GOSFORD HIGH SCHOOL

2009

Trial HSC

MATHEMATICS EXTENSION I

Time Allowed: 2 Hours + 5 minutes reading time

General Instructions:

- Reading Time – 5 minutes.
- Working time – 2 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.
- Each question should be started in a separate writing booklet.

TOTAL MARKS – 84

- Attempt Questions 1 – 7
- All questions are of equal value.

Mathematics Extension 1 Trial Higher School Certificate - 2009

Question 1 (12 Marks) **Marks**

a) Solve $\dfrac{2x-3}{x-2} \geq 1$ **2**

b) The point P divides the segment AB externally in the ratio 3:2. **2**
If A is the point (1, 4), and B is the point (–1, 8), find the coordinates of P.

c) State the domain and range of $y = \cos^{-1}\dfrac{x}{3}$ **2**

d) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \cos^3\theta \sin\theta \, d\theta$ **2**

e) The polynomial $x^4 + 2x$ is divided by $x + 2$. Calculate the remainder. **2**

f) The acute angle between $y = 3x + 5$ and $y = mx + 4$ is 45°. **2**

Find 2 possible values of m.

Question 2 (12 Marks) Begin a new booklet. **Marks**

a) Differentiate $x \tan^{-1} 3x$ with respect to x. **2**

b) Find $\int \sin^2 3x \; dx$ **3**

c) Evaluate $\int_0^3 \frac{x}{\sqrt{x+1}} dx$ using the substitution $x = u^2 - 1$ **3**

d) The polynomial $P(x) = x^5 + ax^3 + bx$ (where a and b are numerical constants), leaves a remainder of 5 when divided by $x - 2$.

(i) Show that $P(x)$ is odd. **1**

(ii) Find the remainder when $P(x)$ is divided by $x + 2$. **1**

e) The equation $x^3 + 2x^2 + 3x + 6 = 0$ has roots α, β and γ. **2**

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

Question 3 (12 Marks) Begin a new booklet. **Marks**

a) (i) Find the derivative of $\sin^{-1}x + \cos^{-1}x$ **1**

(ii) Hence, find the value of $\sin^{-1}x + \cos^{-1}x$ for all x. **1**

b) Taking $x = 0.5$ as a first approximation to the root of $x + \ln x = 0$, use Newton's method to find a second approximation. **2**

c) (i) Sketch the graph of $y = |1 - 2x|$ **1**

(ii) Hence, or otherwise, solve $|1 - 2x| \leq x$ **3**

d) Prove by mathematical induction that **4**

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \ldots n \times 2^{n-1} = 1 + (n-1)2^n$$

for all integers $n \geq 1$

Question 4 (12 Marks) Begin a new booklet. **Marks**

a) (i) Express $\sqrt{3}\cos 2t - \sin 2t$ in the form $A\cos(2t+\alpha)$, **2**

with $A > 0$ and α acute.

(ii) Find, in exact form, the general solutions to **2**

$$\sqrt{3}\cos 2t - \sin 2t = 1$$

b) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ where $a > 0$. The chord PQ passes through the focus.

(i) Show that $pq = -1$. **1**

(ii) Show that the point of intersection T of the tangents to the parabola at P and Q lies on the line $y = -a$. **2**

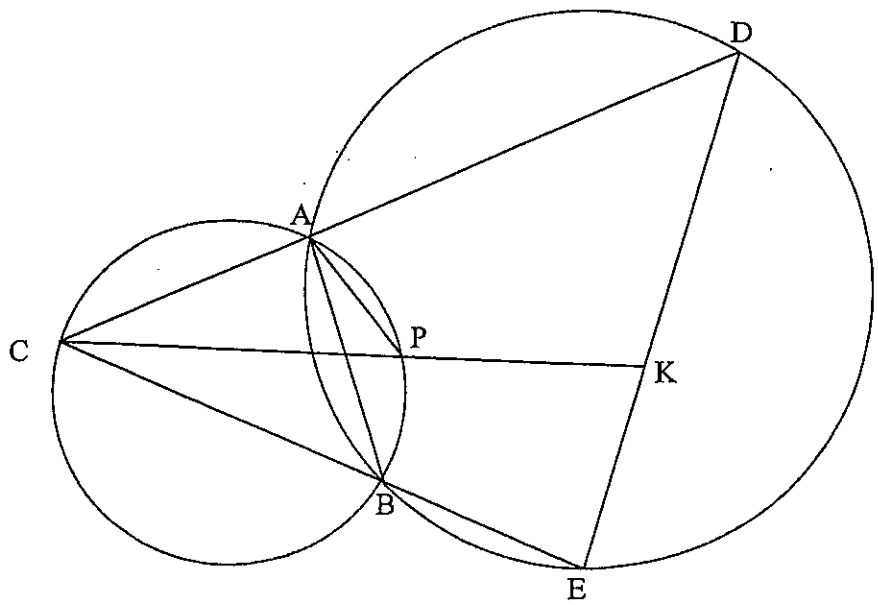
(iii) Show that the chord PQ has length $a(p + \frac{1}{p})^2$. **2**

(Hint: Use the focus–directrix definition of the parabola)

c) In the diagram, two circles intersect at A and B. CAD, CBE, CPK and DKE are straight lines.

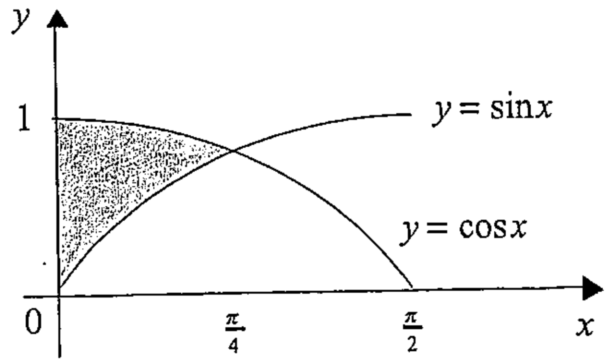
i) Show why $\angle APC = \angle ABC$ **1**

ii) Hence, or otherwise, show that ADKP is a cyclic quadrilateral. **2**



Question 5 (12 Marks) Begin a new booklet. **Marks**

a)



The region bounded by the curves $y = \cos x$ and $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{4}$ is rotated through one complete revolution about the x-axis. Find the volume of the solid. **2**

b) A particle, whose displacement is x, moves in simple harmonic motion such that

$$\frac{d^2x}{dt^2} = -16x. \qquad \text{At time } t = 0,\ x = 1 \text{ and } \frac{dx}{dt} = 4.$$

(i) Show that, for all positions of the particle, **2**

$$\left|\frac{dx}{dt}\right| = 4\sqrt{2 - x^2}$$

(ii) What is the particle's greatest displacement? **1**

(iii) Find x as a function of t. You may assume the general form for x. **2**

c) A rectangular vessel is divided into two equal compartments by a vertical porous membrane. Liquid in one compartment, initially at a depth of 20 cm, passes into the other compartment, initially empty, at a rate proportional to the difference in levels.

(i) If the depth of liquid in one of the vessels at any time t minutes is x cm, show that

$$\frac{dx}{dt} = k(20 - 2x)$$ **1**

(ii) Show that $x = 10(1 - e^{-2kt})$ is a solution to this equation **1**

(iii) If the level in the second compartment rises 2 cm in the first 5 minutes, after what time will the difference in levels be 2 cm? **3**

Question 6 (12 Marks) Begin a new booklet. **Marks**

a) A spherical bubble is expanding so that its volume is increasing at the constant rate of 10 mm^3 per second. What is the rate of increase of the radius when the surface area is 500 mm^2? **2**

b) A particle moves in a straight line. Initially it is 2 m to the right of a fixed point O, and velocity is v m/s where

$$v = \frac{32}{x} - \frac{x}{2}$$

(i) Prove that $\frac{d^2x}{dt^2} = \frac{d}{dx}(\frac{1}{2}v^2)$ **2**

(ii) Find an expression for acceleration in terms of x. **2**

(iii) Show that $t = \int \frac{2x}{64 - x^2} dx$ **3**

and hence show $x^2 = 64 - 60e^{-t}$

(iv) Sketch the graph of x^2 against t and describe the limiting behaviour of the particle. **3**

Question 7 (12 Marks) Begin a new booklet. **Marks**

a) Sketch the graph of $y = \sec x$ for $-\pi \le x \le 2\pi$ **2**

b) The inverse secant $y = \sec^{-1} x$ could be defined as the function

$$x = \sec y \text{ with } 0 \le y \le \pi \quad \text{and } y \ne \frac{\pi}{2}$$

i) Find $\sec^{-1}\sqrt{2}$ **1**

(ii) Sketch the graph of $y = \sec^{-1} x$ **2**

(iii) For $y = \sec^{-1} x$ find $\frac{dx}{dy}$ and hence show that $\frac{dy}{dx} = \left|\frac{1}{x\sqrt{x^2-1}}\right|$ **3**

Why is the absolute value sign appropriate? **1**

(iv) Hence, differentiate $y = \sec^{-1}\frac{x}{a}$ and simplify your answer. **1**

(v) Hence, or otherwise, find $\int \frac{dx}{x\sqrt{25x^2-9}}$ for positive x. **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2009 Ext 1 Trial

Question 1

a) $\dfrac{2x-3}{x-2} \geq 1$

For $x > 2$ Solve
$2x - 3 \geq x - 2$
$x \geq 1$
$\therefore$ All $x > 2$

For $x < 2$ Solve
$2x - 3 \leq x - 2$
$x \leq 1$
$\therefore x \leq 1$

$\therefore$ $\underline{\underline{x \leq 1 \text{ or } x > 2}}$

b) $A(\overset{x_1,\ y_1}{1, 4})$ $\quad B(\overset{x_2\ y_2}{-1, 8})$ $\quad \dfrac{k}{l} = \dfrac{3}{-2}$

$x = \dfrac{kx_2 + lx_1}{k+l}$ $\qquad y = \dfrac{ky_2 + ly_1}{k+l}$

$x = \dfrac{3 \times -1 + -2 \times 4}{1}$ $\qquad y = \dfrac{3 \times 8 + -2 \times 4}{1}$

P is $(-5, 16)$

c) $-1 \leq \dfrac{x}{3} \leq 1$

$\therefore D: \quad -3 \leq x \leq 3$
$R: \quad 0 \leq y \leq \pi$

d) $\left[-\dfrac{\cos^4\theta}{4}\right]_0^{\frac{\pi}{4}}$

$= -\frac{1}{4}\left(\left(\frac{1}{\sqrt{2}}\right)^4 - 1\right)$

$= -\frac{1}{4}\left(\frac{1}{4} - 1\right)$

$= \dfrac{3}{16}$

e) $P(x) = x^4 + 2x$
$P(-2) = (-2)^4 + 2 \times -2$
$= 16 - 4$
$= 12$

f) $m_1 = 3, \quad m_2 = m$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$1 = \left|\dfrac{3 - m}{1 + 3m}\right|$

$|1 + 3m| = |3 - m|$

$\therefore 1 + 3m = 3 - m$ or $1 + 3m = m - 3$
$4m = 2 \qquad 2m = -4$
$m = \frac{1}{2}$ or $m = -2$.

Question 2

a) $y' = x \cdot \frac{3}{1+9x^2} + \tan^{-1} 3x$

$y' = \frac{3x}{1+9x^2} + \tan^{-1} 3x$

b)

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\therefore \int \sin^2 3x \, dx = \frac{1}{2}\int 1 - \cos 6x \, dx$

$= \frac{1}{2}\left(x - \frac{\sin 6x}{6}\right)$

$= \frac{x}{2} - \frac{\sin 6x}{12} + C$

c) $\int_0^3 \frac{x}{\sqrt{x+1}} \, dx$

$x = u^2 - 1$ or $u = \sqrt{x+1}$

$dx = 2u\,du$

$x = 0, \; u = \sqrt{1} = 1$

$x = 3, \; u = \sqrt{4} = 2$

(positive $\sqrt{\ }$ as $\sqrt{x+1}$ is +ve)

$= \int_1^2 \frac{(u^2-1) \cdot 2u\,du}{u}$

$= 2\int_1^2 u^2 - 1 \, du$

$= 2\left[\frac{u^3}{3} - u\right]_1^2$

$= 2\left[\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right)\right]$

$= 2\left(\frac{8}{3} - 2 - \frac{1}{3} + 1\right)$

$= \frac{8}{3}$

Q2 (cont)

d) $P(x) = x^5 + ax^3 + bx$

$P(-x) = (-x)^5 + a(-x)^3 + b(-x)$

$= -x^5 - ax^3 - bx$

$= -P(x)$

$\therefore P(x)$ is odd.

ii) $P(-2) = -P(2)$

$= -5$

e) $x^3 + 2x^2 + 3x + 6 = 0$

$\alpha + \beta + \gamma = -2$

$\alpha\beta + \beta\gamma + \gamma\alpha = 3$

$\alpha\beta\gamma = -6$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{3}{-6}$$

$$= -\frac{1}{2}$$

3 a) i) $y' = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}}$

$= 0$

ii) $\therefore \sin^{-1}x + \cos^{-1}x = \text{constant}$

Sub $x = 0$

$\sin^{-1}0 + \cos^{-1}0 = c$

$0 + \frac{\pi}{2} = c$

$\therefore \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

b) $f(x) = x + \ln x$ $\qquad f(0.5) \doteq -0.193$

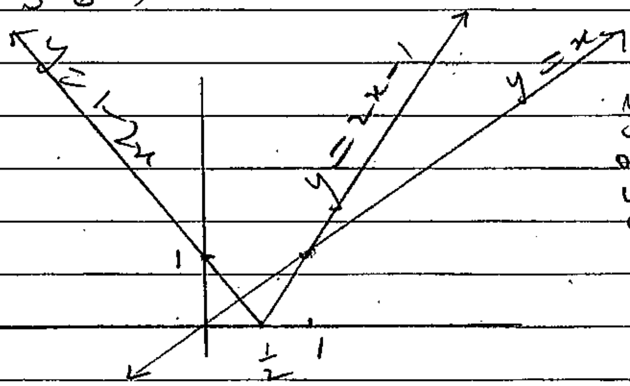
$f'(x) = 1 + \frac{1}{x}$ $\qquad f'(0.5) = 3$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 0.5 - \frac{-0.193}{3}$

$x_2 = 0.564$

c) i)



$y = |1-2x|$ or $y = |2x-1|$

Solve $2x - 1 = x$ $\qquad 1 - 2x = x$

$x = 1$ $\qquad x = \frac{1}{3}$

From graph $\frac{1}{3} \le x \le 1$

d) Prove true for $n=1$: $LHS = 1 \times 2^0 = 1$ $\qquad RHS = 1 + 0 = 1$

Assume true for $n = k$:

$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + k \times 2^{k-1} = 1 + (k-1)2^k$ — $*$

Add $(k+1)^{th}$ term to b/s of $*$

$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + k \times 2^{k-1} + (k+1)2^k = 1 + (k-1)2^k + (k+1)2^k$

$= 1 + 2^k \{k - 1 + k + 1\}$

$= 1 + 2^k \cdot 2k$

$= 1 + k\,2^{k+1}$

which is of the form $1 + (n-1)2^n$ when n is replaced by $k+1$.

$\therefore$ True for all $n \ge 1$.

4 a) i) $\sqrt{3}\cos 2t - \sin 2t \equiv A\cos(2t+\alpha)$
$= A\cos 2t\cos\alpha - A\sin 2t\sin\alpha$

$\therefore \left.\begin{array}{l} A\cos\alpha = \sqrt{3} \\ A\sin\alpha = 1 \end{array}\right\} \Longrightarrow \tan\alpha = \frac{1}{\sqrt{3}}$
$\alpha = \frac{\pi}{6}$

$A^2 = (\sqrt{3})^2 + 1^2 \Longrightarrow A = 2.$

$\therefore \sqrt{3}\cos 2t - \sin 2t \equiv 2\cos\left(2t + \frac{\pi}{6}\right)$

ii) $2\cos(2t + \frac{\pi}{6}) = 1$
$\cos(2t + \frac{\pi}{6}) = \frac{1}{2}$

$\therefore 2t + \frac{\pi}{6} = \pm\frac{\pi}{3} + 2n\pi$

$2t = \frac{\pi}{6} + 2n\pi$ or $-\frac{\pi}{2} + 2n\pi$

$t = \frac{\pi}{12} + n\pi$ or $n\pi - \frac{\pi}{4}$

b) i) Chord PQ: $y = \frac{p+q}{2}x - apq$
Sub $(0, a)$
$a = 0 - apq$
$pq = -1$

ii) Tangent at P: $y = px - ap^2$ —— ①
" " Q: $y = qx - aq^2$ —— ②

① − ② $0 = (p-q)x - a(p^2 - q^2)$
$a(p-q)(p+q) = (p-q)x$
$p \neq q$ $\therefore x = a(p+q)$
Sub into ①
$y = ap(p+q) - ap^2$
$y = apq$
But $pq = -1$ $\therefore y = -a$ as required.

iii) PQ is focal chord $\therefore PQ = PS + SQ$
But PS = dist of P to directrix $= ap^2 + a$
SQ = dist of Q to directrix $= aq^2 + a$
$\therefore PQ = ap^2 + a + aq^2 + a$
$= a(p^2 + 2 + q^2)$
But $q = -\frac{1}{p}$ $\therefore PQ = a\left(p^2 + 2 + \frac{1}{p^2}\right)$
$= a\left(p + \frac{1}{p}\right)^2$

4c) i) $\angle APC = \angle ABC$ Angles in same segment standing on minor arc AC.

ii) $\angle ABC = \angle ADE$ exterior angle of cyclic quadrilateral ADEB

$\therefore \angle APC = \angle ADE$ (Both equal to $\angle ABC$)

But $\angle APC$ is exterior angle of quad ADKP & equals interior angle ADK.

$\therefore$ ADKP is a cyclic quadrilateral.

Q5 a) $V = \pi \int_0^{\frac{\pi}{4}} \cos^2 x \, dx - \pi \int_0^{\frac{\pi}{4}} \sin^2 x \, dx$

$V = \pi \int_0^{\pi/4} \cos^2 x - \sin^2 x \, dx$

$V = \pi \int_0^{\pi/4} \cos 2x \, dx$

$= \pi \left[\frac{\sin 2x}{2} \right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{2} \left(\sin \frac{\pi}{2} - \sin 0 \right)$

$V = \frac{\pi}{2} \; u^3$

b) $\ddot{x} = -16x$

i) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2} v^2\right) = -16x$

$\frac{1}{2} v^2 = -\frac{16x^2}{2} + c$

$v^2 = -16x^2 + K$

$x = 1, \; v = 4 \therefore 16 = -16 + K \implies K = 32$

$v^2 = 32 - 16x^2$

$= 16(2 - x^2)$

$v = \pm 4\sqrt{2 - x^2}$

or $|v| = 4\sqrt{2 - x^2}$

ii) Max value of x for this to exist is $x = \sqrt{2}$

5b) iii) $\ddot{x} = -n^2 x$ with $n = 4$

$\therefore$ $x = a\cos(nt + \alpha)$ is a general solution

$\therefore$ $x = a\cos(4t + \alpha)$ But $a = \sqrt{2}$

$x = \sqrt{2}\cos(4t + \alpha)$

$x = 1$ when $t = 0$

$1 = \sqrt{2}\cos\alpha$

$\frac{1}{\sqrt{2}} = \cos\alpha \implies \alpha = \pm\frac{\pi}{4}$

$x = \sqrt{2}\cos\left(4t \pm \frac{\pi}{4}\right)$

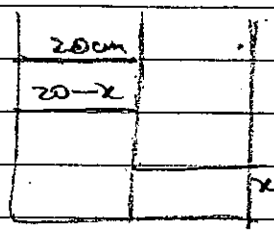
Now $\dot{x} = -4\sqrt{2}\sin\left(4t \pm \frac{\pi}{4}\right)$

$t = 0$, $v = 4$ $\therefore$ $4 = -4\sqrt{2}\sin\left(\pm\frac{\pi}{4}\right)$

$\sin \mp\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

$\therefore$ Negative sign is required.

$\therefore$ $x = \sqrt{2}\cos\left(4t - \frac{\pi}{4}\right)$

c) 

When x in in one compartment $20 - x$ in other. Difference $= (20 - x) - x$

$= 20 - 2x$

$\therefore \frac{dx}{dt} = k(20 - 2x)$

ii) $x = 10(1 - e^{-2kt}) = 10 - 10e^{-2kt}$

Differentiating w.r.t t

$\frac{dx}{dt} = 10\left(0 - e^{-2kt} \cdot -2k\right)$

$= 10\left(2ke^{-2kt}\right) = 20ke^{-2kt}$

But $x = 10 - 10e^{-2kt}$ or $10e^{-2kt} = 10 - x$

$\therefore \frac{dx}{dt} = 2k(10 - x)$

$= k(20 - 2x)$ as required

iii) $t = 5$, $x = 2$ $\therefore$ $2 = 10 - 10e^{-10k}$

$10e^{-10k} = 8$

$e^{-10k} = 0.8$

$-10k = \ln 0.8 \implies k = \frac{\ln .8}{-10} \doteqdot 0.0223$

Diff in levels $= 2cm$ $\therefore 20 - 2x = 2$

$18 = 2x \quad x = 9$

$\therefore$ Sub $x = 9$ in $x = 10(1 - e^{-2kt})$

$$9 = 10(1 - e^{-2kt})$$

$$0.9 = 1 - e^{-2kt}$$

$$e^{-2kt} = 0.1$$

$$-2kt = \ln 0.1$$

$$t = \frac{\ln 0.1}{-2k}$$

$$t = \frac{\ln 0.1}{-0.0446} = 51.59 \text{ minutes.}$$

Q6 a)

$$\frac{dV}{dt} = 10 \text{ mm}^3/s$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$10 = 500 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{10}{500} = \frac{1}{50} \text{ mm/s}$$

b)

$$v = \frac{32}{x} - \frac{x}{2}$$

i) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $\quad \frac{1}{2}v^2 = \frac{1}{2}\left(\frac{32^2}{x^2} - 16 + \frac{x^2}{4}\right)$

$$\frac{1}{2}v^2 = 512x^{-2} - 8 + \frac{x^2}{8}$$

$$\ddot{x} = -1024x^{-3} + \frac{x}{4}$$

$$\ddot{x} = -\frac{1024}{x^3} + \frac{x}{4}$$

ii)

$$\frac{dx}{dt} = \frac{64 - x^2}{2x}$$

$$\frac{dt}{dx} = \frac{2x}{64 - x^2}$$

$$\therefore \quad t = \int \frac{2x}{64-x^2}\,dx$$

$$t = -\ln(64-x^2) + c$$

$t=0, \quad x=2$

$$0 = -\ln 60 + c$$

$$c = \ln 60$$

$$t = \ln 60 - \ln(64-x^2)$$

$$t = \ln \frac{60}{64-x^2}$$

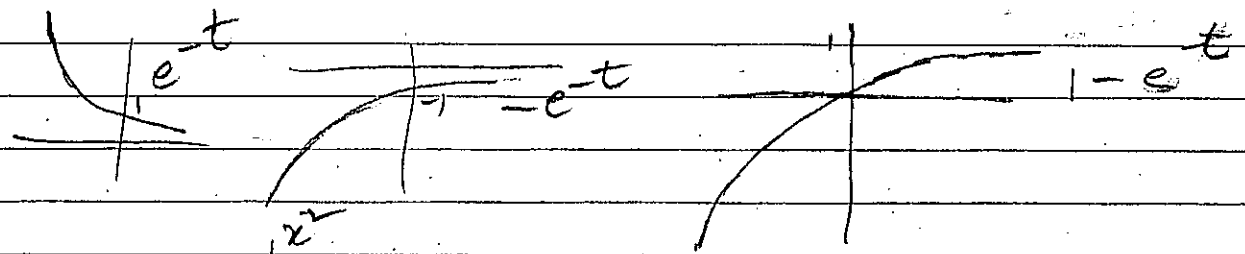
or $\dfrac{60}{64-x^2} = e^{t}$

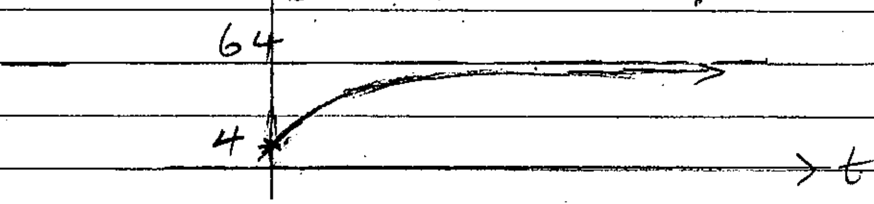
or $\dfrac{64-x^2}{60} = e^{-t}$

$$64 - x^2 = 60e^{-t}$$

$$x^2 = 64 - 60e^{-t}$$

iii)





as $t \to \infty \qquad x^2 \to 64 \qquad x \to 8$

Note initial velocity $v = \frac{32}{2} - \frac{2}{2} = 15$ m/s to right of O.

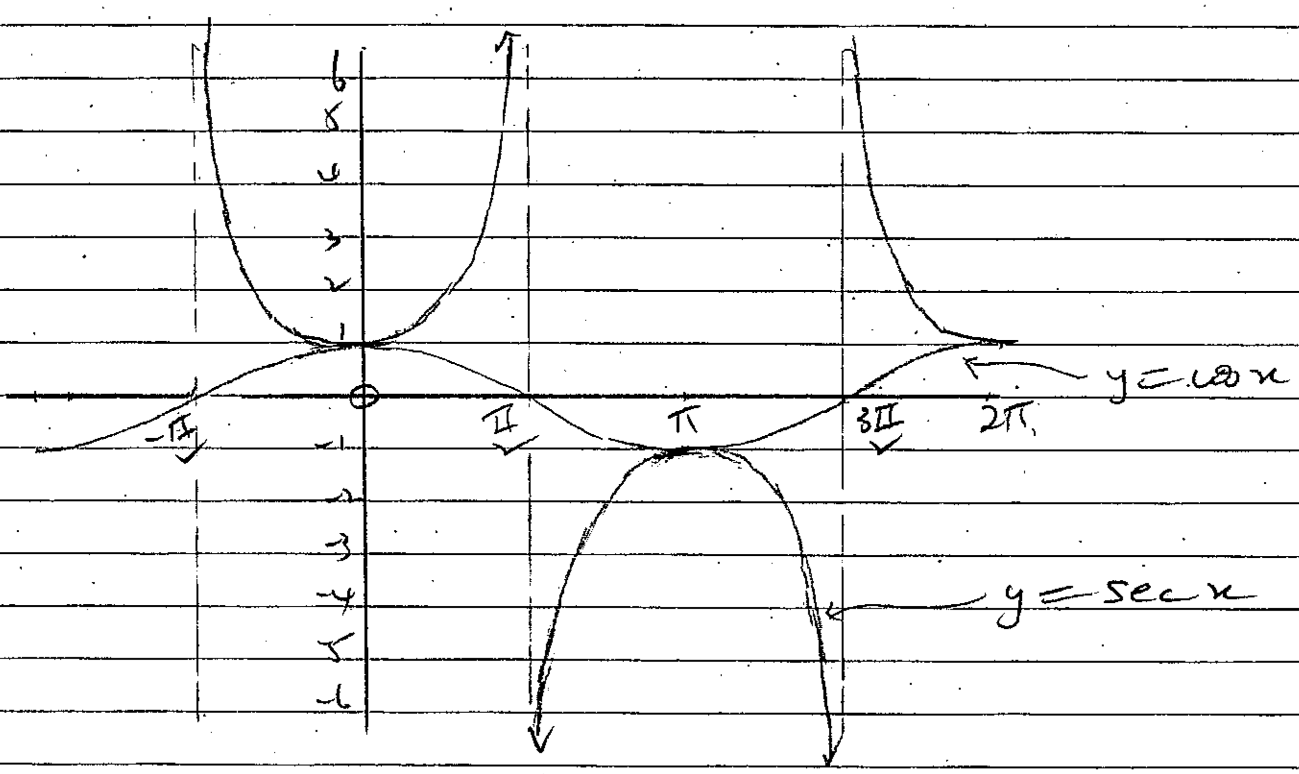
As $t \to \infty$, $x \to 8$, $v \to \frac{32}{8} - \frac{8}{2} \to 0$

as x is always less than 8, v is positive

acc $\to -\frac{1024}{8^3} + \frac{8}{4} = -2+2 \to 0$ from negative. Particle is slowing down.

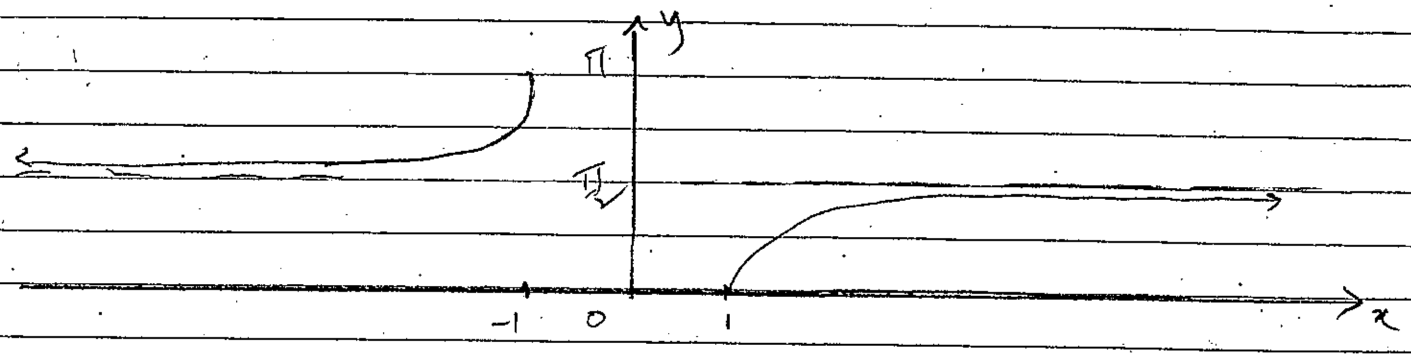
Question 7

a)


6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = \cos x$
$y = \sec x$

Q3 b) i) $\sec^{-1}\sqrt{2} = \theta \Rightarrow \sec\theta = \sqrt{2} \Rightarrow \theta = \frac{\pi}{4}$

ii) $y = \sec^{-1} x$



D: $x \geq 1$ or $x \leq -1$

iii) $x = \sec y$

$\frac{dx}{dy} = \sec y \tan y$ $\qquad \sec^2 y - \tan^2 y = 1$

$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$ $\qquad \tan^2 y = \sec^2 y - 1$

$\qquad \tan y = \pm\sqrt{\sec^2 y - 1}$

$$\frac{dy}{dx} = \frac{1}{\sec y \cdot \pm\sqrt{\sec^2 y - 1}}$$

$$\frac{dy}{dx} = \frac{\pm 1}{x\sqrt{x^2 - 1}} = \frac{1}{|x| \cdot \sqrt{x^2-1}}$$

But Gradient is always positive, ∴ Absolute value sign is required when $x < 0$ to give a positive value to $\frac{dy}{dx}$

iv) $y = \sec^{-1}\frac{x}{a}$

$$\frac{dy}{dx} = \left| \frac{1}{\frac{x}{a}\sqrt{\frac{x^2}{a^2} - 1}} \right| \cdot \frac{1}{a}$$

$$= \left| \frac{a}{x\sqrt{\frac{x^2 - a^2}{a^2}}} \right| \cdot \frac{1}{a}$$

$$\frac{dy}{dx} = \left| \frac{a}{x\sqrt{x^2 - a^2}} \right|$$

v) $\displaystyle\int \frac{dx}{x\sqrt{25x^2-9}}$

$$= \int \frac{dx}{5x\sqrt{x^2-\frac{9}{25}}}$$

$\therefore\ a = \frac{3}{5}$

$$= \frac{1}{5}\cdot\frac{5}{3}\sec^{-1}\frac{x}{3/5}$$

$$= \frac{1}{5}\cdot\frac{5}{3}\sec^{-1}\frac{5x}{3}$$

$$= \frac{1}{3}\sec^{-1}\frac{5x}{3} + c$$

(Once again positive sign required if x is positive, & negative when x is negative)