 \le x \le 2\pi$ **3**

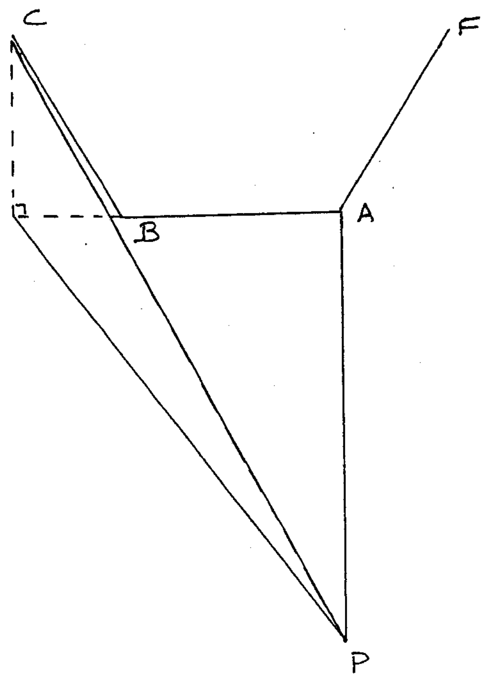
(b) Wheat falls from an auger onto a conical pile at the rate of $20\text{cm}^3\text{s}^{-1}$. The radius of the base of the pile is always equal to half its height.

(i) Show that $V = \frac{1}{12}\pi h^3$ and hence find $\frac{dh}{dt}$ **2**

(ii) Find the rate at which the pile is rising when it is 8 cm deep **1**

(iii) Find the time taken for the pile to reach a height of 8 cm. **2**

(c) In a horizontal triangle APB, AP=2AB, and the angle A is a right angle. On AB stands a vertical and regular hexagon ABCDEF. Prove that PC is inclined to the horizontal at an angle whose tangent is $\frac{\sqrt{3}}{5}$. **4**



Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The speed v m s^{-1} of a particle moving along the X axis is given by $v^2 = 24 - 6x - 3x^2$, where x m is the distance of the particle from the origin.

(i) Show that the particle is executing Simple Harmonic Motion **2**

(ii) Find the amplitude and the period of the motion **2**

(b) P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$

(i) Show that the equation of PQ is given by the equation $y - \frac{1}{2}(p+q)x + apq = 0$. **2**

(ii) Find the condition that PQ passes through the point $(0, -a)$. **1**

(iii) If the focus of the parabola is S, prove that **2**

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$$

(c) One root of $x^3 + px^2 + qx + r = 0$ equals the sum of the two other roots, prove that $p^3 + 8r = 4pq$ **3**

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Find the largest possible domain of positive values for which $f(x) = x^2 - 6x + 13$ has an inverse. 1

(ii) Find the equation of the inverse function, $f^{-1}(x)$. 2

(b) (i) Write down the expansions of $(1+x)^n$ 1

(ii) Using part (i) show that 3

$$^{10}C_0 + {}^{10}C_2 7^2 + {}^{10}C_4 7^4 + {}^{10}C_6 7^6 + {}^{10}C_8 7^8 + {}^{10}C_{10} 7^{10} = 2^9\left(2^{20} + 3^{10}\right)$$

(c) A projectile is fired from O, with velocity $V\ \text{ms}^{-1}$, at an angle of α to the horizontal. After t seconds, its horizontal and vertical displacements from O are x metres and y metres.

(i) Beginning with the acceleration equations $\ddot{x} = 0$ and $\ddot{y} = -g$ show that the equations of motion of the projectile are 2

$$x = Vt\cos\alpha \text{ and } y = \frac{-gt^2}{2} + Vt\sin\alpha$$

(ii) Fire fighters are forced to stay 60 metres away from a dangerous fire burning in a low open tank on horizontal ground. They have two pumps. One which can eject water in any direction at $30\,\text{ms}^{-1}$, is on the ground, while the other, which can eject water at $40\,\text{ms}^{-1}$ but only horizontally, is on a vertical stand 5 m high. 3

Take $g = 10\,\text{ms}^{-2}$.

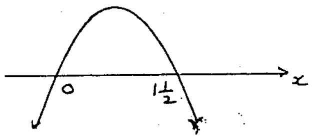
Using the equations of motion derived in (i) find whether or not both pumps can reach the fire.

End of Examination

Ext 1 TRIAL GOSFORD HIGH 2004

Question 1

a) $x(3-2x)>0$



$0<x<1\frac{1}{2}$

b) $\frac{d}{dx}(e^{-x}\cos^{-1}x)$

$= e^{-x}\times\frac{-1}{\sqrt{1-x^2}} + (-e^{-x})\cos^{-1}x$

$= -e^{-x}\left\{\frac{1}{\sqrt{1-x^2}}+\cos^{-1}x\right\}$

c) $P(-2)=11$

$(-2)^3+a(-2)^2-3(-2)+5=11$

$-8+4a+6+5=11$

$4a=11$

$a=2$

d) $2\cos x+\sqrt{3}=0$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = 2n\pi \pm \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= 2n\pi \pm \left(\pi - \cos^{-1}\frac{\sqrt{3}}{2}\right)$

$= 2n\pi \pm \left(\pi - \frac{\pi}{6}\right)$

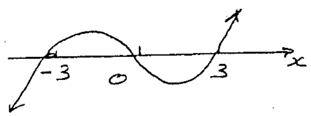
$= 2n\pi \pm \frac{5\pi}{6}$

e) $\frac{x^2-9}{x}\geq 0$

$\frac{x^2(x^2-9)}{x}\geq 0\times x^2$

$x(x^2-9)\geq 0$

$x(x-3)(x+3)\geq 0$



$-3\leq x<0$ OR $x\geq 3$

f) $\int_0^2 \frac{1}{4+x^2}\,dx$

$= \frac{1}{2}\left[\tan^{-1}\frac{x}{2}\right]_0^2$

$= \frac{1}{2}\{\tan^{-1}1-\tan^{-1}0\}$

$= \frac{1}{2}\left(\frac{\pi}{4}-0\right)$

$= \frac{\pi}{8}.$

Question 2.

a) $\int\frac{e^x}{\sqrt{1-e^{2x}}}\,dx$ let $x=\ln u$

$\frac{dx}{du}=\frac{1}{u}$

$dx=\frac{1}{u}du$

$= \int\frac{u}{\sqrt{1-u^2}}\cdot\frac{1}{u}\,du$ Note $u=e^x$

$= \int\frac{du}{\sqrt{1-u^2}}$

$= \sin^{-1}u+c$

$= \sin^{-1}e^x+c$

OR. let $x=\ln u$

$\therefore u=e^x$

$\frac{du}{dx}=e^x$

$du=e^x dx$

$\therefore \int\frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$= \int\frac{du}{\sqrt{1-u^2}}$

$= \sin^{-1}u+c$

$= \sin^{-1}e^x+c$

b) $\cos x = x$

$\therefore \cos x - x = 0$

Let $f(x)=\cos x - x$

$f'(x) = -\sin x - 1$

Now $x_0 = 0.5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 0.5 - \frac{(\cos 0.5 - 0.5)}{(-\sin 0.5 - 1)}$

$= 0.76$

c) $y=e^{2x}$

$\frac{dy}{dx}=2e^{2x}$

at $x=0$, $\frac{dy}{dx}=2e^0=2$

$y=1+4x-x^2$

$\frac{dy}{dx}=4-2x$

at $x=0$, $\frac{dy}{dx}=4$

$\therefore m_1=4,\ m_2=2$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{4-2}{1+4\times 2}\right|$

$= \left|\frac{2}{9}\right|$

$\theta = 12°32'$

d) (i) ${}^6C_2\times{}^7C_2\times{}^3C_1$

$= 15\times 21\times 3$

$= 945$

(ii) ${}^5C_1\times{}^6C_1\times{}^3C_1$

$= 90$

Question 3

(i) $\cos(\alpha+\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(ii) Let $\beta=\alpha$

$\therefore \cos(\alpha+\alpha)$

$= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$= \cos^2\alpha - \sin^2\alpha$

$= 1-\sin^2\alpha-\sin^2\alpha$

$= 1-2\sin^2\alpha$

$\therefore \cos 2\alpha = 1-2\sin^2\alpha$

(iii) $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

$= \lim_{x\to 0}\frac{1-(1-2\sin^2 x)}{x^2}$

$= \lim_{x\to 0}\frac{1-1+2\sin^2 x}{x^2}$

$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2}$

$= 2\cdot\lim_{x\to 0}\frac{\sin x}{x}\lim_{x\to 0}\frac{\sin x}{x}$

$= 2\cdot 1\cdot 1$

$= 2.$

b) $\tan(\alpha-\beta)$

$= \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

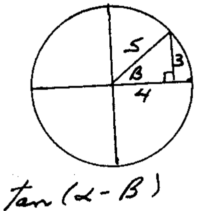
Now $\alpha=\tan^{-1}\frac{5}{12}$

$\therefore \tan\alpha = \frac{5}{12}$

$\beta = \cos^{-1}\frac{4}{5}.$

$\therefore \cos\beta = \frac{4}{5}$

$\therefore \tan\beta = \frac{3}{4}$



Hence $\tan(\alpha-\beta)$

$= \frac{\frac{5}{12}-\frac{3}{4}}{1+\frac{5}{12}\times\frac{3}{4}}$

$= \frac{20-36}{48+15}$

$= \frac{-16}{63}$

c) (i) $(-1,7)$ $(5,-2)$

$k:1$

$x = \frac{1\times(-1)+k\times 5}{k+1}$

$= \frac{-1+5k}{k+1}$

$y = \frac{1\times 7+k(-2)}{k+1}$

$= \frac{7-2k}{k+1}$

$\therefore P$ is $\left(\frac{-1+5k}{k+1},\frac{7-2k}{k+1}\right)$

(ii) P lies on $5x-4y=1$

$\therefore 5\left(\frac{5k-1}{k+1}\right)-4\left(\frac{7-2k}{k+1}\right)=1$

$25k-5-28+8k=k+1$

$33k-33=k+1$

$32k=34$

$k=\frac{34}{32}$

$k=\frac{17}{16}.$

$\therefore AP:PB = 17:16$

d) $P(X=4)$

$= {}^{10}C_4\,q^{10-4}p^4$

$= {}^{10}C_4\,q^6p^4$

$= {}^{10}C_4\,(0.4)^6(0.6)^4$

$= 0.1115$ To 4 dec. pl.

e) Step 1.

If $n=1$. L.H.S. $=1\times 1!$

$=1$

R.H.S. $=(1+1)!-1$

$=2!-1$

$=1$

$\therefore$ L.H.S. = R.H.S.

Result is true for $n=1$.

Step 2

Assume that the result is true for $n=k$

i.e.

$1\times 1!+2\times 2!+\dots+k(k!)$

$=(k+1)!-1$

Step 3.

Prove that the result is true for $n=k+1$

i.e.

$1\times 1!+2\times 2!+\dots+(k+1)(k+1)!$

$=(k+1+1)!-1$

i.e.

$1\times 1!+2\times 2!\dots+(k+1)(k+1)!$

$=(k+2)!-1$

Proof:

L.H.S

$=1\times 1!+2\times 2!+\dots+(k+1)k$

$=1\times 1!+2\times 2!+\dots+k\,k!+(k+1)(k+1)!$

$=(k+1)!-1+(k+1)(k+1)!$

$=(k+1)!+(k+1)(k+1)!-1$

$=(k+1)!(1+k+1)-1$

$=(k+1)!(k+2)-1$

$=(k+2)!-1$

= R.H.S.

Hence the result is true for $n=k+1$ if it is true for $n=k$

Step 4:

Since the result is true for $n=1$ then it is true for $n=1+1$ i.e. for $n=2$ and thus for $n=3$ and so on for all positive integral values of n.

Question 4

a) $T_{k+1} = {}^nC_k\, a^{n-k} b^k$

$\therefore$ for $\left(3x+\left(\frac{-5}{x^3}\right)\right)^8$

T_{k+1}

$= {}^8C_k (3x)^{8-k}\left(-\frac{5}{x^3}\right)^k$

$= {}^8C_k \frac{3^{8-k} x^{8-k} (-1)^k 5^k}{(x^3)^k}$

$= {}^8C_k (-1)^k 3^{8-k} 5^k \frac{x^{8-k}}{x^{3k}}$

$= {}^8C_k (-1)^k 3^{8-k} 5^k x^{8-4k}$

We want

$8-4k=0$

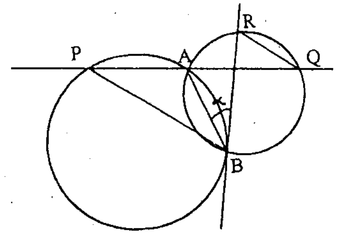
$\therefore 4k=2$

$\therefore T_{2+1}$

$= {}^8C_2 (-1)^2 3^{8-2} 5^2 x^0$

$= 28 \times 3^6 \times 5^2$

$= \underline{510300}$



Join A to B and let $\angle ABR = \alpha$

$\angle APB = \angle ABR$ (alt. seg. theorem)

$\angle AQR = \angle ABR$ ($\angle$'s at the circumference standing on the same arc are equal)

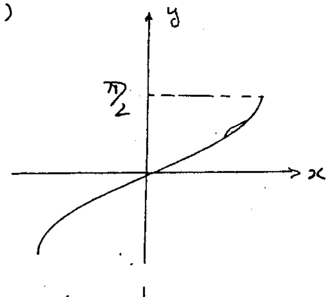
$= \alpha$

$\therefore \angle APB = \angle AQR$

but these are alt. $\angle$'s

$\therefore PB \parallel QR$

c)



Now $y = \sin^{-1} x$

$\therefore x = \sin y$

$x^2 = \sin^2 y$

$\therefore V = \pi \int_0^{\pi/2} x^2\, dy$

$= \pi \int_0^{\pi/2} \sin^2 y\, dy$

$= \pi \int_0^{\pi/2} \frac{1}{2}(1-\cos 2y)\, dy$

$= \frac{\pi}{2} \int_0^{\pi/2} (1-\cos 2y)\, dy$

$= \frac{\pi}{2}\left[y - \frac{1}{2}\sin 2y\right]_0^{\pi/2}$

$= \frac{\pi}{2}\left\{\left(\frac{\pi}{2} - \frac{1}{2}\sin 2\times\frac{\pi}{2}\right) - \left(0 - \frac{1}{2}\sin 0\right)\right\}$

$= \frac{\pi}{2}\left\{\frac{\pi}{2} - \frac{1}{2}\sin\pi - (0-0)\right\}$

$= \frac{\pi}{2}\left(\frac{\pi}{2} - 0\right)$

$= \underline{\frac{\pi^2}{4}}$ cubic units

d) $\ddot{x} = \frac{1}{(x+3)^2}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{1}{(x+3)^2}$

$= (x+3)^{-2}$

$\therefore \frac{1}{2}v^2 = \int (x+3)^{-2}\, dx$

$\frac{1}{2}v^2 = \frac{(x+3)^{-1}}{-1\times 1} + c$

$\frac{1}{2}v^2 = \frac{-1}{x+3} + c$

at $t=0$, $x=0$, $v=0$

$\therefore 0 = \frac{-1}{3} + c$

$c = \frac{1}{3}$

$\therefore \frac{1}{2}v^2 = \frac{-1}{x+3} + \frac{1}{3}$

$\frac{1}{2}v^2 = \frac{-3+x+3}{3(x+3)}$

$\frac{1}{2}v^2 = \frac{x}{3(x+3)}$

$v^2 = \frac{2x}{3(x+3)}$

$v = \pm\sqrt{\frac{2x}{3(x+3)}}$

as $a>0$ for all real values of x then $v>0$ for all real x

$\therefore v = \sqrt{\frac{2x}{3(x+3)}}$

Question 5.

a) $v^2 = 24 - 6x - 3x^2$

(i) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left\{12 - 3x - \frac{3}{2}x^2\right\}$

$= -3 - 3x$

$= -3(x+1)$

let $y = x+1$

$\therefore \frac{dy}{dt} = \frac{dx}{dt}$ & $\frac{d^2y}{dt^2} = \frac{d^2x}{dt^2}$

i.e. $\dot{y} = \dot{x}$

and $\ddot{y} = \ddot{x}$

$\therefore \ddot{y} = -3y$ which is S.H.M. centre

$y = 0$

i.e. $x+1 = 0$

$x = -1$

(ii) Particle is at rest when $v=0$

i.e. $24 - 6x - 3x^2 = 0$

$x^2 + 2x - 8 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ OR $x = 2$

$\therefore$ length of path $= 2a$

$= 6$

$\therefore a = 3$

Amplitude $= 3$ Period $= \frac{2\pi}{n}$

$= \frac{2\pi}{\sqrt{3}}$

b) (i)

$m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$

$= \frac{a(p^2-q^2)}{2a(p-q)}$

$= \frac{(p-q)(p+q)}{2(p-q)}$

$= \frac{p+q}{2}$

Equation of PQ is

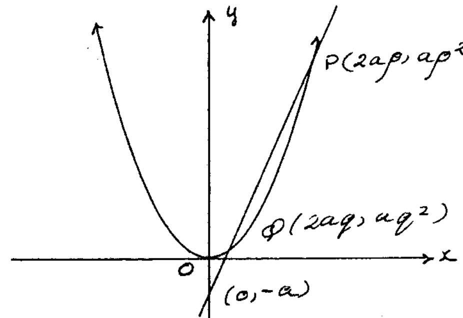
$y - y_1 = m(x - x_1)$

$y - ap^2 = \frac{1}{2}(p+q)(x - 2ap)$

$y - ap^2 = \frac{1}{2}(p+q)x - 2ap\times\frac{1}{2}(p+q)$

$y - ap^2 = \frac{1}{2}(p+q)x - ap^2 - apq$

$\therefore y - \frac{1}{2}(p+q)x + apq = 0$



(ii) If PQ passes through $(0,-a)$ then $(0,-a)$ satisfies the equation of PQ

$\therefore -a - \frac{1}{2}(p+q)\times 0 + apq = 0$

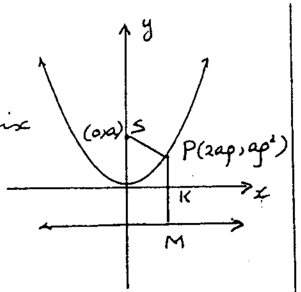
$-a + apq = 0$

$\underline{pq = 1}$

(iii)

$SP = PM$

by the focus-directrix definition



$SP = MK + KP$

$= a + ap^2$

$= a(1+p^2)$

Similarly $SQ = a(1+q^2)$

$\therefore \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(1+p^2)} + \frac{1}{a(1+q^2)}$

But from (ii) $q = \frac{1}{p}$.

$\therefore \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(1+p^2)} + \frac{1}{a(1+\frac{1}{p^2})}$

$= \frac{1}{a(1+p^2)} + \frac{p^2}{a(1+p^2)}$

$= \frac{1+p^2}{a(1+p^2)}$

$= \frac{1}{a}$ as required

c) let the roots be α, β & $\alpha+\beta$

$\therefore \alpha + \beta + (\alpha+\beta) = -\frac{b}{a}$

i.e. $2\alpha + 2\beta = -\frac{p}{1}$

$\alpha + \beta = -\frac{p}{2}$

But $\alpha+\beta$ is a root of

$x^3 + px^2 + qx + r = 0$

$\therefore \left(-\frac{p}{2}\right)^3 + p\left(-\frac{p}{2}\right)^2 + q\left(-\frac{p}{2}\right) + r = 0$

$-\frac{p^3}{8} + \frac{p^3}{4} - \frac{pq}{2} + r = 0$

$-p^3 + 2p^3 - 4pq + 8r = 0$

$p^3 + 8r = 4pq$

Question 6.

a) $\cos x - \sqrt{3}\sin x = 1$

$\frac{1-t^2}{1+t^2} - \sqrt{3} \times \frac{2t}{1+t^2} = 1$

where $t = \tan\frac{x}{2}$.

$\therefore 1 - t^2 - 2\sqrt{3}t = 1 + t^2$

$-2\sqrt{3}t = 2t^2$

$0 = 2t^2 + 2\sqrt{3}t$

$0 = 2t(t+\sqrt{3})$

$2t = 0$ or $t + \sqrt{3} = 0$

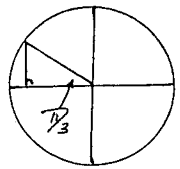
$t = 0$ $\quad t = -\sqrt{3}$

$\tan\frac{x}{2} = 0$ or $\tan\frac{x}{2} = -\sqrt{3}$

where $0 \le \frac{x}{2} \le \pi$

$\frac{x}{2} = 0, \pi$

$x = 0, 2\pi$



$\frac{x}{2} = \frac{2\pi}{3}$

$x = \frac{4\pi}{3}$

$\therefore x = 0, 2\pi, \frac{4\pi}{3}$

b) (i) $V = \frac{1}{3}\pi r^2 h$

But $r = \frac{1}{2}h$

$\therefore V = \frac{1}{3}\pi \times \left(\frac{1}{2}h\right)^2 h$

$= \frac{1}{3}\pi \times \frac{1}{4}h^2 \cdot h$

$= \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$= \frac{\pi}{12} \times 3h^2 \frac{dh}{dt}$

Now $\frac{dV}{dt} = 20\ cm^3/s$.

$\therefore 20 = \frac{\pi}{12} \times 3h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{80}{\pi h^2}$

(ii) $\frac{dh}{dt} = \frac{80}{\pi h^2}$

If $h = 8$, $\frac{dh}{dt} = \frac{80}{\pi \times 64}$

$= \frac{5}{4\pi}$ cm/sec

(iii) $\frac{dh}{dt} = \frac{80}{\pi h^2}$

$\therefore \frac{dt}{dh} = \frac{\pi h^2}{80}$

$t = \frac{\pi}{80}\int h^2\,dh$

$t = \frac{\pi}{80}\frac{h^3}{3} + c$

at $t = 0$, $h = 0$

$\therefore 0 = 0 + c$

$0 = c$

$t = \frac{\pi h^3}{240}$

when $h = 8$

$t = \frac{\pi \times 8^3}{240}$

$= \frac{32\pi}{15}$ sec

$\doteqdot 6{\cdot}7$ sec.

c)

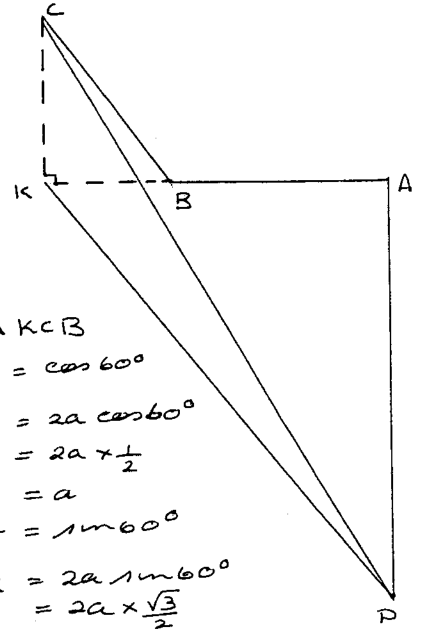
Let $AB = 2a$ units

$\therefore AP = 4a$ units

Each $\angle$ of a regular hexagon

$= \frac{(6-2)\times 180^\circ}{6}$

$= 120^\circ$



In $\triangle KCB$

$\frac{KB}{2a} = \cos 60^\circ$

$KB = 2a\cos 60^\circ$

$= 2a \times \frac{1}{2}$

$= a$

$\frac{CK}{2a} = \sin 60^\circ$

$CK = 2a\sin 60^\circ$

$= 2a \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}a$

In $\triangle KAP$

$(KP)^2 = (3a)^2 + (4a)^2$ Pythagoras' Theorem

$(KP)^2 = 9a^2 + 16a^2$

$= 25a^2$

$\therefore KP = 5a$

$\therefore \tan \angle CPK = \frac{\sqrt{3}a}{5a}$

$= \frac{\sqrt{3}}{5}$

Question 7

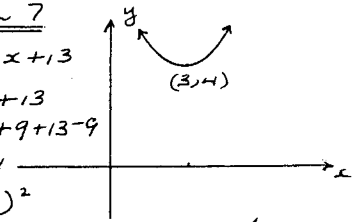
$f(x) = x^2 - 6x + 13$

$y = x^2 - 6x + 13$

$y = x^2 - 6x + 9 + 13 - 9$

$y = (x-3)^2 + 4$

$y - 4 = (x-3)^2$



$x \geq 3$ is the largest domain for which $f(x)$ has an inverse

(ii) $(y-4) = (x-3)^2$

$\therefore$ Inverse is

$(x-3)^2 = (y-4)$

$y - 3 = \pm\sqrt{x-3}$

$y = 3 \pm \sqrt{x-3}$

Note for $f(x)$

Domain $x \geq 3$

Range $y \geq 4$

$\therefore$ for $f^{-1}(x)$

Domain $x \geq 4$

Range $y \geq 3$

$\therefore f^{-1}(x) = 3+\sqrt{x-4}$

c(i) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

(ii) Let $n = 10$ and $x = 7$

$\therefore (1+7)^{10} = {}^{10}C_0 + {}^{10}C_1 7 + {}^{10}C_2 7^2 + \dots + {}^{10}C_{10} 7^n$

Let $n = 10$, $x = -7$

$\therefore (1+(-7))^{10} = {}^{10}C_0 - {}^{10}C_1 7 + {}^{10}C_2 7^2 - {}^{10}C_3 7^3 + \dots - {}^{10}C_9 7^9 + {}^{10}C_{10} 7^{10}$

Add

$\therefore (1+7)^{10} + (1-7)^{10} = 2\,{}^{10}C_0 + 2\,{}^{10}C_2 7^2 + \dots + 2\,{}^{10}C_{10} 7^{10}$

$8^{10} + (-6)^{10} = 2\{{}^{10}C_0 + {}^{10}C_2 7^2 + \dots + {}^{10}C_{10} 7^{10}\}$

$(2^3)^{10} + (2\times 3)^{10} = 2\{{}^{10}C_0 + {}^{10}C_2 7^2 + \dots + {}^{10}C_{10} 7^{10}\}$

$\frac{2^{30} + 2^{10}\times 3^{10}}{2} = {}^{10}C_0 + {}^{10}C_2 7^2 + \dots + {}^{10}C_{10} 7^{10}$

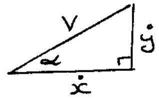
$\frac{2^{10}(2^{20}+3^{10})}{2} = {}^{10}C_0 + {}^{10}C_2 7^2 + \dots + {}^{10}C_{10} 7^{10}$

$\therefore {}^{10}C_0 + {}^{10}C_2 7^2 + {}^{10}C_4 7^4 + {}^{10}C_6 7^6 \times {}^{10}C_8 7^8 + {}^{10}C_{10} 7^{10}$

$= 2^9(2^{20}+3^{10})$

c (i)

at $t=0, x=0, y=0, \dot{x} = V\cos\alpha, \dot{y} = V\sin\alpha$



$\ddot{x} = 0$

$\dot{x} = \int 0\,dt$

$\dot{x} = C_1$

at $t=0$, $\dot{x} = V\cos\alpha$

$V\cos\alpha = C_1$

$\dot{x} = V\cos\alpha$

$x = \int (V\cos\alpha)\,dt$

$x = Vt\cos\alpha + C_2$

at $t=0, x=0$

$\therefore 0 = 0 + C_2$

$0 = C_2$

$x = Vt\cos\alpha$

$\ddot{y} = -g$

$\dot{y} = \int -g\,dt$

$\dot{y} = -gt + C_3$

at $t=0$, $\dot{y} = V\sin\alpha$

$V\sin\alpha = 0 + C_3$

$V\sin\alpha = C_3$

$\dot{y} = -gt + V\sin\alpha$

$y = \int(-gt + V\sin\alpha)\,dt$

$y = -\frac{gt^2}{2} + Vt\sin\alpha + C_4$

at $t=0, y=0$

$\therefore 0 = 0 + 0 + C_4$

$0 = C_4$

$y = -\frac{gt^2}{2} + Vt\sin\alpha.$

(ii) PUMP 1

$V = 30$

$\angle$ of firing $= \alpha$

$\ddot{x} = 0$

$\dot{x} = 30\cos\alpha$

$x = 30t\cos\alpha$

$\ddot{y} = -10$

$\dot{y} = -10t + 30\sin\alpha$

$y = -5t^2 + 30t\sin\alpha$

Maximum range occurs when $\alpha = 45^\circ$

$\therefore x = 30t\cos 45^\circ$, $y = -5t^2 + 30t\sin 45^\circ$

$x = 30t\times\frac{1}{\sqrt{2}}$ $\qquad = -5t^2 + 30t\times\frac{1}{\sqrt{2}}$

when $y=0$, $-5t^2 + \frac{30t}{\sqrt{2}} = 0$

$-5t\left(t - \frac{6}{\sqrt{2}}\right) = 0$

$\therefore t = 0$ or $t = \frac{6}{\sqrt{2}}$

$= 3\sqrt{2}$

when $t = 3\sqrt{2}$, $x = 30\times 3\sqrt{2}\times\frac{1}{\sqrt{2}}$

$= 90$

$\therefore$ Pump A will reach the fire

PUMP 2

$\alpha = 0$

$V = 40$

$\therefore x = 40t\cos 0$, $y = -5t^2 + 40t\sin 0^\circ$

$x = 40t$ $\qquad y = -5t^2$

when $y = -5$

$-5 = -5t^2$

$\therefore t^2 = 1$

$t = 1$ Note $t > 0$

If $t = 1$, $x = 40\times 1$

$x = 40$

$\therefore$ Pump 2 does not reach the fire

* It has to reach 60 metres to reach the fire