GOSFORD EXT. 1 2005

Total Marks – 84
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

QUESTION 1 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Solve $x - \frac{1}{x} < 0$ **2**

b) For what values of x is $(2-x)(2x-1)(x+3) \leq 0$? **2**

c) A committee of 3 has to be chosen from 4 males and 5 females. The committee must have at least 1 male and 1 female. How many different committees can be chosen? **2**

d) Find $\int_0^1 x(2x-1)^4\, dx$ using the substitution $u = 2x - 1$ **3**

e) The equation $x^3 + 2x^2 - 3x + 5 = 0$ has the roots α, β and γ

i) Find $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$, and $\alpha\beta\gamma$ **2**

ii) Hence find the value of $(\alpha - 1)(\beta - 1)(\gamma - 1)$ **1**

QUESTION 2 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Find $\frac{d}{dx}\ln\left(\frac{2x}{(x-1)^2}\right)$ **1**

b) The gradient function of a certain curve is $\left(x^2 + 25\right)^{-1}$ **2**
Find the equation of the curve if it passes through the point $\left(5, \frac{\pi}{2}\right)$.

c) i) Sketch the curve $y = 3\sin^{-1} 2x$. State its domain and range. **1**

ii) Find the exact area bounded by the curve $y = 3\sin^{-1} 2x$, the x axis and the line $x = \frac{1}{2}$. **3**

d) Evaluate $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3 - x^2}}$ **2**

e) Find the quotient *Q(x)* and remainder *R(x)* when the polynomial **3**

$P(x) = 2x^4 - 3x^3 - x^2 + 2x + 1$ is divided by $x^2 + 2x - 1$.

QUESTION 3 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) $\log_e x + \sin x = 0$ has a root close to $x = 0.5$. Using one application of Newton's method, find a better approximation to the root **2**

b) $\int \cos^2\left(x - \frac{\pi}{4}\right)dx$ **2**

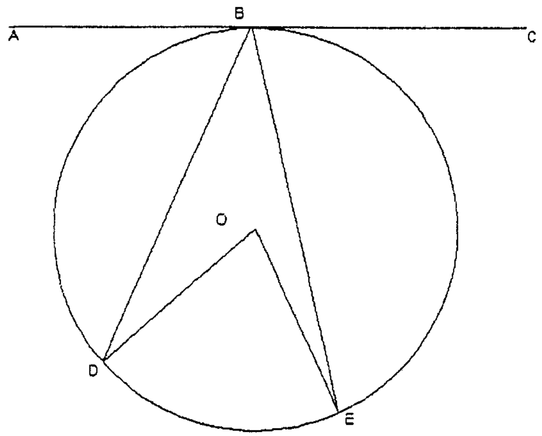
c) Find the coefficient of x^9 in $\left(5x^2 - \frac{1}{2x}\right)^{12}$ **2**

d) A particle is moving with acceleration $\ddot{x} = -9x$ and is initially stationary at $x = 4$.

i) Find v^2 as a function of x. **2**

ii) What is the particle's maximum speed? **1**

e) In the diagram below, O is the centre of the circle, AC is a tangent at B and D and E are points on the circumference. If $\angle ABD = 80°$ and $\angle DBE = 40°$, find the size of $\angle BEO$, giving reasons. **3**



QUESTION 4 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) i) Prove the ratio of the $(k+1)$th term to the k th term in the expansion of $\left(2x + \frac{3}{x}\right)^{12}$ simplifies to $\frac{39-3k}{2k}$ **2**

ii) Hence find the greatest coefficient of the expansion. **2**

b) i) Express $\sqrt{3}\cos 2t - \sin 2t$ in the form $R\cos(2t+\alpha)$ where $0<\alpha<\frac{\pi}{2}$ **2**

ii) Hence or otherwise find all positive solutions of $\sqrt{3}\cos 2t - \sin 2t = 0$ **2**

c) A particle moves in a straight line and is x metres from a fixed point O after t seconds where $x = 5 + \sqrt{3}\cos 2t - \sin 2t$.

i) Prove that the acceleration of the particle is $-4(x-5)$. **2**

ii) Between which two points does the particle oscillate? You may use your answers from part (b) **1**

iii) When does the particle first pass through the point $x=5$? **1**

QUESTION 5 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Andy Roddick estimates that his chances of beating Rodger Federer are $\frac{1}{3}$

i) If 5 matches are played what is the probability Andy has exactly 3 wins. **1**

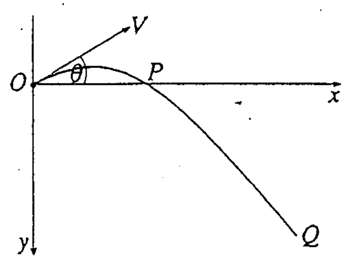
ii) How many matches must Andy play so that the probability he wins at least one match is greater than 0.9? **2**

b) A is the point (-4, 1) and B is the point (2, 4). Q is the point which divides AB internally in the ratio 2:1 and R divides AB externally in the ratio 2:1. P(x, y) is a variable point which moves so that PA = 2PB

(i) Find the co-ordinates of Q and R. **2**

(ii) Show that the locus of P is a circle with QR as diameter. **2**

c)



A projectile is fired from 0 with speed $V\ ms^{-1}$ at an angle of elevation of θ to the horizontal. After t seconds, its horizontal and vertical displacements from 0 as shown are x m and y m.

i) Given that $\ddot{x} = 0$ and $\ddot{y} = -g$, prove the equations of motion are $x = Vt\cos\theta$ and $y = -\frac{1}{2}gt^2 + Vt\sin\theta$. **2**

ii) Find the time taken to reach P **1**

iii) The projectile falls to Q where its angle of depression from 0 is θ. Prove that in its flight from 0 to Q that P is the half way point in terms of time. **2**

Marks

QUESTION 6 (12 MARKS) Begin a NEW sheet of writing paper.

a) Use the identity $(1+x)^8(1+x)^8 = (1+x)^{16}$ to show:

$$\binom{8}{0}^2 + \binom{8}{1}^2 + \binom{8}{2}^2 + \dots + \binom{8}{8}^2 = \binom{16}{8}$$ **3**

b) Prove, by Mathematical induction, that for all positive integral values of n,

$$2\times 1! + 5\times 2! + 10\times 3! + \dots\dots + \left(n^2+1\right)n! = n(n+1)!$$ **4**

c) i) Draw a sketch of $y = x$ and $y = \frac{3-x^2}{2}$ marking the coordinates of the points of intersection. **2**

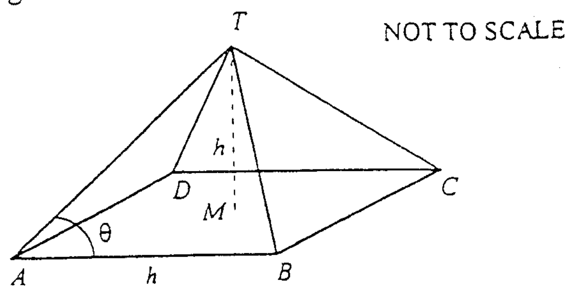
ii) Given $f(x) = \frac{3-x^2}{2}$ find the largest possible domain such that this function has an inverse. **1**

iii) State the domain and range of the inverse function. **1**

iv) Sketch the inverse function also on the same diagram **1**

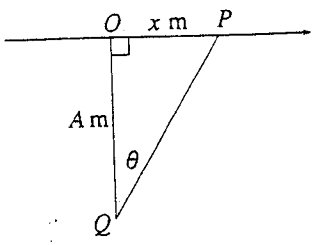
QUESTION 7 (12 MARKS) Begin a NEW sheet of writing paper. Marks

a) The diagram shows a right square pyramid with base ABCD, vertex T and altitude TM. It is given that TM=AB=h units.



Show that if $\angle TAB = \theta^\circ$ then $\cos\theta^\circ = \frac{1}{\sqrt{6}}$ 3

b) P is a point oscillating in simple harmonic motion on an x axis, the centre of motion being the origin, 0. The amplitude of the motion is A m, the period 2π seconds, and when t=0, the point is at 0 moving in the positive direction



i) Express x as a sine function of t. 1

ii) OQ is perpendicular to the axis, OQ=A and $\angle OQP = \theta$
Show that $x = A\tan\theta$ and deduce that $\frac{dx}{d\theta} = A\left(1+\sin^2 t\right)$ 2

iii) Find $\frac{d\theta}{dt}$ as a function of t. 2

iv) Find the first time at which θ is increasing at a rate of $\frac{2}{7}$ radians/sec 1

c) The parabola $y^2 = x$ and $x^2 = 8y$ intersect at the origin and the point (a,b)

i) Find the values of a and b 1

ii) Prove that the curves divide the rectangle whose vertices are $(0,0)$ $(a,0)$ (a,b) $(0,b)$ into three regions of equal area. 2

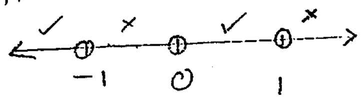
EXT 1 TRIAL 2005 SOLUTIONS

QUESTION 1.

a) $x \neq 0$ critical point

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$



Award 1 $x \neq 0$ and $-1 < x < 1$ and no errors

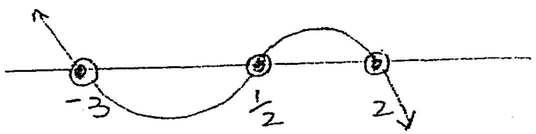
test $x = 2$ test $x = -\frac{1}{2}$

$x - \frac{1}{x} < 0$ false $\quad -\frac{1}{2} + 2 < 0$ false

[2]

$\therefore\ x < -1$ or $0 < x < 1$

b)



$x = -3$

$x = \frac{1}{2}$

$x = 2$

[2]

$-3 \leq x \leq \frac{1}{2}$ and $x \geq 2$

c) 1 M 2 F $\quad {}^4C_1 \times {}^5C_2 = 40$ [2]

or

1 F 2 M $\quad {}^4C_2 \times {}^5C_1 = 30$

total $= 70$

d) $u = 2x - 1$.

$$\int_{-1}^{1} u^4 . \left(\frac{u+1}{2}\right) . \frac{du}{2}$$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$$\frac{1}{4}\int_{-1}^{1} u^5 + u^4 \, du$$

if $x = 1 \quad u = 1$

$x = 0 \quad u = -1$

[3]

$= \frac{1}{4}\left[\frac{u^6}{6} + \frac{u^5}{5}\right]_{-1}^{1} \quad = \frac{1}{}$

[2]

e) $\alpha + \beta + \gamma = -2 \quad \alpha\beta + \alpha\gamma + \beta\gamma = -3 \quad \alpha\beta\gamma = -5$

(-1 each error).

$(\alpha - 1)(\beta - 1)(\gamma - 1) = \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma)$ -1

$= -5 - (-3) + (-2) - 1$

$= -5.$ [1]

(correct answer only here. No half marks)

QUESTION 2

a) $\frac{d}{dx}\left[\ln(2x) - \ln(x-1)^2\right] = \frac{1}{2x} . 2 - \frac{2}{x-1}$

$= \frac{1}{x} - \frac{2}{x-1}$ [1]

(No part marks. correct or award 0) OR $= \frac{-x-1}{x(x-1)}$

b) $\frac{dy}{dx} = \frac{1}{x^2+25}$

$y = \int \frac{1}{x^2+25} dx$

$= \frac{1}{5} \tan^{-1} \frac{x}{5} + c$

when $x = 5 \quad y = \frac{\pi}{2}$

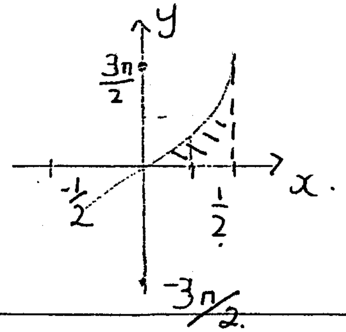
$\frac{\pi}{2} = \frac{1}{5} \tan^{-1} 1 + c$

$\frac{\pi}{2} = \frac{1}{5} \times \frac{\pi}{4} + c$

$\frac{9\pi}{20} = c$

$y = \frac{1}{5} \tan^{-1} \frac{x}{5} + \frac{9\pi}{20}$ [2]

2c)

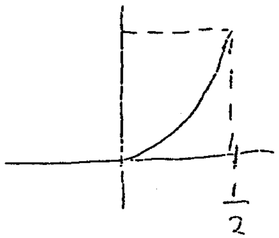


D: $-1 \le 2x \le 1 \quad \therefore -\frac{1}{2} \le x \le \frac{1}{2}$

R: $-\frac{\pi}{2} \le \frac{y}{3} \le \frac{\pi}{2}$

$-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$ [1]

ii)



$\int_0^{\frac{1}{2}} 3\sin^{-1}(2x)\,dx$ [3]

$= \text{rectangle} - \frac{1}{2}\int_0^{\frac{3\pi}{2}} \sin\frac{y}{3}\,dy$

$y = 3\sin^{-1}(2x)$

$\frac{y}{3} = \sin^{-1}(2x$

$2x = \sin\left(\frac{y}{3}\right)$

Award [1] for this
[-1 each error]

$= \frac{1}{2} \times \frac{3\pi}{2} + \frac{3}{2}\left[\cos\frac{y}{3}\right]_0^{\frac{3\pi}{2}}$

$= \frac{3\pi}{4} + \frac{3}{2}\left[\cos\frac{\pi}{2} - \cos 0\right]$

$= \frac{3}{4}(\pi - 2)$ units

or $3\pi/4 - 3/2$.

2d) $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3-x^2}} = \left[\sin^{-1}\frac{x}{\sqrt{3}}\right]_0^{\sqrt{3}}$

$= \sin^{-1}\frac{\sqrt{3}}{\sqrt{3}} - \sin^{-1}0$

$= \sin^{-1} 1$

$= \frac{\pi}{2}$ [2]

2e)

$$\begin{array}{r|l} & 2x^2 - 7x + 15 \\ \hline x^2+2x-1 & 2x^4 - 3x^3 - x^2 + 2x + 1 \\ & 2x^4 + 4x^3 - 2x^2 \\ \hline & -7x^3 + x^2 + 2x \\ & -7x^3 - 14x^2 + 7x \\ \hline & 15x^2 - 5x + 1 \\ & 15x^2 + 30x - 15 \\ \hline & -35x + 16 \end{array}$$

$Q(x) = 2x^2 - 7x + 15 \qquad R(x) = -35x + 16$

Award [2] if one error.

a) $f(x) = \log_e 0.5 + \sin 0.5$

$= -0.214$

$f'(x) = \frac{1}{x} + \cos x$

$= \frac{1}{0.5} + \cos 0.5$

$= 2.8776$

$\text{approx} = 0.5 - \frac{-0.214}{2.8776} = 0.574$

b)

$\frac{1}{2}\int 1 + \cos 2(x - \pi/4)\, dx$

$\cos 2x = 2\cos^2 x - 1$

$\frac{1 + \cos 2x}{2} = \cos^2 x$

$\frac{1}{2}\left[x + \frac{1}{2}\sin 2\left(x - \frac{\pi}{4}\right)\right] + c$

c) $T_{k+1} = {}^{12}C_k \, (5x^2)^{12-k} \cdot \left(\frac{-1}{2x}\right)^k$

$= {}^{12}C_k \cdot 5^{12-k} \cdot x^{24-2k} \cdot (-1)^k \cdot \left(\frac{1}{2}\right)^k \cdot x^{-k}$

now $x^{24-2k} \cdot x^{-k} = x^9$

$24 - 3k = 9$

$k = 5$

$\therefore$ coefficient ${}^{12}C_5 \cdot 5^7 \cdot (-1)^5 \cdot \left(\frac{1}{2}\right)^5$ [2]

$= -1933593.75$

d) i) $\ddot{x} = -9x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -9x$

$\frac{1}{2}v^2 = -\frac{9x^2}{2} + c$

when $x = 4$ $v = 0$

$0 = -72 + c$

$c = 72$

$\frac{1}{2}v^2 = -\frac{9x^2}{2} + 72$

$\therefore v^2 = -9x^2 + 144$

ii) max speed acceleration $= 0$ at centre of motion

$x = 0$

$v^2 = 144$

$v = 12$ m/sec

e) join D to E

$\angle BED = 80°$ $\angle$ in alternate segment

$\angle DOE = 80°$ 2 x angle at circumf.

$DO = OE$ (equal radii)

$\therefore \angle OED = 50°$ (equal $\angle$'s isosceles $\triangle$)

$\angle BEO = \angle BED - \angle OED$

$= 80 - 50$

$= 30°$

QUESTION 4.

$$\frac{T_{k+1}}{T_k} = \frac{{}^{12}C_k (2x)^{12-k} \cdot \left(\frac{3}{x}\right)^k}{{}^{12}C_{k-1} (2x)^{13-k} \cdot \left(\frac{3}{x}\right)^{k-1}}$$

$$= \frac{{}^{12}C_k}{{}^{12}C_{k-1}} \quad \frac{\frac{3}{x}}{2x}$$

$$= \frac{12!}{(12-k)!\,k!} \cdot \frac{(13-k)!\,(k-1)!}{12!} \cdot \frac{3}{2x^2}$$

$$= \frac{3(13-k)}{2kx^2}$$

[2]

$$= \frac{39-3k}{2k}$$

$$\frac{T_{k+1}}{T_k} \geq 1$$

$$\frac{39-3k}{2k} \geq 1$$

$k \leq 7\frac{4}{5}$

so $k = 7$.

greatest coeff

${}^{12}C_7 \cdot 2^5 \cdot 3^7$ [2]

$= 55\,427\,328$

b) i) $\sqrt{3}\cos 2t - \sin 2t = R\cos(2t+\alpha)$

$R^2 = \sqrt{3}^2 + 1^2$

$R = 2.$

$\frac{\sqrt{3}}{2}\cos 2t - \frac{1}{2}\sin 2t = \cos 2t\cos\alpha - \sin 2t \sin\alpha$

$\therefore \cos\alpha = \frac{\sqrt{3}}{2}$

$\alpha = \frac{\pi}{6}$

$\therefore \quad 2\cos\left(2t + \frac{\pi}{6}\right)$

ii) $2\cos\left(2t + \frac{\pi}{6}\right) = 0$

$\cos\left(2t + \frac{\pi}{6}\right) = 0$

$2t + \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$2t = \frac{\pi}{3}, \frac{8\pi}{6}, \frac{14\pi}{6}$

$t = \frac{\pi}{6}, \frac{4\pi}{6}, \frac{7\pi}{6} \ldots$

$t = \left(\frac{3n+1}{6}\right)\pi$

[2]

OR

$2t + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{2}$

$2t = 2n\pi \pm \frac{\pi}{2} - \frac{\pi}{6}$

$2t = 2n\pi + \frac{\pi}{2} - \frac{\pi}{6}$

$= 2n\pi + \frac{\pi}{3}$ $\quad n\pi + \frac{\pi}{6}$

$2t = 2n\pi - \frac{\pi}{2} - \frac{\pi}{6}$

$= 2n\pi - \frac{2\pi}{3}$

$t = n\pi - \frac{\pi}{3}$

c) $x = 5 + \sqrt{3}\cos 2t - \sin 2t \quad (1)$

$\dot{x} = -2\sqrt{3}\sin 2t - 2\cos 2t$

$\ddot{x} = -4\sqrt{3}\cos 2t + 4\sin 2t$ [2]

$= -4(\sqrt{3}\cos 2t - \sin 2t)$

$= -4(x-5)$ from (1)

ii) $x = 5 + 2\cos\left(2t + \frac{\pi}{6}\right)$

centre is 5 amplitude 2 [1]

$\therefore$ moves between 3 and 7

iii) $x = 5 \quad 5 = 5 + 2\cos\left(2t + \frac{\pi}{6}\right)$

$2\cos\left(2t + \frac{\pi}{6}\right) = 0$ [1]

$\therefore t = \frac{\pi}{6}$

a) $p = \frac{1}{3}$ $\quad q = \frac{2}{3}$

$^5C_3\, p^3 q^2 = {}^5C_3 \left(\frac{1}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^2$ ✓ [1]

$= \frac{40}{243}$

ii) $1 - \text{no success} \geq 0.9$

$1 - {}^nC_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n \geq 0.9$

$\frac{2}{3}^n \leq 0.1$

$\log_e \left(\frac{2}{3}\right)^n \leq \log_e 0.1$ [2]

$n \geq \frac{\log_e 0.1}{\log_e 2/3} \geq 5.4$

at least 6 matches

5b) $\overset{A}{(-4, 1)}\ \overset{B}{(2, 4)}$ $\quad$ 2 : 1 $\quad \overset{A}{(-4, 1)}\ \overset{B}{(2, 4)}$ $\quad$ 2 : 1

$Q = (0, 3)$ $\quad R(8, 7)$ $\quad$ (2)

ii) $PA = 2PB$

$\sqrt{(x+4)^2 + (y-1)^2} = 2\sqrt{(x-2)^2 + (y-4)^2}$

$x^2 + 8x + 16 + y^2 - 2y + 1 = 4[x^2 - 4x + 4 + y^2 - 8y + 16]$

$\therefore 3x^2 - 24x + 3y^2 - 30y + 63 = 0$

$x^2 - 8x + y^2 - 10y = -21$

$(x-4)^2 + (y-5)^2 = 20$

midpoint $QR = (4, 5)$ which is

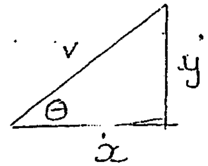
length of $QR = \sqrt{8^2 + 4^2}$

$= \sqrt{80}$

$= 4\sqrt{5}$

radius of circle $= 2\sqrt{5}$ so QR diameter

c). $\ddot{y} = -g$



$\dot{y} = -gt + c$

$\dot{y} = -gt + v\sin\theta$

$y = -\frac{1}{2}gt^2 + vt\sin\theta + c_2$

when $t = 0$ $y = 0$ $c_2 = 0$

$y = -\frac{1}{2}gt^2 + vt\sin\theta$

$\sin\theta = \frac{\dot{y}}{v}$ $\quad \cos\theta = \dot{x}/v$

$v\sin\theta = \dot{y}$ $\quad v\cos\theta = \dot{x}$

$\ddot{x} = 0$ [2]

$\dot{x} = v\cos\theta$

$x = vt\cos\theta + c_3$

$t = 0$ $x = 0$ $c_3 = 0$

$x = vt\cos\theta$

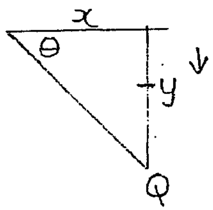
ii) $y = 0$

$0 = -\frac{1}{2}gt^2 + vt\sin\theta$

$0 = -gt + 2v\sin\theta$

$t = \frac{2v\sin\theta}{g}$ [1]

iii)



$\tan\theta = \frac{-y}{x}$

$= \frac{+\frac{1}{2}gt^2 - vt\sin\theta}{vt\cos\theta}$

$vt\cos\theta \cdot \tan\theta = +\frac{1}{2}gt^2 - vt\sin\theta$

$v\cos\theta \cdot \frac{\sin\theta}{\cos\theta} = +\frac{1}{2}gt - v\sin\theta$

$2v\sin\theta = \frac{1}{2}gt$

$\frac{4v\sin\theta}{g} = t$ $\quad$ so twice the time to Q

QUESTION 6

a) $(1+x)^8(1+x)^8 = (1+x)^{16}$

ie $({}^8C_0 + {}^8C_1 x + {}^8C_2 x^2 + \dots {}^8C_8 x^8)({}^8C_0 + {}^8C_1 x + \dots {}^8C_8 x^8)$

$= {}^{16}C_0 + {}^{16}C_1 x + \dots {}^{16}C_{16} x^{16}$ [3]

RHS coeff of $x^8 = {}^{16}C_8$

LHS coeff of $x^8 = {}^8C_0 . {}^8C_8 + {}^8C_1 . {}^8C_7 + {}^8C_2 . {}^8C_6 \dots$

$= ({}^8C_0)^2 + ({}^8C_1)^2 + ({}^8C_2)^2 + \dots ({}^8C_8)^2$

b) Prove true for n=1

LHS = $2 \times 1!$ RHS = $1(1+1)!$

$= 2$ $= 2$

$\therefore$ true for $n=1$

Step (2) Assume true for $n=k$

$2\times 1! + 5\times 2! + 10\times 3! + \dots + (k^2+1)k! = k(k+1)!$

step (3) prove true for $n=k+1$ if true for $n=k$.

$= 2\times 1! + 5\times 2! + \dots + (k^2+1)k! + ((k+1)^2+1)(k+1)!$

$= k(k+1)! + (k^2+2k+1+1)(k+1)!$

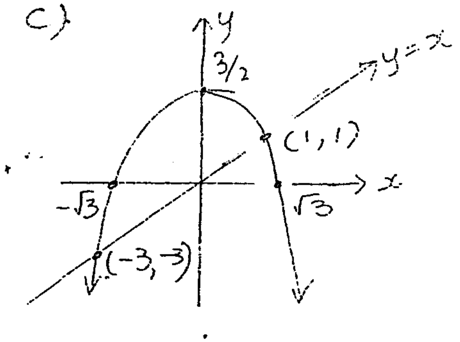
$= (k+1)!(k+k^2+2k+2)$

$= (k+1)!(k+2)(k+1)$

$= (k+1)(k+2)!$

step (4) since it is true for n=1 then it is true for n=1+1, n=2, since it is true for n=k then it is true for n=k+1 and for all integral values of n $n \geq 1$.

c)



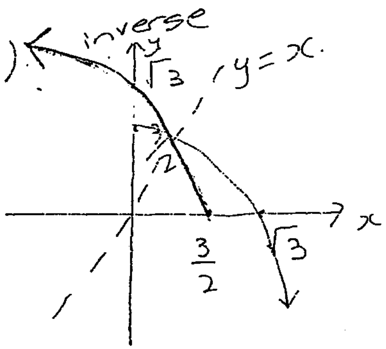
[2]

ii) $x \geq 0$ [1]

iii) D : $x \leq \frac{3}{2}$ [½]

R : $y \geq 0$ [½]

iv)



[1]

QUESTION 7.

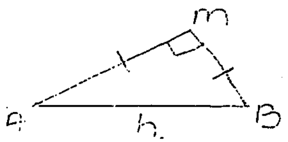
a) $\cos\theta = \dfrac{AT^2 + AB^2 - TB^2}{2 \times AT \times AB}$

$= \dfrac{AT^2 + h^2 - AT^2}{2 \times AT \times AB}$

since $AT = TB$

$= \dfrac{h^2}{2 \times AT \times AB}$

need to find AT^2 in terms of h.

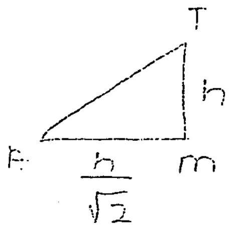


$Am = mB$ square pyramid

$\therefore\ Am^2 + Am^2 = h^2$

$2Am^2 = h^2$

$Am = \dfrac{h}{\sqrt{2}}$



$AT^2 = h^2 + \dfrac{h^2}{2}$

$= \dfrac{3h^2}{2}$

$AT = h\sqrt{\dfrac{3}{2}}$

[3]

$\cos\theta = \dfrac{h^2}{2 \times h\sqrt{\frac{3}{2}} \times h}$

$= \dfrac{1}{2 \times \frac{\sqrt{3}}{\sqrt{2}}} = \dfrac{\sqrt{2}}{2\sqrt{3}} = \dfrac{1}{\sqrt{6}}$

b) $x = A\sin nt$ $\quad T = \dfrac{2\pi}{n}$ so $2\pi = \dfrac{2\pi}{n}$

$= A\sin t$ [1] $\quad n = 1$

ii) $\tan\theta = \dfrac{x}{A}$

$A\tan\theta = x$

$\dfrac{dx}{d\theta} = A\sec^2\theta$

$= A(1 + \tan^2\theta)$

$= A\left(1 + \dfrac{x^2}{A^2}\right)$

$= A\left(1 + \dfrac{A^2\sin^2 t}{A^2}\right)$ [2]

$= A(1 + \sin^2 t)$

iii) $\dfrac{d\theta}{dt} = \dfrac{d\theta}{dx} \cdot \dfrac{dx}{dt}$

$= \dfrac{1}{A(1+\sin^2 t)} \cdot A\cos t$

$= \dfrac{\cos t}{1 + \sin^2 t}$

iv) Find when $\dfrac{\cos t}{1+\sin^2 t} = \dfrac{2}{7}$

$7\cos t = 2 + 2\sin^2 t$

$7\cos t = 2 + 2(1 - \cos^2 t)$

$2\cos^2 t + 7\cos t - 4 = 0$

Q7 ctd.

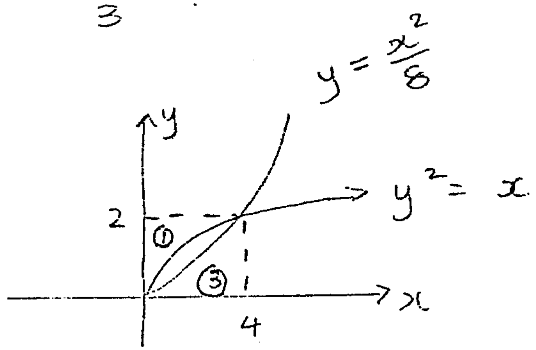
$(2\cos t - 1)(\cos t + 4) = 0$

$\cos t = \frac{1}{2}$ $\quad$ $\cos t = -4$

no solution

$t = \frac{\pi}{3}$

(c)



$x^2 = y^4$ $\quad$ $\therefore\ 8y = x^2$

$8y = y^4$

$0 = y^4 - 8y$

$0 = y(y^3 - 8)$

$y = 0$ or $y = 2$

$x = 0$ $\quad$ $x = 4$

$\therefore\ a = 4 \quad b = 2$

$A_1 = \int_0^2 y^2\,dy$

$= \frac{8}{3}\,u$

$A_3 \int_0^4 \frac{x^2}{8}\,dx$

$= \frac{8}{3}\,u$

$A_2 = \text{rectangle} - A_1 - A_3$

$= 8 - \frac{8}{3} - \frac{8}{3}$

$= \frac{8}{3}$

$\therefore$ rectangle divided in 3 equal areas