NAME ____________________

2012

GOSFORD HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper

Total marks – 70

Section I

- 10 marks
- Attempt Questions 1 – 10
- Multiple Choice
- Use the answer sheet provided at the end of this paper for this section
- Allow about 15 minutes for this section

Section II

- 60 marks
- Attempt Questions 11 – 14
- Show all necessary working
- Answer this section in the booklets provided
- Start each Question in a new booklet
- Allow about 1 hour 45 minutes for this section

SECTION I **Multiple Choice** **10 marks**

(use the provided answer sheet)

Question 1 A café menu contains 4 different entrees, 8 different main courses and 5 different deserts. How many different 3 course meals does the café offer?

A) $4!\times 8!\times 5!$ B) $4\times 8\times 5$

C) ${}^{17}C_3$ D) ${}^{17}P_3$

Question 2 $\lim\limits_{x\to\infty}\left[\dfrac{x+2}{1-x}\right]=?$

A) 1 B) -2

C) -1 D) 2

Question 3 The exact value of $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$ is

A) $-\dfrac{\pi}{6}$ B) $\dfrac{5\pi}{6}$

C) $-\dfrac{\pi}{3}$ D) $\dfrac{2\pi}{3}$

Question 4 The equation of the chord of contact of the tangents to the parabola $x^2=8y$ from the point $(3,-2)$ is

A) $3x-4y+8=0$ B) $3x-8y+16=0$

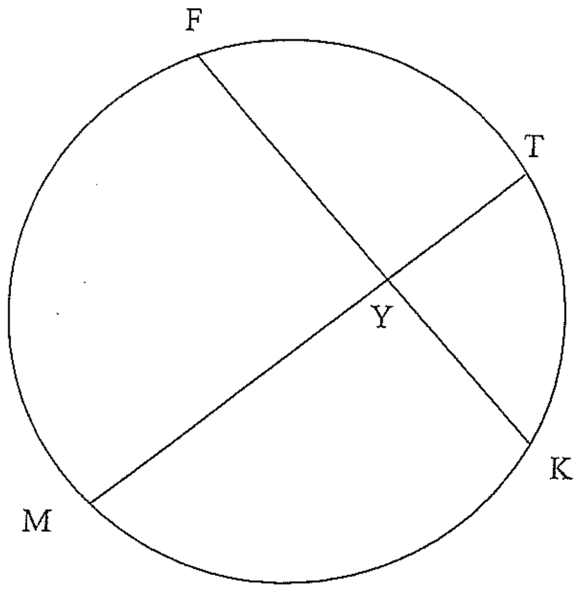
C) $3x-8y-8=0$ D) $3x-4y+16=0$

Question 5 $\sin 2x = ?$

A) $\dfrac{1-\tan^2 x}{1+\tan^2 x}$ B) $\dfrac{2\tan x}{1+\tan^2 x}$

C) $\dfrac{2\tan x}{1-\tan^2 x}$ D) $\dfrac{1+\tan^2 x}{1-\tan^2 x}$

Question 6 In the diagram below $MT = 9$, $TY = a$, $FY = x$ and $YK = y$



Which one of the following statements is true

A) $xy = 9a$ B) $\dfrac{x}{y} = \dfrac{9-a}{a}$

C) $x(x+y) = a(9-a)$ D) $xy = a(9-a)$

Question 7 If n is an integer then the general solution to the equation $\cos\theta = \cos\beta$ is given by

A) $\theta = 2n\pi \pm \beta$ B) $\theta = n\pi + \beta$

C) $\theta = 2n\pi \pm \cos^{-1}\beta$ D) $\theta = n\pi + \cos^{-1}\beta$

Question 8 $\int \sin^2 3x \, dx =$

A) $\frac{1}{2}\left[\frac{1}{6}\sin 6x - x\right] + c$

B) $6\sin 3x \cos 3x + c$

C) $\frac{1}{2}\left[x + \frac{1}{6}\sin 6x\right] + c$

D) $\frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right] + c$

Question 9 For $0 < x < 1$, $\frac{d}{dx}\left[\sin^{-1}\left(\frac{1}{x}\right)\right] = ?$

A) $\frac{-1}{x\sqrt{x^2-1}}$

B) $\frac{x}{\sqrt{x^2-1}}$

C) $\frac{-1}{\sqrt{x^2-1}}$

D) $\frac{-x}{\sqrt{x^2-1}}$

Question 10 $f(x) = x(x-4)$, for $x \le 2$

Which of the following represents $f^{-1}(x)$

A) $f^{-1}(x) = 2 - \sqrt{x+4}$

B) $f^{-1}(x) = 2 + \sqrt{x+4}$

C) $f^{-1}(x) = 2 \pm \sqrt{x+4}$

D) $f^{-1}(x) = \frac{1}{x(x-4)}$, for $x \le 2$

SECTION II

Question 11 **15 marks** ***(start a new booklet)***

a) Find the primitive of $\dfrac{1}{4+9x^2}$ (2)

b) Solve $2\sin^2 x = \sin 2x$, for $0 \le x \le \pi$ (2)

c) In how many ways can the letters of the word ENGINEER be arranged

(i) without restriction? (1)

(ii) if the vowels must be together? (2)

d) (i) Show that $(p-q)^2 = 2(p^2+q^2)-(p+q)^2$ (1)

(ii) If $P(2p, p^2)$ and $Q(2q, q^2)$ are two points on the parabola $x^2 = 4y$, find the coordinates of M, the midpoint of PQ, in terms of p and q (1)

(iii) If P and Q are restricted to move on the parabola so that $p - q = 1$, using (i) or otherwise, find the Cartesian equation of the locus of M. (2)

e) (i) Show that the curves $y = \sin x$ and $y = \cos x$ intersect at $P\left(\dfrac{\pi}{4}, \dfrac{1}{\sqrt{2}}\right)$. (1)

(ii) Show that if α is the acute angle between these curves at P, then $\tan\alpha = 2\sqrt{2}$ (3)

Question 12 **15 marks** ***(start a new booklet)***

a) (i) Show that there is a root to the equation $1-2x+2\sin x=0$ between $x=0\cdot 8$ and $x=1\cdot 8$ (1)

(ii) Using $x=1.2$ as a first approximation to the solution, apply Newton's Method once to obtain a closer approximation to the root.

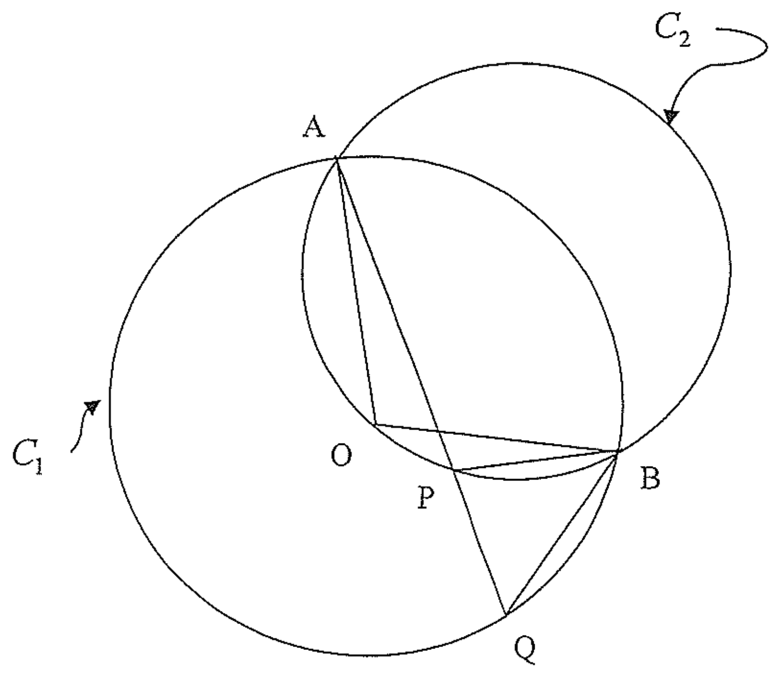
Give your answer correct to 2 d.p. (3)

b) The diagram below shows two unequal circles C_1 and C_2.

O is the centre of C_1 and the circle C_2 passes through O.

The two circles intersect at A and B.

Q and P lie on the circles C_1 and C_2 respectively, such that A, P and Q are collinear.



(i) If $\angle AQB = x$,

express $\angle AOB$ in terms of x, giving reason(s) for your answer. (1)

(ii) Hence, or otherwise, show that PB = PQ. (3)

c) Evaluate $\int_3^4 x\sqrt{x-3}$ using the substitution $u = x - 3$ (3)

d) The polynomial $P(x) = Ax^3 + Bx^2 + 2Ax + C$ has real roots $\sqrt{p}, \dfrac{1}{\sqrt{p}}$ and α

(i) Explain why $\alpha = -\dfrac{C}{A}$ (1)

(ii) Show that $A^2 + C^2 = BC$ (3)

Question 13 **15 marks** ***(start a new booklet)***

a) (i) Show that $(k+1)^2(k+4)=k^3+6k^2+9k+4$ (1)

(ii) Use mathematical induction to prove that

$$\sum_{k=1}^{n}\frac{1}{k(k+1)(k+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$ for all positive integral values of n (4)

b) The velocity (v m/s) of a particle moving along the x axis is given by

$$v^2=8x-x^2-7$$

(i) Find the acceleration of the particle. (2)

(ii) Explain why the motion of the particle is Simple Harmonic. (1)

(iii) State the centre of the motion and the maximum speed of the particle. (2)

c) In a hive of bees it is found that the number (N) of bees affected by a virus at any time (t), in months, is given by

$$N=\frac{600}{4+Ae^{-0\cdot5t}}$$

(i) If initially there are 50 infected bees, find the value of the constant A. (1)

(ii) Find the time taken for there to be 90 bees infected by the virus. (2)

(iii) Find the rate at which the infection is spreading when there are 90 bees infected by the virus. (2)

Question 14 **15 marks** ***(start a new booklet)***

a) (i) Write $\sqrt{3}\cos\theta - \sin\theta$ in the form $R\cos(\theta+\alpha)$ where $R>0$ and α is acute. (2)

(ii) Solve $\sqrt{3}\cos\theta - \sin\theta = 1$ for $-\pi \le \theta < \pi$. (2)

b) A spherical balloon with radius r m, volume $V\text{m}^3$ and surface area $A\,\text{m}^2$ is expanding so its volume is increasing at a constant rate of $7\cdot2\ \text{m}^3/\text{s}$.

Given $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ find the rate of increase of the surface area when the radius of the sphere is $1\cdot2\,\text{m}$. (3)

c) (i) Find the domain and range of $y = \tan^{-1}(e^x)$ (2)

(ii) Show that $\frac{dy}{dx} = \frac{1}{2}\sin 2y$ (3)

d) The velocity v m/s of a particle is given by $v = 1 + e^{-x}$

Initially, the particle is at the origin and its velocity is 2 m/s .

Find the time taken by the particle to reach a velocity of $1\frac{1}{2}$ m/s (3)

END OF EXAMINATION

Name:______________________ Teacher:__________________

Multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely, using a black pen.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ◯ B ● C ◯ D ◯

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ◯ D ◯

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ◯ D ◯

Start here →

1. A ◯ B ◯ C ◯ D ◯
2. A ◯ B ◯ C ◯ D ◯
3. A ◯ B ◯ C ◯ D ◯
4. A ◯ B ◯ C ◯ D ◯
5. A ◯ B ◯ C ◯ D ◯
6. A ◯ B ◯ C ◯ D ◯
7. A ◯ B ◯ C ◯ D ◯
8. A ◯ B ◯ C ◯ D ◯
9. A ◯ B ◯ C ◯ D ◯
10. A ◯ B ◯ C ◯ D ◯

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq 1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2012 GHS TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

SOLUTIONS

SECTION 1

Multiple Choice

(1)	B	(2)	C
(3)	B	(4)	A
(5)	B	(6)	D
(7)	A	(8)	D
(9)	A	(10)	A

SECTION 2

Question 11

a) $\int \frac{1}{4+9x^2}\,dx = \frac{1}{9}\int \frac{1}{\frac{4}{9}+x^2}\,dx$

$= \frac{1}{9}\times\frac{3}{2}\tan^{-1}\left(\frac{3x}{2}\right)+c$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)+c$

b) $2\sin^2 x = \sin 2x \qquad 0 \le x \le \pi$

$2\sin^2 x = 2\sin x\cos x$

$2\sin^2 x - 2\sin x\cos x = 0$

$2\sin x(\sin x - \cos x) = 0$

$\therefore \sin x = 0$ and/or $\sin x - \cos x = 0$

$x = 0, \pi \qquad \sin x = \cos x \quad) \div \cos x^{*}$

$\tan x = 1$

$x = \frac{\pi}{4}$

$\therefore$ Solutions are $x = 0, \frac{\pi}{4}, \pi$

Note $\cos x = 0$

$x = \frac{\pi}{2}$ is not a solution.

c) (i) Nº. of arrangements $= \frac{8!}{3!2!}$

$= 3360$

c) (ii) Vowels $\rightarrow$ 3 E's & 1 I can be arranged in $\frac{4!}{3!}$ ways

i.e $\therefore$ 4 ways.

Vowels together and 4 consonants (2 alike) can be arranged in $\frac{5!}{2!}$ ways $= 60$ ways.

$\therefore$ Nº. of arrangements $= 4\times 60$

$= 240$ ways

d) (i) R.H.S. $= 2(p^2+q^2)-(p+q)^2$

$= 2p^2+2q^2-p^2-2pq-q^2$

$= p^2-2pq+q^2$

$= (p-q)^2$

$=$ L.H.S.

(ii) Midpoint $M = \left(\frac{2p+2q}{2}, \frac{p^2+q^2}{2}\right)$

$= \left(p+q, \frac{p^2+q^2}{2}\right)$

(iii) Parametric Equations of locus of M are $x = p+q$, $y = \frac{p^2+q^2}{2}$

$2y = p^2+q^2$

Now using (i) and $p-q=1$. $\quad (p-q)^2 = 2(p^2+q^2)-(p+q)^2$

$(1)^2 = 2(2y)-x^2$

$1 = 4y-x^2$

or $x^2 = 4y-1$ is the Cartesian locus of M.

e) (i) When $x = \frac{\pi}{4}$, $\sin x = \sin\frac{\pi}{4}$ and $\cos x = \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$ $\qquad$ $= \frac{1}{\sqrt{2}}$

$\therefore$ Curves intersect at $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

(ii) $y = \sin x$. $\qquad$ $y = \cos x$

$\frac{dy}{dx} = \cos x$. $\qquad$ $\frac{dy}{dx} = -\sin x$.

$m_1 = \cos\frac{\pi}{4}$ at P $\qquad$ $m_2 = -\sin\frac{\pi}{4}$ at P

$m_1 = \frac{1}{\sqrt{2}}$ $\qquad$ $m_2 = -\frac{1}{\sqrt{2}}$

$$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}}\right|$$

$$= \left|\frac{\frac{2}{\sqrt{2}}}{1 - \frac{1}{2}}\right|$$

$$= \left|\frac{2}{\sqrt{2}} \times \frac{2}{1}\right|$$

$$= \frac{4}{\sqrt{2}}\left(\times\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$= 2\sqrt{2}$ as required.

Question 12

a) (i) Let $P(x) = 1 - 2x + 2\sin x$

$P(0.8) = 1 - 2(0.8) + 2\sin(0.8)$

$\doteqdot 0.835 > 0$

$P(1.8) = 1 - 2(1.8) + 2\sin(1.8)$

$\doteqdot -0.652 < 0$

$\therefore$ Since $P(0.8)$ and $P(1.8)$ are opposite in sign and $P(x)$ is continuous in the domain $0.8 < x < 1.8$ then a root of $P(x)$ exists in the interval.

(ii) $P(x) = 1 - 2x + 2\sin x$ $\qquad$ $P'(x) = -2 + 2\cos x$

$P(1.2) = 1 - 2(1.2) + 2\sin(1.2)$ $\qquad$ $P'(1.2) = -2 + 2\cos(1.2)$

$= 0.464078171$ $\qquad$ $= -1.275284491$

Let x_1 be improved approximation

$$x_1 = 1.2 - \frac{P(1.2)}{P'(1.2)}$$

$$= 1.2 - \frac{0.464078171}{-1.275284491}$$

$= 1.56$ (2 d.p)

Note: Check $P(1.56) = -0.120$ indicating improved approximation

b) (i) If $A\hat{Q}B = x$

$A\hat{O}B = 2x$ ($\angle$ at the centre of circle C_1 is twice the angle at the circumference standing on the same arc ABQ

b) (ii) $A\hat{P}B = A\hat{O}B$ (angles at the circumference standing on the same arc (AB) of circle C_2 are equal)

$= 2x$

$P\hat{B}Q + A\hat{Q}B = A\hat{P}B$ (exterior ∠ of a △ (PBQ) theorem.

$P\hat{B}Q + x = 2x$

$P\hat{B}Q = x$

$= P\hat{Q}B$

$\therefore PB = PQ$ (equal sides opposite equal ∠'s of isosceles △ PBQ)

c) $u = x - 3$ when $x = 4,\ u = 1$

$\frac{du}{dx} = 1$ $\quad x = 3,\ u = 0$

$\frac{dx}{du} = 1$

$$\therefore \int_3^4 x\sqrt{x-3}\,dx = \int_0^1 (u+3)\cdot u^{\frac{1}{2}} \times 1\,du.$$

$$= \int_0^1 u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\,du$$

$$= \left[\frac{2u^{\frac{5}{2}}}{5} + 3 \times \frac{2u^{\frac{3}{2}}}{3}\right]_0^1$$

$$= \frac{2}{5} + 2$$

$$= 2\frac{2}{5}$$

d) $P(x) = Ax^3 + Bx^2 + 2Ax + C$

(i) Product of Roots $= -\frac{d}{a}$

$\sqrt{p} \times \frac{1}{\sqrt{p}} \times \alpha = -\frac{C}{A}$

$\therefore \alpha = -\frac{C}{A}$

(ii) Now α is root & therefore satisfies

$$\therefore A \times \left(-\frac{C}{A}\right)^3 + B\left(-\frac{C}{A}\right)^2 + 2A\left(-\frac{C}{A}\right) + C = 0$$

$$\frac{-C^3}{A^2} + \frac{BC^2}{A^2} - 2C + C = 0$$

$$\therefore -C^3 + BC^2 - CA^2 = 0$$

$$BC^2 = C^3 + CA^2$$

$$BC = C^2 + A^2 \text{ for } C \neq 0$$

<u>Question 13</u>

(i) L.H.S. $= (k+1)^2(k+4)$

$= (k^2+2k+1)(k+4)$

$= k^3 + 4k^2 + 2k^2 + 8k + k + 4$

$= k^3 + 6k^2 + 9k + 4$

$=$ R.H.S.

(ii) Prove true for $n = 1$

L.H.S $= \frac{1}{1(2)(3)}$ $= \frac{1}{6}$

R.H.S. $= \frac{1(1+3)}{4(2)(3)}$ $= \frac{1}{6}$ $=$ L.H.S.

$\therefore$ True for $n = 1$

Assume $\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\cdots\cdots+\frac{1}{k(k+1)(k+2)}=\frac{k(k+3)}{4(k+1)(k+2)}$

Prove true for $n=k+1$, if true for $n=k$

Prove $\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\cdots\cdots+\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}=\frac{(k+1)(k+4)}{4(k+2)(k+3)}$

$$\begin{aligned}
L.H.S. &= \frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)} \quad \text{(using assumption)}\\
&= \frac{k(k+3)^2+4}{4(k+1)(k+2)(k+3)}\\
&= \frac{k(k^2+6k+9)+4}{4(k+1)(k+2)(k+3)}\\
&= \frac{k^3+6k^2+9k+4}{4(k+1)(k+2)(k+3)}\\
&= \frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)}\\
&= \frac{(k+1)(k+4)}{4(k+2)(k+3)}\\
&= R.H.S.
\end{aligned}$$

$\therefore$ true for $n=k+1$, if true for $n=k$

Since true for $n=1$, $\therefore$ true for $n=1+1=2$

Since true for $n=2$, $\therefore$ true for $n=2+1=3$

and so on

$\therefore$ True, by induction, for all given 'n'.

(i) Acceleration $=\frac{d}{dx}\left[\frac{1}{2}v^2\right]$

$$\begin{aligned}
\ddot{x} &= \frac{d}{dx}\left[\frac{1}{2}(8x-x^2-7)\right]\\
&= \frac{1}{2}(8-2x)\\
&= 4-x\\
\ddot{x} &= -(x-4)
\end{aligned}$$

(ii) Which is of the form $\ddot{X}=-n^2X$ where $X=x-4$ and $n=1$.

$\therefore$ Motion is Simple Harmonic

Since acceleration is opposite in sign and proportional to displacement from $X=0$; ie $x=4$

(iii) Centre of Motion occurs when $\ddot{x}=0$

ie when $x=4$

Max. Speed occurs at centre of motion

$$\begin{aligned}
\therefore V_{max}^2 &= 8(4)-(4)^2-7\\
&= 32-16-7\\
&= 9\\
V_{max} &= 3\text{ m/s}
\end{aligned}$$

c) $N=\frac{600}{4+Ae^{-0.5t}}$

(i) when $t=0$, $N=50$

$$50=\frac{600}{4+A}$$

$$4+A=12$$

$$A=8$$

$$4 + 8e^{-0.5t}$$

$$\therefore \quad N = \frac{150}{1 + 2e^{-0.5t}}.$$

$$\therefore \quad 90 = \frac{150}{1 + 2e^{-0.5t}}$$

$$1 + 2e^{-0.5t} = \frac{15}{9}$$

$$2e^{-0.5t} = \frac{2}{3}$$

$$e^{-0.5t} = \frac{1}{3} \quad \text{.........} (A)$$

$$-0.5t = \ln\left(\frac{1}{3}\right).$$

$$t = \frac{\ln 3}{0.5} \approx 2.2 \text{ months.}$$

(iii)

$$N = 150\left(1 + 2e^{-0.5t}\right)^{-1}$$

$$\frac{dN}{dt} = -150\left(1 + 2e^{-0.5t}\right)^{-2} \times \left(-e^{-0.5t}\right)$$

$$= -150\left(1 + 2 \times \frac{1}{3}\right)^{-2} \times \left(-\frac{1}{3}\right) \quad \text{using } (A)$$

$$= 50 \times \left(\frac{5}{3}\right)^{-2}.$$

$$= 50 \times \frac{9}{25}.$$

$$= 18 \text{ bees / month.}$$

Question ..

a) (i) $\sqrt{3}\cos\theta - \sin\theta \equiv R\cos(\theta + \alpha)$

$$\equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha.$$

Equating Coefficients

$R\sin\alpha = 1$

$R\cos\alpha = \sqrt{3}$ $\longrightarrow$ $\tan\alpha = \frac{1}{\sqrt{3}}$

$$\alpha = \frac{\pi}{6}.$$

$$\therefore R\sin\frac{\pi}{6} = 1$$

$$R \times \frac{1}{2} = 1$$

$$R = 2.$$

$$\therefore \sqrt{3}\cos\theta - \sin\theta = 2\cos\left(\theta + \frac{\pi}{6}\right)$$

(ii) $\therefore 2\cos\left(\theta + \frac{\pi}{6}\right) = 1 \qquad -\pi \le \theta < \pi$

$$\cos\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}.$$

$$\theta + \frac{\pi}{6} = -\frac{\pi}{3}, \frac{\pi}{3}$$

$$\theta = -\frac{\pi}{2}, \frac{\pi}{6}$$

b) $\frac{dV}{dt} = 7.2$

$A = 4\pi r^2 \longrightarrow \frac{dA}{dr} = 8\pi r$

$= 9.6\pi$ when $r = 1.$

$V = \frac{4}{3}\pi r^3 \longrightarrow \frac{dV}{dr} = 4\pi r^2$

$= 4\pi \times (1.2)^2$

$= 5.76\pi$

$$\frac{dA}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \times \frac{dA}{dr}$$

$\frac{}{dt} = 7.2 \times \frac{1}{5.76\pi} \times \ldots$

$= 12\ m^2/s$

c) (i) Domain : x is any Real Number

Noting $e^x > 0$ for all x.

Range $0 < y < \frac{\pi}{2}$

(ii) $y = \tan^{-1}(e^x)$

$\frac{dy}{dx} = \frac{1}{1+(e^x)^2} \times e^x$

$= \frac{e^x}{1+(e^x)^2}$

But $e^x = \tan y$.

$\therefore \frac{dy}{dx} = \frac{\tan y}{1+\tan^2 y}$

$= \frac{\tan y}{\sec^2 y}$

$= \frac{\sin y}{\cos y} \times \cos^2 y$

$= \sin y \cos y$

$= \frac{1}{2}\sin 2y$ as required.

a) $V = 1 + e$

when $V = 1\frac{1}{2}$, $1\frac{1}{2} = 1 + e^{-x}$

$\frac{1}{2} = e^{-x}$

$e^x = 2$ ..... (A)

$x = \ln 2$

Need to find t when $x = \ln 2$

$\therefore \frac{dx}{dt} = 1 + e^{-x}$

$\frac{dt}{dx} = \frac{1}{1+e^{-x}}$

$t = \int \frac{1}{1+e^{-x}}\, dx$

$= \int \frac{e^x}{e^x+1}\, dx$

$t = \ln(e^x+1) + c$

when $t = 0, x = 0$

$\therefore 0 = \ln 2 + c$

$c = -\ln 2$

$\therefore t = \ln(e^x+1) - \ln 2$

$= \ln\left(\frac{e^x+1}{2}\right)$

$t = \ln\left(\frac{2+1}{2}\right)$ when $V = 1\frac{1}{2}$, $e^x = 2$

$= \ln\left(\frac{3}{2}\right)$ seconds