GOSFORD HIGH SCHOOL

2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

General Instructions:

- Reading time – 5minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Each question should be started on a new page.
- All necessary working should be shown in every question

Total marks: - 84

- Attempt Questions 1 -7
- All questions are of equal value

Question 1: (12 marks)

(a) Solve $\dfrac{2x}{x-3} \leq 1,\ x \neq 3$. (3)

(b) Find $\dfrac{d}{dx}(x\cos^{-1}x)$. (2)

(c) If α, β, γ are the roots of $2x^3 + 4x^2 - 6x + 3 = 0$ find:

(i) $\alpha + \beta + \gamma$. (1)

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$. (2)

(d) The interval joining the points A (2,1) and B (-3, -4) cuts the x-axis at C. Find the ratio in which the point C divides AB. (4)

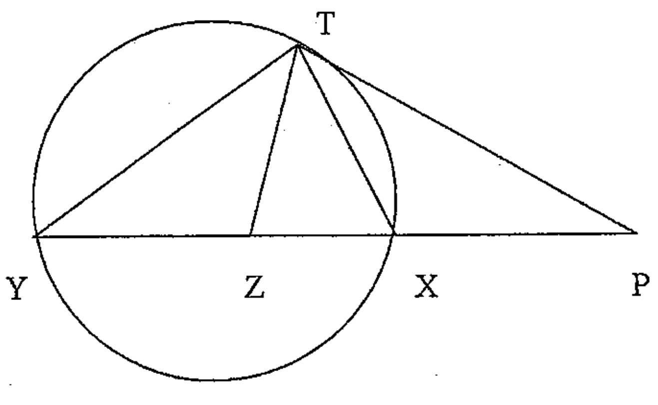
Question 2: (12 marks)

(a) Evaluate $\displaystyle\int_0^{\sqrt{2}} \frac{dx}{\sqrt{2-x^2}}$ (2)

(b) Use the substitution $u = 1 + x$ to find $\displaystyle\int_0^1 \frac{x}{\sqrt{1+x}}\,dx$ in simplest surd form. (4)

(c) The acute angle between the lines $2x - y + 1 = 0$ and $kx + y + 5 = 0$ is 45^0. Find the value(s) of k. (3)

(d) PT is a tangent to the circle and PXY is a secant. Z is a point on PXY such that TX = TZ. Prove that $\angle YTZ = \angle XPT$. (3)



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Question 3: (12 marks)

(a) Prove that $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$ (2)

(b) (i) Draw a neat sketch of $y=2\sin^{-1}3x$ and state its domain and range. (2)

(ii) Find the volume of the solid of revolution obtained by rotating the curve $y=2\sin^{-1}3x$ about the y-axis between $y=0$ and $y=\frac{\pi}{2}$. (4)

(c) Prove by the principle of Mathematical Induction that

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\ldots\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}. \quad (4)$$

Question 4: (12 marks)

(a) Show that $\frac{dv}{dt}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ (1)

(b) A particle moves in a straight line so that its acceleration 'a' in m/s^2 is given by $a=-e^{\frac{-x}{2}}$. When $t=0$, $x=0$ and $v=2m/s$.

(i) Show that the velocity of the particle is given by $v=2e^{\frac{-x}{4}}$. (3)

(ii) Find the displacement and velocity of the particle in terms of 't'. (3)

(c) The point $P(2p,p^2)$ lies on the parabola $x^2=4y$ whose focus is S.

(i) Find the equation of the tangent to the parabola at P. (2)

(ii) The tangent at P meets the x-axis at Q. The point R divides the interval joining SQ externally in the ratio 3: 2. Show that as P moves on the parabola $x^2=4y$ the locus of R is a straight line. (3)

Question 5: (12 marks)

(a) Evaluate $\lim\limits_{x\to0}\frac{\sin3x}{5x}$. (1)

(b) Find the general solution to $\cos2\theta=\cos\theta$. (3)

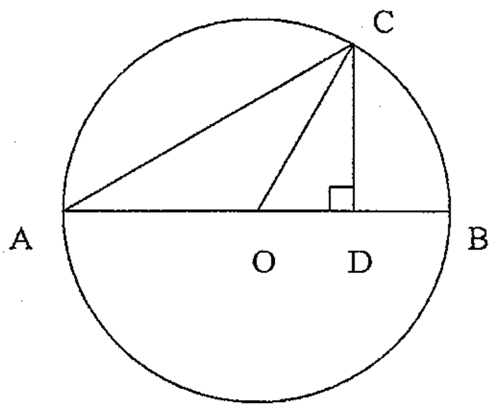
(c) For the function $f(x)=e^{x-1}$ find the inverse function $f^{-1}(x)$ and hence show that $f\left[f^{-1}(x)\right]=f^{-1}\left[f(x)\right]=x$. (3)

(d) (i) The equation $x^3-kx^2+2=0$ has exactly one root between $x=0$ and $x=1$. Prove that $k>3$. (2)

(ii) The equation $x^3 - 4x^2 + 2 = 0$ has a root between $x = 0$ and $x = 1$. Using a first approximation of this root as $x = 0.7$, use one application of Newton's Method to find a better approximation correct to 2 decimal places. (3)

Question 6: (12 marks)

(a) AB is a diameter of a circle centre O, radius 2 units. CD ⊥ AB.



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(i) If $\angle CAO = \theta$, explain why $\angle DOC = 2\theta$. (1)

(ii) Show that the perimeter 'P' of ΔDOC is given by:
$P = 2 + 2(\sin 2\theta + \cos 2\theta)$. (2)

(iii) Express $\sin 2\theta + \cos 2\theta$ in the form $R\sin(2\theta + \alpha)$ where α is acute. (2)

(iv) Find the maximum value of the perimeter 'P' and the value of θ for which the perimeter is a maximum. (2)

(b) A particle moving in Simple Harmonic Motion starts at the centre of oscillation O with an initial velocity of $3\,cm/s$. If it has a period of $\frac{\pi}{2}$ seconds, find:

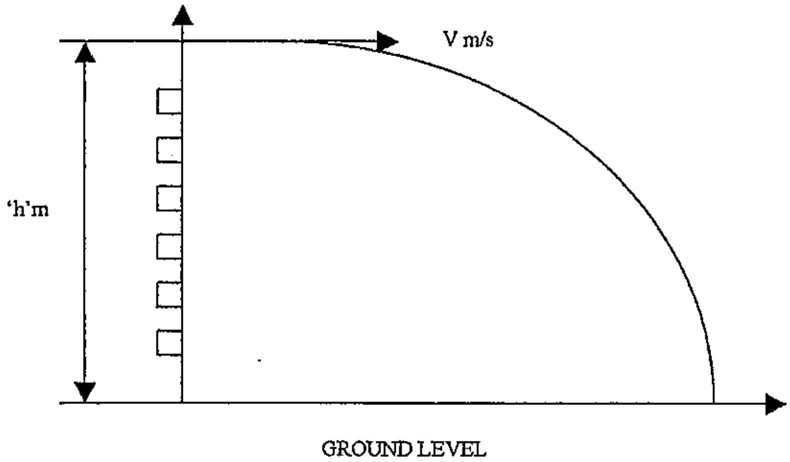
(i) the value of 'n'. (1)

(ii) the amplitude 'a' of the motion. (1)

(iii) the time taken for the particle to first reach a displacement of $x = -0.375\,cm$. (3)

Question 7: (12 marks)

(a) An object is projected horizontally from the top of a building which is 'h' metres above ground level with a velocity of $V\,ms^{-1}$. (Assume there is no air resistance and take the foot of the building to be the origin).



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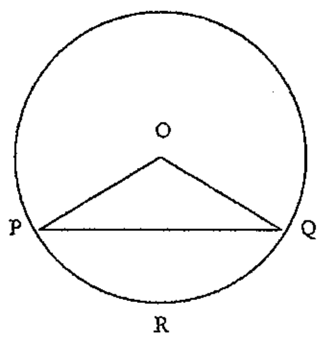
(i) Show that the object strikes the ground when $t=\sqrt{\dfrac{2h}{g}}$ at a horizontal displacement of $V\sqrt{\dfrac{2h}{g}}$ metres from the foot of the building (2)

(ii) Another object is projected horizontally from a height 'k' times the height of the first object for some constant 'k'. If it is to strike the ground at the same point as the first object calculate the speed of projection in terms of 'V' and 'k'. (2)

(iii) If in part (i) the building is 40 metres high and the speed of projection is $20\,ms^{-1}$ find the object's velocity when it strikes the ground and the angle at which it strikes the ground to the nearest degree. (Take $g=10\,ms^{-2}$) (2)

(b) The chord PQ cuts off a minor segment PRQ in a circle centre O radius 'r' units. The angle POQ is 'x' radians.



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It is given that 'r' and 'x' vary so that the area of the minor segment is constant at 50 cm^2.

(i) Show that $r = \dfrac{10}{\sqrt{x - \sin x}}$. (2)

(ii) If 'x' is increasing at a rate of 0.05 rads/sec, find the rate at which the radius is decreasing when x = 1.5 rads correct to 2 decimal places. (4)

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2007 Ext 1 Trial HSC Solutions

Question 1.

a) $\dfrac{2x}{x-3} \leq 1$

$x \neq 3$.

If $\dfrac{2x}{x-3} = 1$

$2x = x - 3$

$x = -3$ (3)

Since this is the sol^n to the eq^n it can't be a sol^n to the ineq^n.

$\times$ ● ✓ ○ $\times$ (number line: -3, 3)

By testing pts in the different regions on the no. line the sol^n is

$-3 \leq x < 3$.

b) Let $y = x \cos^{-1}x$ ($u = x$, $v = \cos^{-1}x$)

$\dfrac{dy}{dx} = vu' + uv'$ (2)

$= \cos^{-1}x \cdot 1 + x \cdot \dfrac{-1}{\sqrt{1-x^2}}$

$= \cos^{-1}x - \dfrac{x}{\sqrt{1-x^2}}$

c) $2x^3 + 4x^2 - 6x + 3 = 0$

(i) $\alpha + \beta + \gamma = \dfrac{-b}{a}$

$= \dfrac{-4}{2}$ (1)

$= -2$.

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

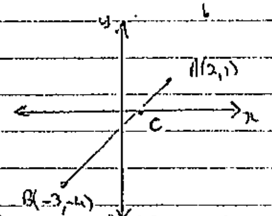
$= \dfrac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \dfrac{c/a}{-d/a}$

$= \dfrac{-6/2}{-3/2}$ (2)

$= +2$.

d)



Gradient of AB is 1.

$\therefore$ Eq^n of AB is

$y - 1 = 1(x-2)$

$y = x - 1$

If $y = 0$, $x = 1$

$\therefore$ C is the pt $(1, 0)$

Let the ratio be $k : 1$

Using $x = \dfrac{lx_1 + kx_2}{k + l}$ (4)

$1 = \dfrac{1 \times 2 + k \times -3}{k+1}$

$1 = \dfrac{2 - 3k}{k+1}$

$k + 1 = 2 - 3k$

$4k = 1$

$k = 1/4$

$\therefore k : 1 = \tfrac{1}{4} : 1$

$\therefore$ the ratio is $1 : 4$.

Question 2.

a) $\displaystyle\int_0^{\sqrt{2}} \frac{dx}{\sqrt{2 - x^2}}$

$= \left[\sin^{-1}\left(\dfrac{x}{\sqrt{2}}\right)\right]_0^{\sqrt{2}}$ (2)

$= \sin^{-1}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right) - \sin^{-1} 0$

$= \dfrac{\pi}{2} - 0$

$= \dfrac{\pi}{2}$.

b) If $u = 1 + x$ If $x = 0$, $u = 1$

$du = dx$ If $x = 1$, $u = 2$.

$\therefore \displaystyle\int_0^1 \frac{x}{\sqrt{1+x}}\,dx = \int_1^2 \frac{u-1}{\sqrt{u}}\,du$

$= \displaystyle\int_1^2 u^{1/2} - u^{-1/2}\,du$ (4)

$= \left[\dfrac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^2$

$= \left(\dfrac{2}{3} \cdot 2^{3/2} - 2 \cdot 2^{1/2}\right) - \left(\dfrac{2}{3} \cdot 1^{3/2} - 2 \cdot 1^{1/2}\right)$

$= \dfrac{2}{3} \cdot 2\sqrt{2} - 2\sqrt{2} - \dfrac{2}{3} + 2$

$= \dfrac{4}{3}\sqrt{2} - 2\sqrt{2} + \dfrac{4}{3}$

$= \dfrac{4}{3}\sqrt{2} - \dfrac{6}{3}\sqrt{2} + \dfrac{4}{3}$

$= \dfrac{4 - 2\sqrt{2}}{3}$

c) If $2x - y + 1 = 0$, $y = 2x + 1$

If $kx + y + 5 = 0$, $y = -kx - 5$

$\therefore M_1 = 2$ & $M_2 = -k$.

$\tan\theta = \left|\dfrac{M_1 - M_2}{1 + M_1 M_2}\right|$ (3)

$\therefore 1 = \left|\dfrac{2 + k}{1 - 2k}\right|$

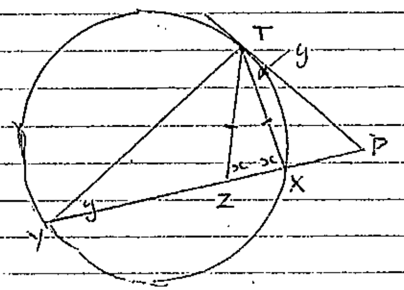
$\dfrac{2+k}{1-2k} = 1$ or $\dfrac{2+k}{1-2k} = -1$

$2 + k = 1 - 2k$ $2 + k = -1 + 2k$

$3k = -1$ $3 = k$

$k = -\dfrac{1}{3}$ or $k = 3$

d)



Let $\angle TZX$ & $\angle TXZ$ be $x°$ (base $\angle$s of isosc $\triangle$ TXZ), & let $\angle TYZ$ be $y°$. (3)

$\therefore \angle XTP = y°$ ($\angle$ in the alternate segment theorem)

$\angle YTZ = x° - y°$ (ext. $\angle$ of a $\triangle$ theorem)

& $\angle XPT = x° - y°$ (ext $\angle$ of a $\triangle$ theorem)

$\therefore \angle YTZ = \angle XPT$.

Question 3.

(a) $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$

LHS $= \frac{2\sin\theta\cos\theta}{1+2\cos^2\theta - 1}$

$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$ (2)

$= \frac{\sin\theta}{\cos\theta}$

$=$ RHS

(b) (i) If $y = 2\sin^{-1}3x$

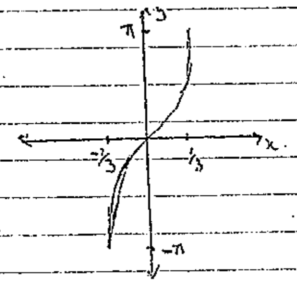
$\frac{y}{2} = \sin^{-1}3x$

$\therefore D: -1 \le 3x \le 1$

$-\frac{1}{3} \le x \le \frac{1}{3}$

$R: -\frac{\pi}{2} \le \frac{y}{2} \le \frac{\pi}{2}$

$-\pi \le y \le \pi$ (2)



(ii) $V = \pi\int_0^b x^2\,dy$

If $\frac{y}{2} = \sin^{-1}3x$

$3x = \sin\frac{y}{2}$

$x = \frac{1}{3}\sin\frac{y}{2}$

$\therefore x^2 = \frac{1}{9}\sin^2\frac{y}{2}$

$V = \pi\int_0^{\pi/2} \frac{1}{9}\sin^2\frac{y}{2}\,dy$ (4)

$= \frac{\pi}{9}\int_0^{\pi/2} \frac{1}{2} - \frac{1}{2}\cos y\,dy$

$= \frac{\pi}{9}\left[\frac{y}{2} - \frac{1}{2}\sin y\right]_0^{\pi/2}$

$= \frac{\pi}{9}\left\{\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right) - \left(0 - \frac{1}{2}\sin 0\right)\right\}$

$= \frac{\pi}{9}\left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi^2}{36} - \frac{\pi}{18}$ units^3.

c) Prove true for $n=1$

LHS $= \frac{1}{1\times 3} = \frac{1}{3}$

RHS $= \frac{1}{2\times 1+1} = \frac{1}{3}$

$\therefore$ True for $n=1$.

Assume $S(k)$ true

ie. $\frac{1}{1\times 3} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$.

Prove $S(k+1)$ true

ie $\frac{1}{1\times 3} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}$

LHS $= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$

$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$

$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$

$=$ RHS.

P.T.O

Therefore if the statement is true for $n=k$ it is true for $n=k+1$. Since it is true for $n=1$ it is true for $n=2$. Since true for $n=2$ it is true for $n=3$ and so on.

$\therefore$ the statement is true for all positive integers n.

Question 4.

a) $\frac{dv}{dt} = \frac{dv}{dx}\times\frac{dx}{dt}$

$= \frac{dv}{dx}\times v$ (1)

$= \frac{dv}{dx}\times\frac{d}{dv}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$.

b) (i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -e^{-x/2}$

$\frac{1}{2}v^2 = \int -e^{-x/2}\,dx$

$= 2e^{-x/2} + c$

When $x=0$, $v=2$.

$\therefore \frac{(2^2)}{2} = 2e^0 + c$

$2 = 2 + c$

$c = 0$

$\therefore \frac{1}{2}v^2 = 2e^{-x/2}$ (3)

$v^2 = 2(2e^{-x/2})$

$v^2 = 4e^{-x/2}$

$v = \pm 2e^{-x/4}$

$v = 2e^{-x/4}$ since $v=2$ when $x=0$.

(ii) $\frac{dx}{dt} = 2e^{-x/4}$

$\frac{dx}{dt} = \frac{2}{e^{x/4}}$

$\therefore \frac{dt}{dx} = \frac{e^{x/4}}{2}$

$t = \int \frac{1}{2}e^{x/4}\,dx$

$= \int 2\times\frac{1}{4}e^{x/4}\,dx$

$= 2e^{x/4} + k$.

If $t=0$, $x=0$

$\therefore 0 = 2e^0 + k$

$k = -2$.

$t = 2e^{x/4} - 2$

$t+2 = 2e^{x/4}$ (3)

$\frac{t+2}{2} = e^{x/4}$

$\ln\left(\frac{t+2}{2}\right) = \frac{x}{4}$

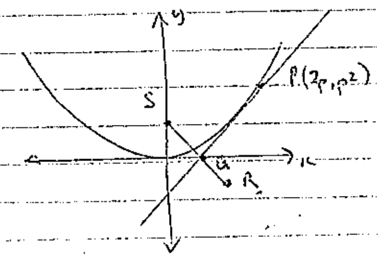
$x = 4\ln\left(\frac{t+2}{2}\right)$

$x = 4\left[\ln(t+2) - \ln 2\right]$

$\frac{dx}{dt} = 4\left[\frac{1}{t+2} - 0\right]$

$v = \frac{4}{t+2}$.

c)



(i) If $x^2 = 4y$

$y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{2x}{4}$

When $x = 2p$

$m = \frac{4p}{4}$

$= p$.

∴ Eqn of tangent is (2)

$y - p^2 = p(x - 2p)$

$y - p^2 = px - 2p^2$

$y = px - p^2$.

(ii) When $y = 0$,

$px - p^2 = 0$

$p(x - p) = 0$

$x = p$.

So Q is the pt $(p, 0)$

S: If $x^2 = 4ay$

& $x^2 = 4y$

$a = 1$

∴ S is the pt $(0,1)$ (3)

$S(0,1)$ & $Q(p,0)$

$3 : -2$

$x = \frac{-2\times0 + 3\times p}{1}$ $y = \frac{-2\times1 + 3\times0}{1}$

$= 3p$ $= -2$

∴ R is the pt $(3p, -2)$

∴ the locus of R is the straight line

$y = -2$.

Question 5.

a) $\lim_{x\to0} \frac{\sin 3x}{5x}$

$= \lim_{x\to0} \frac{\sin 3x}{3x} \times \frac{3}{5}$

$= 1 \times \frac{3}{5}$ (1)

$= \frac{3}{5}$

b) $\cos 2\theta = \cos\theta$

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$ (3)

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = 1$.

$\theta = 2n\pi \pm \cos^{-1}(-\frac{1}{2})$ or $2n\pi \pm \cos^{-1}(1)$

$\theta = 2n\pi \pm \frac{2\pi}{3}$ or $2n\pi$.

c) $f(x) = e^{x-1}$

Let $y = e^{x-1}$

∴ $x = e^{y-1}$

$\ln x = y - 1$

$y = \ln x + 1$. (3)

∴ $f^{-1}(x) = \ln x + 1$.

$f[f^{-1}(x)] = e^{\ln x + 1 - 1}$

$= e^{\ln x}$

$= x$

$f^{-1}[f(x)] = \ln(e^{x-1}) + 1$

$= x - 1 + 1$

$= x$

∴ $f[f^{-1}(x)] = f^{-1}[f(x)] = x$

d) (i) $x^3 - kx^2 + 2 = 0$

If there is a root between 0 & 1

$f(0)$ & $f(1)$ must be opposite in sign

$f(0) = 2$. which is positive

$f(1) = 1 - k + 2$

$= 3 - k$. (2)

∴ $3 - k < 0$

$k > 3$.

(ii) $x^3 - 4x^2 + 2 = 0$

Let $f(x) = x^3 - 4x^2 + 2$

$f'(x) = 3x^2 - 8x$

$a_2 = a_1 - \frac{f(a_1)}{f'(a_1)}$

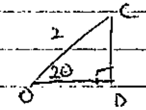
$= 0.7 - \frac{f(0.7)}{f'(0.7)}$ (3)

$= 0.7 - \frac{0.383}{-4.13}$

$= 0.792736077$

$\doteqdot 0.79$ (2 d.p.)

(ii)



$OC = 2$ units

$\frac{OD}{OC} = \cos 2\theta$ $\frac{CD}{OC} = \sin 2\theta$

$OD = 2\cos 2\theta$ $CD = 2\sin 2\theta$

∴ $P = 2 + 2\sin 2\theta + 2\cos 2\theta$ (2)

$= 2 + 2(\sin 2\theta + \cos 2\theta)$.

(iii) $\sin 2\theta + \cos 2\theta = a\sin 2\theta + b\cos 2\theta$

where $a = b = 1$.

$R = \sqrt{a^2 + b^2}$ $\tan\alpha = \frac{b}{a}$

$= \sqrt{2}$ $= 1$

∴ $\alpha = \pi/4$ (2)

∴ $\sin 2\theta + \cos 2\theta = \sqrt{2}\left(\sin 2\theta + \frac{\pi}{4}\right)$

(iv) $P = 2 + 2(\sin 2\theta + \cos 2\theta)$

$= 2 + 2\sqrt{2}\left(\sin 2\theta + \frac{\pi}{4}\right)$

Max value of $\sin\left(2\theta + \frac{\pi}{4}\right)$ is 1

∴ Max $P = 2 + 2\sqrt{2} \times 1$

$= 2 + 2\sqrt{2}$ units

If $\sin\left(2\theta + \frac{\pi}{4}\right) = 1$

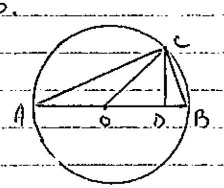
$2\theta + \frac{\pi}{4} = \frac{\pi}{2}$

$2\theta = \frac{\pi}{4}$ (2)

$\theta = \frac{\pi}{8}$

Question 6.

a)



(i) If $\angle CAO = \theta$ (1)

$\angle DOC = 2\theta$

(angle at the centre of a circle is twice the angle at the circumf).

b) (i) Since the motion is SHM

$T = \frac{2\pi}{n}$

$\therefore \frac{2\pi}{n} = \frac{\pi}{2}$ (1)

$n = 4$

(ii) Since the motion is SHM

$v^2 = n^2(a^2 - x^2)$

If $x=0$, $v=3$, $n=4$

$\therefore 9 = 16(a^2 - 0)$

$a^2 = \frac{9}{16}$

$a = \pm\frac{3}{4}$ (1)

$\therefore$ Amplitude is $\frac{3}{4}$ cm.

(iii) Let $x = a\sin(nt+\alpha)$ describe the motion.

If $t=0$, $x=0$,

$\therefore 0 = a\sin(0+\alpha)$

$\alpha = 0$.

So $x = \frac{3}{4}\sin 4t$.

When $x = -0.375$

$-0.375 = \frac{3}{4}\sin 4t$

$-1.5 = 3\sin 4t$

$\sin 4t = -\frac{1}{2}$.

$\therefore 4t = \frac{7\pi}{6}, \frac{11\pi}{6}, \ldots$

$t = \frac{7\pi}{24}, \frac{11\pi}{24}, \ldots$

$\therefore$ The particle first reaches -0.375cm after $\frac{7\pi}{24}$ seconds.

Question 7.

a) (i) $\ddot{x} = 0$ $\quad \ddot{y} = -g$

$\dot{x} = V$ $\quad \dot{y} = -gt$

$x = Vt$ $\quad y = -\frac{1}{2}gt^2 + h$

If $y=0$, $0 = -\frac{1}{2}gt^2 + h$

$\frac{1}{2}gt^2 = h$

$t^2 = \frac{2h}{g}$

$t = \pm\sqrt{\frac{2h}{g}}$ (2)

$\therefore t = \sqrt{\frac{2h}{g}}$ as $-\sqrt{\frac{2h}{g}}$ is meaningless.

When $t = \sqrt{\frac{2h}{g}}$

$x = Vt$

$= V\sqrt{\frac{2h}{g}}$.

(ii) If object is ~~fired~~ projected from a height of kh

$\therefore y = -\frac{1}{2}gt^2 + kh$

When $y=0$, $0 = -\frac{1}{2}gt^2 + kh$

$\frac{1}{2}gt^2 = kh$

$gt^2 = 2kh$

$t^2 = \frac{2kh}{g}$

$t = \pm\sqrt{\frac{2kh}{g}}$

$\therefore t = \sqrt{\frac{2kh}{g}}$ as $-\sqrt{\frac{2kh}{g}}$ is meaningless.

ie $t = \sqrt{k}\cdot\sqrt{\frac{2h}{g}}$

Let the speed be V_1

$\therefore X = V_1\sqrt{k}\sqrt{\frac{2h}{g}}$

When $X = x$ (2)

$V_1\sqrt{k}\sqrt{\frac{2h}{g}} = V\sqrt{\frac{2h}{g}}$

$V_1 = \frac{V}{\sqrt{k}}$.

(iii) If $V=20$, $h=40$, $g=10$

$x = 20t$, $y = -5t^2 + 40$

When $y=0$, $-5t^2 + 40 = 0$

$5t^2 = 40$

$t^2 = 8$

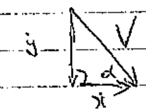
$t = \pm 2\sqrt{2}$

$\therefore t = 2\sqrt{2}$ as $-2\sqrt{2}$ is meaningless

$\dot{x} = 20$, $\dot{y} = -10t$

$= -10\cdot 2\sqrt{2}$

$= -20\sqrt{2}$



(2)

$V^2 = (-20\sqrt{2})^2 + 20^2$

$= 800 + 400$

$= 1200$

$V = \pm 20\sqrt{3}$

$|\tan\alpha| = \left|\frac{20\sqrt{2}}{20}\right|$

$\tan\alpha = \sqrt{2}$

$\alpha = 55^\circ$ (nearest deg.)

$\therefore$ The object strikes the ground at $20\sqrt{3}$ ms^{-1} at an angle of 55°.

b) (i) Area of segment $= \frac{1}{2}r^2x - \frac{1}{2}r^2\sin x$

$A = \frac{1}{2}r^2(x - \sin x)$

If $A = 50$

$50 = \frac{1}{2}r^2(x - \sin x)$

$100 = r^2(x - \sin x)$

$\frac{100}{x - \sin x} = r^2$

$r = \pm\frac{10}{\sqrt{x - \sin x}}$ (2)

So $r = \frac{10}{\sqrt{x - \sin x}}$ as the −ve soln is meaningless

ie $r = 10(x - \sin x)^{-1/2}$

(ii) $\frac{dr}{dt} = \frac{dr}{dx} \times \frac{dx}{dt}$ (1)

$\frac{dr}{dx} = 10 \times -\frac{1}{2}(x - \sin x)^{-3/2} \times (1 - \cos x)$

$= \frac{-5(1 - \cos x)}{(x - \sin x)^{3/2}}$

When $x = 1.5$

$\frac{dr}{dx} = \frac{-5(1 - \cos 1.5)}{(1.5 - \sin 1.5)^{3/2}}$

$\doteq -13.044$ (3 dp) (2)

$\therefore \frac{dr}{dt} \doteq -13.044 \times 0.05$

$= -0.65$ (2dp) (1)

$\therefore$ The radius is decreasing at approx 0.65 cm/s.