

Student Name/ Number: .....



**2006**

**YEAR 12**

**TRIAL HIGHERSCHOOL CERTIFICATE EXAMINATION**

# **MATHEMATICS EXTENSION 1**

## **General Instructions**

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question

## **Total marks - 84**

- Attempt Questions 1-7
- All questions are of equal value.

**Total Marks – 84**

**Attempt Questions 1-7**

**All Questions are of equal value**

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page.

---

- QUESTION 1. (12 MARKS)** Begin a NEW sheet of writing paper. **Marks**
- a) Solve  $\frac{x}{x^2 - 4} < 0$  **2**
- b) For what values of  $x$  is  $3^{2x} - (1 + \sqrt{3}) \times 3^x + \sqrt{3} = 0$  ? **2**
- c) Write the general solution to  $\sqrt{3} \tan \theta - 1 = 0$  in exact radian form **2**
- d) Find  $\int \sec^2 x \cdot \tan^2 x dx$  using the substitution  $u = \tan x$  **2**
- e) Calculate the acute angle between the lines  $2x - y - 1 = 0$  and  $x - 2y + 1 = 0$ . Give your answer to the nearest degree. **2**
- f) A parabola has the parametric equation  $x = \sin \theta, y = \cos 2\theta$ . What is the cartesian equation of this parabola? **2**

**QUESTION 2 (12 MARKS)** Begin a NEW sheet of writing paper.

**Marks**

- a) With reference to the table of standard integrals find **2**  
$$\int \frac{\tan 2x}{\cos 2x} dx$$
- b) A particle is moving along the  $x$  axis. Its velocity  $V$  at position  $x$  is given by  $V = \sqrt{8x - x^2}$ . Find the acceleration when  $x = 3$  **2**
- c) On the same axes sketch the graphs of  $y - 2x = 0$  and  $y = -\cos x$  for  $-\pi \leq x \leq \pi$ . Use the graph to deduce the number of solutions to  $2x + \cos x = 0$  **2**
- d) Find the coordinates of the point which divides the interval joining  $(3, -2)$  and  $(-5, 4)$  *externally* in the ratio 5:2 **2**
- e)  $(x - k)$  is a factor of  $x^2 - 5x + (2k + 2)$ . Find the value(s) of  $k$ . **2**
- f) The equation  $2x^3 + 12x^2 + 6x - 20 = 0$  has roots  $\alpha - d, \alpha$  and  $\alpha + d$ . **1**
- i) Find the value of  $\alpha$  **1**
- ii) Find a value of  $d$ . **1**

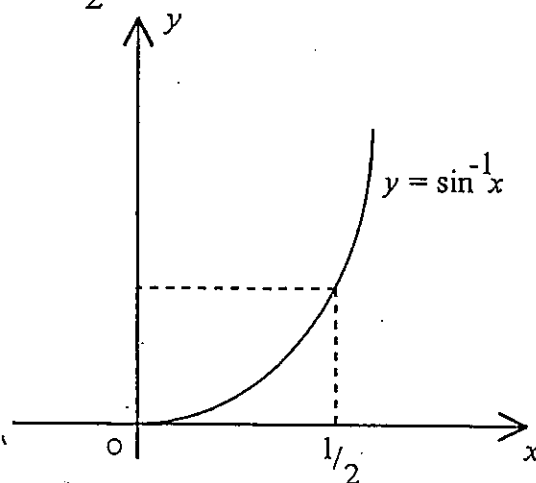
**QUESTION 3 (12 MARKS)** Begin a NEW sheet of writing paper.

**Marks**

a) i) Show that  $x^3 + x^2 + x - 8 = 0$  has a root between  $x = 1$  and  $x = 2$ . **1**

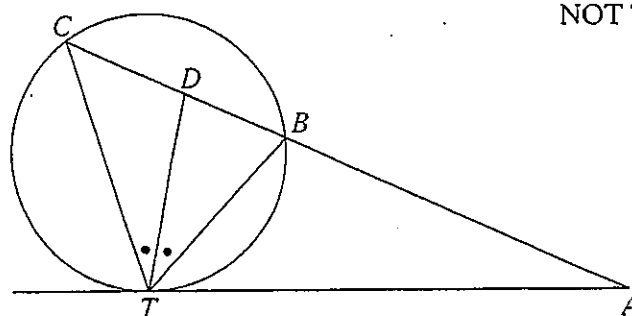
ii) Starting with  $x = 2$  as the first approximation to the root of  $x^3 + x^2 + x - 8 = 0$ , use one application of Newton's method to find a better approximation to the root. **2**

b) Find the exact area bounded by the curve  $y = \sin^{-1} x$ , the  $x$  axis and the ordinate  $x = \frac{1}{2}$ , as shown in the diagram **3**



c) Differentiate  $(x^2 + 2x + 2)e^{-x}$  and hence evaluate  $\int_1^2 x^2 e^{-x} dx$  to 3 decimal places. **3**

d) TA is a tangent to a circle. Line ABCD intersects the circle at B and C. Line TD bisects  $\angle BTC$ . Prove  $AT = AD$  **3**

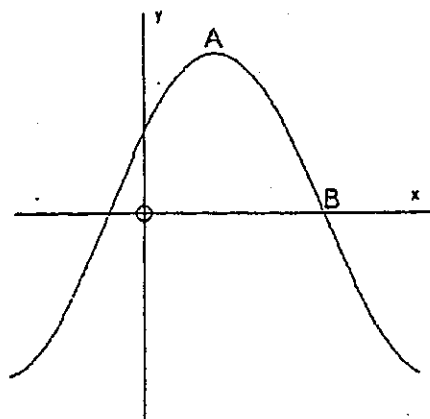


NOT TO SCALE

**QUESTION 4 (12 MARKS)** Begin a NEW sheet of writing paper.

**Marks**

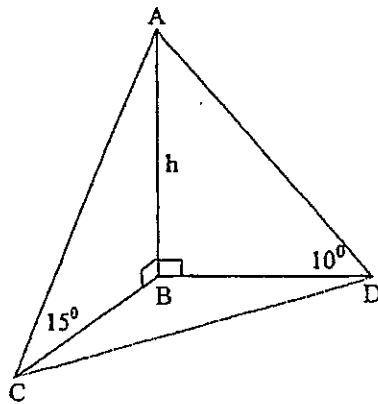
- a)  $P(2ap, ap^2)$  is a point on the parabola  $x^2 = 4ay$
- Show the equation of the normal at  $P$  is  $x + py = 2ap + ap^3$  **2**
  - This normal cuts the  $y$ -axis at  $R$ . State the coordinates of  $R$ . **1**
  - From  $P$ , a line  $PT$  is drawn perpendicular to the directrix, meeting it at  $T$ . State the coordinates of  $T$ . **1**
  - If  $M$  is the midpoint of  $RT$ , find the coordinates of  $M$ . **1**
  - Find the locus of  $M$  and show that it is a parabola with vertex at the focus of the original parabola. **2**
- b)
- Express  $\sqrt{3} \sin x + \cos x$  in the form  $R \sin(x + \alpha)$ ;  $R > 0$ ,  $0 < \alpha < \frac{\pi}{2}$  **2**
  - The graph of  $y = \sqrt{3} \sin x + \cos x$  is shown here. Find the coordinates of  $A$  and  $B$  if  $A$  is a maximum turning point and  $B$  is where the curve cuts the  $x$  axis **3**



**QUESTION 5 (12 MARKS)** Begin a NEW sheet of writing paper

a) A B represents Kincumber mountain, height  $h$  metres. From points C and D in the same plane as the base of the mountain, the angles of elevation of the top of the mountain (A) are  $15^\circ$  and  $10^\circ$  respectively. From the base of the mountain, the bearings of the points C and D are  $230^\circ$  and  $100^\circ$  respectively.

- i) Find the size of angle CBD 1
- ii) Show  $BD = h \cot 10^\circ$  1
- iii) If CD is 450 metres find the height of the mountain 3



- b) i) Prove that  $\frac{d}{dx} \left( \frac{x}{\sqrt{1-x^2}} \right) = \frac{1}{(1-x^2)^{\frac{3}{2}}}$  2
- ii) Hence find the derivative of  $\tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$  2
- iii) What restrictions are there on  $x$  1
- iv) By considering a right angled triangle with a 1 unit hypotenuse show that  $\tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) = \sin^{-1} x$  for the domain  $0 < x < 1$  2

**QUESTION 6 (12 MARKS)** Begin a NEW sheet of writing paper.

a) i) If  $f(x) = 2 - \sqrt{x}$ ,  $x \geq 0$  and  $g(x) = (x - 2)^2$  for all  $x$  find the values of  $x$  for which  $f[g(x)] = x = g[f(x)]$  2

ii) Find  $f^{-1}(x)$  giving its domain. 1

b) Use mathematical induction to prove that for all positive integers  $n \geq 1$  4

$$1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2 = \frac{1}{3}n(2n - 1)(2n + 1)$$

c) A particle is moving in a straight line with Simple Harmonic Motion. At time  $t$  seconds it has displacement  $x$  metres from a fixed point O, velocity  $v$  m/s and its acceleration  $a$   $ms^{-2}$  is given by  $a = -4x + 4$ . Initially the particle is 2m to the right of O and moving away from O with speed  $2\sqrt{3}ms^{-1}$

i) Use integration to show  $v^2 = -4x^2 + 8x + 12$  2

ii) Hence find the centre of the motion 1

iii) If  $x = 1 + 2 \cos(2t + \alpha)$  for  $0 < \alpha < 2\pi$  find the exact value of  $\alpha$ . 2

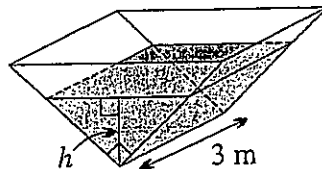
**QUESTION 7 (12 MARKS)** Begin a NEW sheet of writing paper.

Marks

a) Whilst playing in the US Open Tennis Andre Agassi can serve a ball from the height of 1.8 metres. He hits the ball in a horizontal direction at a speed of 35m/s.

- i) Using  $g = 10ms^{-2}$ , derive expressions for the horizontal displacement  $x$  metres and the vertical displacement  $y$  metres, of the tennis ball after time  $t$  seconds of being hit. 2
- ii) Find how long before the ball hits the ground. 1
- iii) Find how far the ball will travel before bouncing. 1
- iv) By how much will the ball clear the net which is 0.95 metres high and 14 metres away from the service line. 2

b) Due to council water restrictions a new tank in the shape of an isosceles triangular prism was installed. The tank was 3 metres long. A hose was used to fill the tank at a constant rate of 2 litres /second. The depth of water was  $h$  cm at time  $t$  seconds



- i) Find an expression for the volume of water in the tank ( in  $cm^3$ /second.) The depth of water is  $h$  cm 1
- ii) Find the rate at which the depth of the water is changing when  $h = 20$  cm 2



**QUESTION 7 CONTINUED**

c) A can of soft drink has an initial temperature of  $18^{\circ}\text{C}$ . To chill it Kim places it in her freezer that has a constant temperature of  $-19^{\circ}\text{C}$ . The cooling rate of the soft drink is proportional to the difference between the temperature of the freezer and the temperature of the soft drink,  $T$ , that is  $T = -19 + Ae^{-kt}$

- i) Find the value of  $A$ . **1**
- ii) After 5 minutes in the freezer the temperature of the drink is  $3^{\circ}\text{C}$ . Find the time it will take for the drink to reach a freezing temperature of  $0^{\circ}$  **2**

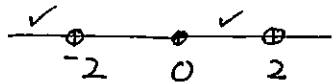
**END OF EXAMINATION**

QUESTION 1.

EX 1 1 2006 TRIAL, GNS.

a)  $\frac{x}{x^2-4} < 0$  c.p  $x \neq \pm 2$

Solve  $x = 0$



test  $x = 1$

$$\frac{1}{-3} \leq 0 \text{ true}$$

$$\underline{x < -2} \quad \underline{0 < x < 2}$$

test  $x = -3$

$$\frac{-3}{5} \leq 0 \text{ true}$$

b) let  $m = 3^x$

but  $3^x = m$

$$m^2 - (1 + \sqrt{3})m + \sqrt{3} = 0$$

$$3^x = \sqrt{3} \quad 3^x = 1$$

$$(m - \sqrt{3})(m - 1) = 0$$

$$\underline{x = \frac{1}{2}} \quad \underline{x = 0}$$

$$m = \sqrt{3} \quad m = 1$$

c)  $\sqrt{3} \tan \theta = 1$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = n\pi + \alpha \text{ in radians}$$

$$0 = n\pi + \frac{\pi}{6}$$

d)  $\int u^2 du$

$$u = \tan x$$

$$= \frac{u^3}{3} + c$$

$$\frac{du}{dx} = \sec^2 x$$

$$du = \sec^2 x dx$$

$$= \frac{1}{3} \tan^3 x + c$$

e)  $2x - 1 = y$

$$x + 1 = 2y$$

$$m_1 = 2$$

$$\frac{1}{2}x + \frac{1}{2} = y$$

$$m_2 = \frac{1}{2}$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} \right|$$

$$= \left| \frac{1\frac{1}{2}}{2} \right|$$

$$\theta = 37^\circ$$

f)  $x = \sin \theta$

$$y = \cos 2\theta$$

$$= 1 - 2 \sin^2 \theta$$

$$= 1 - 2x^2$$

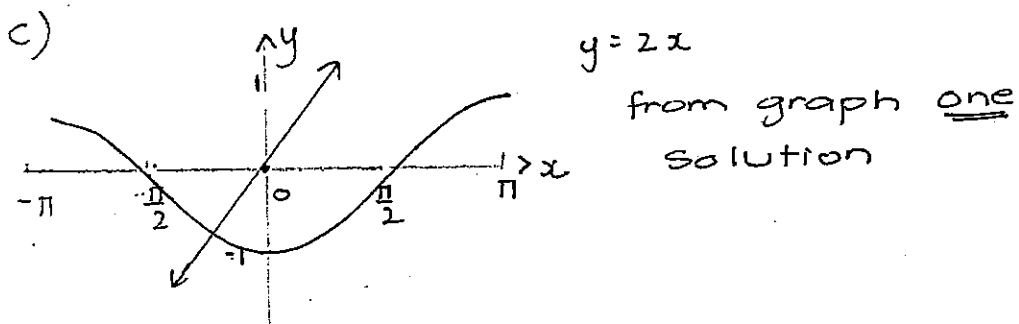
answer  $y = 1 - 2x^2$

QUESTION 2.

a) 
$$\int \frac{\tan 2x}{\cos 2x} dx = \int \sec 2x \tan 2x$$

$$= \frac{1}{2} \sec 2x + c..$$

b)  $v^2 = 8x - x^2$   
 $\frac{1}{2} v^2 = 4x - \frac{x^2}{2}$   
 $\frac{d}{dx} \left( \frac{1}{2} v^2 \right) = 4 - x$   
 at  $x = 3$   $\ddot{x} = 4 - 3$   
 $= 1 \text{ m/s}^2$



d)  $(3, -2)$   $(-5, 4)$   
 $5 : 2$   
 $x = \frac{6 - 25}{3}$   $y = \frac{4 - 20}{3} \Rightarrow x = \frac{-19}{3}$   $y = \frac{-16}{3}$

e)  $f(x) = x^2 - 5x + (2k+2)$   
 $f(k) = k^2 - 5k + 2k + 2 = 0$   
 $k^2 - 3k + 2 = 0$   
 $(k-2)(k-1) = 0$   $k = 2$  or  $k = 1$ .

f)  $\alpha + \beta + \gamma = -\frac{b}{a}$   $\alpha \beta \gamma = -\frac{d}{a}$   
 $a-d + a + a + d = -6$   $(a-d)(a)(a+d) = 10$   
 $3a = -6$   $(-2-d) \times -2(-2+d) = 10$   
 $a = -2$   $-8 + 2d^2 = 10$   
 $2d^2 = 18$   
 $d^2 = 9$   
 $d = \pm 3$

### QUESTION 3.

a)  $f(x) = x^3 + x^2 + x - 8$

$f(1) = -5$

$f(2) = 8 + 4 + 2 - 8$   
 $= 6$

$f(1)$  and  $f(2)$  have opposite signs and  $f(x)$  is continuous

$\therefore$  a root exists between 1 & 2

ii) approx  

$$= x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2 - \frac{6}{17}$$

$$= \underline{1.647}$$

$f'(x) = 3x^2 + 2x + 1$

$f'(2) = 12 + 4 + 1$   
 $= 17$

b)  $y = \sin^{-1} x$      $x = \frac{1}{2}$      $x = 0$   
 $y = \frac{\pi}{6}$      $y = 0$

Area = rectangle - area to y axis

$= \frac{1}{2} \times \frac{\pi}{6} - \int_0^{\frac{\pi}{6}} \sin y \, dy$

$= \frac{\pi}{12} - \left[ -\cos y \right]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{12} + \cos \frac{\pi}{6} - \cos 0$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

c)  $\frac{d}{dx} \left[ (x^2 + 2x + 2)e^{-x} \right]$   
 $= (2x + 2) \cdot e^{-x} - e^{-x}(x^2 + 2x + 2)$   
 $= e^{-x} [2x + 2 - x^2 - 2x - 2]$   
 $= -x^2 e^{-x}$

$\int_1^2 x^2 e^{-x} \, dx = \left[ -(x^2 + 2x + 2)e^{-x} \right]_1^2$   
 $= -e^{-2}(4 + 4 + 2) + e^{-1}(1 + 2 + 2)$   
 $= -10e^{-2} + 5e^{-1}$   
 $= 0.486$

d) Let  $\angle DTB = \beta$   
 $\angle BTA = \angle TCB$  ( $\angle$  in alternate segment) =  $\alpha$

$\angle TDA$  is exterior  $\angle$  of  $\triangle TCD$

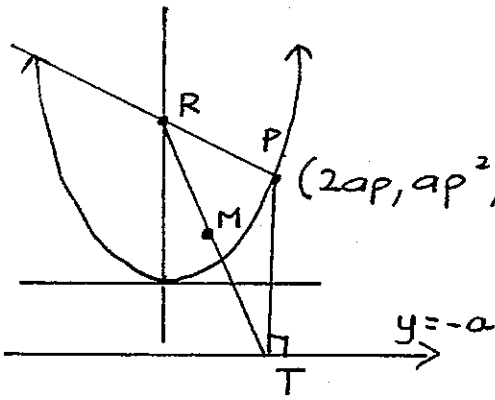
$\therefore \angle TDA = \alpha + \beta$

similarly  $\angle DTA = \alpha + \beta$

$\therefore \triangle ADT$  is isosceles  $\triangle$  (equal base  $\angle$ 's)

$\therefore AT = AD$

# QUESTION . . 4



$$i) y = \frac{x^2}{4a}$$

$$\frac{dy}{dx} = \frac{2x}{4a} \text{ at } x = 2ap$$

$$m = p$$

$$\text{normal } m = -\frac{1}{p}$$

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$py - ap^3 = -x + 2ap$$

$$x + py = 2ap + ap^3$$

$$ii) \text{ y axis } x = 0 \quad py = 2ap + ap^3$$

$$(0, 2a + ap^2) = R.$$

$$iii) T \text{ directrix } y = -a$$

$$T (2ap, -a)$$

$$iv) R (0, 2a + ap^2) \quad T (2ap, -a)$$

$$M = (ap, \frac{a + ap^2}{2})$$

$$v) x = ap \quad y = \frac{a + ap^2}{2}$$

$$p = \frac{x}{a} \quad y = \frac{a}{2} + \left(\frac{x}{a}\right)^2 \cdot \frac{a}{2}$$

$$y = \frac{a^2 + x^2}{2a}$$

$$2ay - a^2 = x^2$$

$$2a(y - \frac{a}{2}) = x^2 \Rightarrow \text{vertex } (0, \frac{a}{2}).$$

$$b) \sqrt{3} \sin x + \cos x = R \sin(x + \alpha)$$

(1 mark R)

$$R = 2$$

$$\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = \sin x \cos \alpha + \cos x \sin \alpha$$

$$\cos \alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{6}$$

(1 mark  $\alpha$ )

$$\therefore \sqrt{3} \sin x + \cos x = 2 \sin(x + \frac{\pi}{6})$$

$$ii) A = (\frac{\pi}{2} - \frac{\pi}{6}, 2)$$

(1 mark each for  $x$  and  $y$ )

$$= (\frac{\pi}{3}, 2)$$

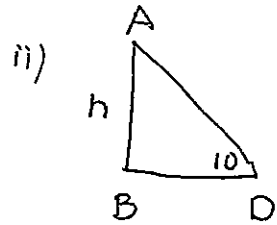
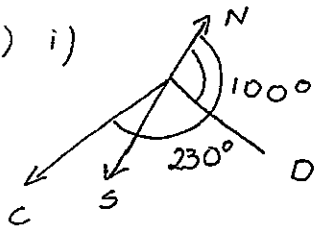
$$B = \pi - \frac{\pi}{6}$$

(1 mark for  $B = \frac{5\pi}{6}$ )

$$= \frac{5\pi}{6}$$

QUESTION 5

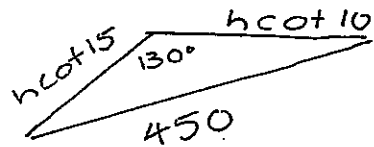
2) i)  $\angle CBD = 130^\circ$  (1)



$$\tan 10^\circ = \frac{h}{BD}$$

$$BD = \frac{h}{\tan 10^\circ} = h \cdot \cot 10^\circ$$

iii)  $BC = h \cdot \cot 15^\circ$



$$450^2 = h^2 \cot^2 15 + h^2 \cot^2 10 - 2h^2 \cot 10 \cot 15 \cos 130^\circ$$

$$450^2 = h^2 [\cot^2 15 + \cot^2 10 - 2 \cot 10 \cot 15 \cos 130^\circ]$$

$$450^2 = h^2 \times 73.3 \quad \checkmark$$

$$h^2 = 2762.589$$

$$h = 52.56 \text{ m} \quad \checkmark$$

$$\begin{aligned} \text{b) } \frac{d}{dx} \left( \frac{x}{\sqrt{1-x^2}} \right) &= \frac{\sqrt{1-x^2} \cdot 1 - x \cdot \frac{1}{2} (1-x^2)^{-1/2} \cdot (-2x)}{(1-x^2)} \\ &= \frac{(1-x^2)^{1/2} + \frac{x^2}{(1-x^2)^{1/2}}}{(1-x^2)} \\ &= \frac{(1-x^2) + x^2}{(1-x^2)^{3/2}} \\ &= \frac{1}{(1-x^2)^{3/2}} \end{aligned}$$

ii) Note  $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

$$\begin{aligned} \frac{d}{dx} \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) &= \frac{1}{1 + \frac{x^2}{1-x^2}} \times \frac{1}{(1-x^2)^{3/2}} \\ &= \frac{1-x^2}{1-x^2+x^2} \times \frac{1}{(1-x^2)^{3/2}} \end{aligned}$$

$$\begin{aligned} &= (1-x^2) \times \frac{1}{(1-x^2)^{3/2}} \\ &= \frac{1}{\sqrt{1-x^2}} \end{aligned}$$

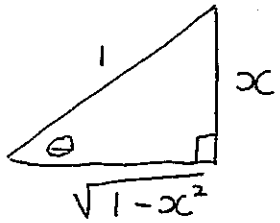
QUESTION 5 ctd.

b iii)  $\tan^{-1} x$  exists for all real  $x$

$\therefore \frac{x}{\sqrt{1-x^2}}$  must be real

$$\therefore -1 < x < 1$$

iv)



$$\sin \theta = \frac{x}{1}$$

$$\sin^{-1} x = \theta$$

$$\text{and } \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) = \theta$$

for  $0 < x < 1$

$$\text{so } \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) = \theta = \sin^{-1} x$$

QUESTION 6

i)  $f(x) = 2 - \sqrt{x} \quad x \geq 0$

$g(x) = (x-2)^2$

$f[g(x)] = 2 - \sqrt{(x-2)^2}$   
 $= 2 - |x-2|$   
 $= 2 - (x-2)$   
 $= 2 + (x-2)$   
 $= x$

for  $x \leq 2$

NOTE

$\sqrt{(x-2)^2} = \pm(x-2)$

If  $x > 2$

$2 - (x-2) = 4 - x$

This is not needed as told  $f[g(x)] = x$

If  $x \leq 2$

$2 - (x-2)$

which is required.

$g[f(x)] = (2 - \sqrt{x} - 2)^2$   
 $= (-\sqrt{x})^2$   
 $= x$

$\therefore f[g(x)] = g[f(x)] = x$

for  $x \geq 0$  and  $x \leq 2$ . or  $0 \leq x \leq 2$

ii)  $y = 2 - \sqrt{x}$

$x = 2 - \sqrt{y}$

$\sqrt{y} = (2-x)$

$y = (2-x)^2$

$f^{-1}(x) = (2-x)^2$

Domain  $x \leq 2 \quad y \geq 0$

b)  $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$

• prove true for  $n=1$

LHS =  $1^2 = 1$       RHS =  $\frac{1}{3} \cdot 1 \cdot (2-1)(2+1) = 1$

hence true for  $n=1$

• assume true for  $n=k$

$1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{1}{3}k(2k-1)(2k+1)$

• prove true for  $n=k+1$  if true for  $n=k$

$1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 + (2k+1)^2$

$= \frac{1}{3}k(2k-1)(2k+1) + (2k+1)^2$

$= \frac{1}{3}k(2k-1)(2k+1) + \frac{3}{3}(2k+1)^2$

$= \frac{1}{3}(2k+1) \{ k(2k-1) + 3(2k+1) \}$

$= \frac{1}{3}(2k+1) \{ 2k^2 + 5k + 3 \}$

$= \frac{1}{3}(2k+1) \{ k+1 \} \{ 2k+3 \}$

$= \frac{1}{3}(k+1)(2(k+1)-1)(2(k+1)+1)$

= RHS.

since it is true for  $n=1$ , true for  $n=1+1$  i.e  $n=2$ , if it is true for  $n=k$ , then true for  $n=k+1$  and so on for all positive integers  $n \geq 1$



### QUESTION 6 (c)

$$\ddot{x} = -4x + 4$$

$$\frac{1}{2}v^2 = \int -4x + 4 dx$$

$$\frac{1}{2}v^2 = -2x^2 + 4x + c$$

at  $x = 2$   $v = 2\sqrt{3}$

$$6 = -8 + 8 + c$$

$$c = 6$$

$$\frac{1}{2}v^2 = -2x^2 + 4x + 6$$

$$v^2 = -4x^2 + 8x + 12$$

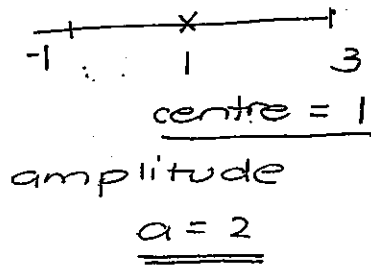
ii) at  $v = 0$

$$4x^2 - 8x - 12 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \quad x = -1$$



iii) when  $t = 0$   $x = 2$

$$x = 1 + 2\cos(\alpha)$$

$$\therefore 2 = 1 + 2\cos(\alpha)$$

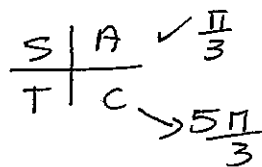
$$1 = 2\cos(\alpha)$$

$$\frac{1}{2} = \cos \alpha$$

$$\alpha = \frac{\pi}{3} \text{ or } \alpha = \frac{5\pi}{3}$$

check which one?

$$v = -4\sin(2t + \alpha)$$



when  $t = 0$   $v = 2\sqrt{3}$

$$2\sqrt{3} = -4\sin \alpha$$

$$\therefore \sin \alpha = -\frac{\sqrt{3}}{2}$$

$\alpha = \frac{5\pi}{3}$  to satisfy both  $x$  and  $v$



### QUESTION 7

a)  $\ddot{y} = -10$

$$\dot{y} = -10t + c$$

when  $t = 0$   $\dot{y} = v \sin \theta$

$$\therefore c = v \sin \theta$$

$$c = 35 \sin \theta$$

$$= 0$$

$$\dot{y} = -10t$$

$$y = -5t^2 + c_1$$

at  $t = 0$   $y = 1.8 \therefore c_1 = 1.8$

$$y = -5t^2 + 1.8$$

Horizontal

$$\dot{x} = v \cos \theta$$

$$= 35 \cos \theta$$

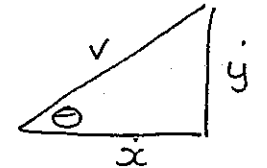
$$= 35$$

$$x = 35t$$

ii) when  $y = 0$

$$0 = -5t^2 + 1.8$$

$$t = 0.6 \text{ sec.}$$



$$v \sin \theta = \dot{y}$$

$$\cos \theta = \frac{\dot{x}}{v}$$

$$v \cos \theta = \dot{x}$$

iii) find  $x$  at  $t = 0.6$

$$x = 35 \times 0.6$$

$$= 21 \text{ metres.}$$

iv) Find  $y$  when

$$x = 14 \text{ find } t \text{ for}$$

$$14 = 35 \times t$$

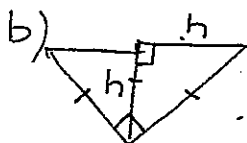
$$t = 0.4 \text{ sec.}$$

at  $t=0.4$

$y = -5(0.4)^2 + 1.8$   
 $= 1 \text{ metre.}$

$\therefore$  Ball clears net by

$1 - 0.95 = .05 \text{ metre}$   
 or 5 cm.



$\tan 45 = 1$

$\therefore$  base is also  $h$ .

Area =  $\frac{1}{2} \times 2h \times h$   
 $= h^2$

Volume =  $h^2 \times 300$   
 $= 300h^2 \text{ cm}^3/\text{sec}^3$

NOTE  
 UNITS  
 $h$  is cm  
 need 3m  
 as 300cm

ii)  $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$        $\frac{dV}{dt} = 2000 \text{ cm}^3$

$2000 = 600h \times \frac{dh}{dt}$

$\frac{2000}{600h} = \frac{dh}{dt}$

$\frac{20}{6 \times 20} = \frac{dh}{dt} \Rightarrow \frac{1}{6} \text{ cm/sec.}$

c)  $T = -19 + Ae^{-kt}$

i) at  $t=0$   $T=18$

$18 = -19 + Ae^0$

①

$37 = A$

ii)  $T = -19 + 37e^{-kt}$

$t=5$   $T=3$

$3 = -19 + 37e^{-5k}$

$e^{-5k} = \frac{22}{37}$

$k = -\frac{1}{5} \log_e \frac{22}{37}$

$\approx 0.103975091$

$0 = -19 + 37e^{-kt}$

$\frac{19}{37} = e^{-kt}$

$\log_e \left( \frac{19}{37} \right) = -0.103975091t$

$t \approx 6.4 \text{ mins or } 6 \text{ min } 25 \text{ sec}$