GOSFORD HIGH SCHOOL

2017 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I Pages 2 – 6

10 Marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 14

60 Marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 Marks
Attempt Questions 1 – 10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1) If $\cos x = \frac{2}{5}$ and $0 < x < \frac{\pi}{2}$, what is the value of $\cos 2x$?

(A) $\frac{4}{5}$

(B) $\frac{17}{25}$

(C) $-\frac{17}{25}$

(D) $-\frac{4}{5}$

2) The equation $ax^3 + bx^2 + cx + d = 0$, where $a \neq 0, b \neq 0, c \neq 0$ and $d \neq 0$, has roots α, β and γ. Which of the following is an expression for $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$?

(A) $-\frac{b}{d}$

(B) $\frac{b}{d}$

(C) $-\frac{c}{d}$

(D) $\frac{c}{d}$

3) In how many ways can the letters of the word MATTYJAY be arranged in a line?

(A) 40 320

(B) 20 160

(C) 10 080

(D) 5 040

4) Which of the following is an expression for the inverse of the function $f(x) = \log_5(x+3) - 2$?

(A) $f^{-1}(x) = \log_5(x+2) - 2$

(B) $f^{-1}(x) = 5^{x+2} - 3$

(C) $f^{-1}(x) = 5^{x} - 1$

(D) $f^{-1}(x) = 10^{x} - 3$

5) Which expression is equal to

$$\int \sin^2 2x \, dx$$

(A) $\frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + c$

(B) $\frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right) + c$

(C) $\frac{\sin^3 2x}{6} + c$

(D) $-\frac{\cos^3 2x}{6} + c$

6) Which of the following is an expression for $\frac{d}{dx}(\tan^{-1}(2x+1))$?

(A) $\frac{1}{4x^2 + 4x + 2}$

(B) $\frac{1}{2x^2 + 2x + 1}$

(C) $\frac{1}{4x^2 + 2}$

(D) $\frac{1}{2x^2 + 1}$

7) What is the general solution of the equation

$$2\sin^2 x - 5\sin x - 3 = 0$$

(A) $n\pi - (-1)^n \frac{\pi}{3}$

(B) $n\pi + (-1)^n \frac{\pi}{3}$

(C) $n\pi - (-1)^n \frac{\pi}{6}$

(D) $n\pi + (-1)^n \frac{\pi}{6}$

8) A spherical hailstone is forming in a cloud with its radius increasing by 2 mm per second. At what rate is the volume of the stone increasing when the radius is 5 mm

(A) $\frac{500\pi}{3}$

(B) 200π

(C) 100π

(D) 50π

9) The original temperature of a body is 100°C and the temperature of the surroundings is 20°C. 10 minutes later the body is 70°C. Which of the following is the temperature T°C at time t minutes later?

(A) $T = 20 + 100e^{-0.069t}$

(B) $T = 20 + 100e^{0.069t}$

(C) $T = 20 + 80e^{-0.047t}$

(D) $T = 20 + 80e^{0.047t}$

10) Assume that the tides rise and fall in simple harmonic motion. At low tide a channel is 6 m deep and at high tide 18 m deep. Low tide is at 6 am with the next high tide at 4 pm.
Which equation models the depth of the water d m at time t hours after 6 am?

(A) $d = -6\cos\frac{\pi}{5}t$

(B) $d = -6\cos\frac{\pi}{10}t$

(C) $d = -12\cos\frac{\pi}{5}t$

(D) $d = -12\cos\frac{\pi}{10}t$

Section II

60 Marks
Attempt Questions 11 – 14.
Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) $A(-2,8)$ and $B(4,-7)$ are two points. Find the coordinates of the point P that divides the interval AB internally in the ratio $5:2$. **2**

b) Find in simplest exact form the value of **3**

$$\int_{-1}^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}}\,dx$$

c) Use the substitution $t = \tan\frac{x}{2}$ to show that **3**

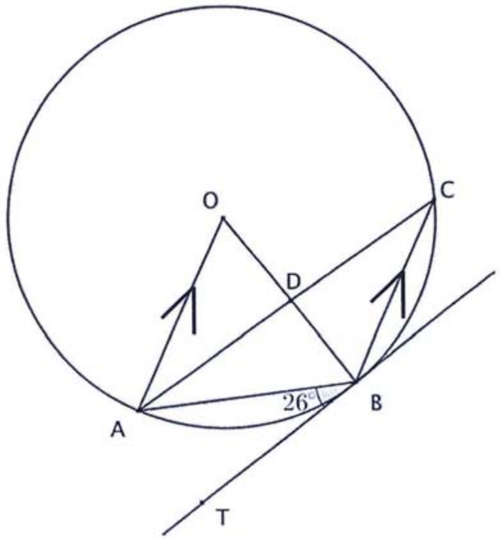
$$\sec x + \tan x = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

d) Find the term independent of x in the expansion of **3**

$$\left(x - \frac{3}{x^2}\right)^{12}$$

Question 11 Continues On Next Page

e)



The points A, B and C lie on a circle with centre O. The lines AO and BC are parallel, and OB and AC intersect at D. Also, $\angle TBA = 26°$, as shown in the diagram.
Copy or trace the diagram into your writing booklet.

i) Find the size of $\angle ACB$, giving reasons **1**

ii) Find the size of $\angle AOB$, giving reasons **1**

iii) Find $\angle BDC$. Justify your answer. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Find correct to the nearest degree the acute angle between the lines $2x - y = 0$ and $x + 3y + 5 = 0$ **2**

b) i) Find the domain and range of the function **2**

$$f(x) = -\sin^{-1}(x+2)$$

ii) Sketch the graph of the function $f(x) = -\sin^{-1}(x+2)$ showing the coordinates of the end points. **2**

c) Show that $(x-2)$ is a factor of **1**

$$P(x) = x^3 + 2x^2 - 5x - 6$$

d) Sketch the polynomial **2**

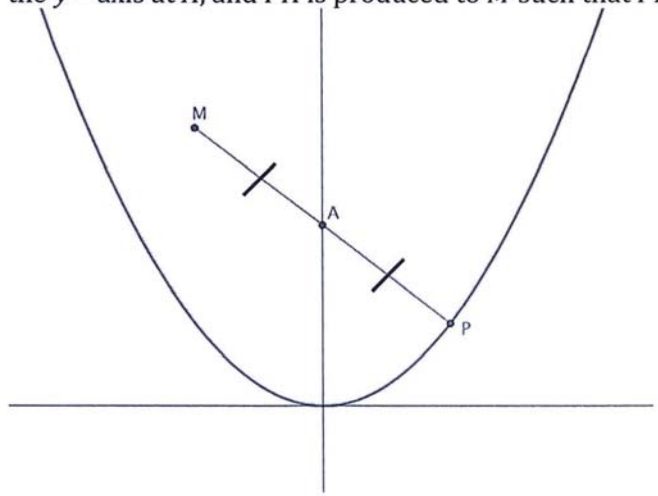
$$P(x) = (x+2)^2(1-x)$$

e) Use the substitution $u = x + 2$ to find **2**

$$\int x\sqrt{x+2}\,dx$$

Question 12 Continues On Next Page

f) The normal at $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ meets the y-axis at A, and PA is produced to M such that $PA = AM$



i) Prove that M has coordinates **2**

$$(-2ap, 4a + ap^2)$$

ii) Hence find the locus of M **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) Prove by mathematical induction that, for all $n \geq 2$ 3

$$3^n > n^2$$

b) i) Given that the function $f(x) = x^2 - \sin x$ is continuous for all real x, show that the equation $f(x) = 0$ has a root between $x = \frac{1}{2}$ and $x = 1$ 1

ii) Use one application of Newton's method with an initial approximation 0.8 to find the next approximation to this root, giving your answer correct to 2 decimal places. 2

c) A particle is moving in a straight line. At time t seconds it has displacement x metres to the right of a fixed point O on the line and velocity v ms^{-1} given by

$$v = \frac{8 - x^2}{x}$$

Initially the particle is 1 metre to the right of O.

i) Show that 3

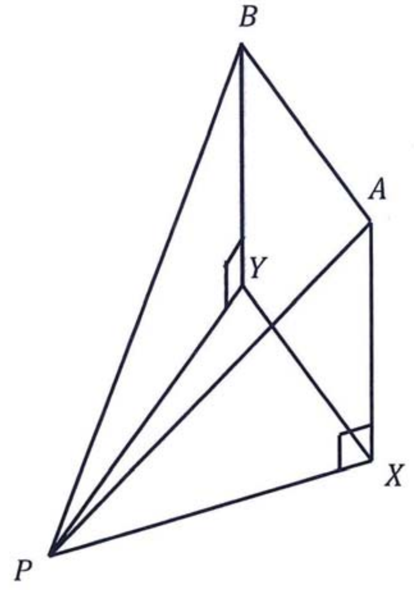
$$x = \sqrt{8 - 7e^{-2t}}$$

ii) Find the limiting position of the particle 1

iii) Describe the motion of the particle as $t \to \infty$ 2

Question 13 Continues on Next Page

d) A plane flies horizontally at a height h m over a distance of 8 000 m from a point A, vertically above X to a point B, vertically above Y.
An observer standing at P notices the angle of elevation to the plane at A was 5° and when the plane was at B the angle of elevation was 13°.



The observer notes that the initial bearing of the plane was 037°T and the final bearing was 290°T.

i) Show that $PX = h \cot 5$ 1

ii) Hence, find the value of h (to the nearest metre). 2

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) i) Write down the expansion of $(1+x)^n$ is ascending powers of x 1

ii) Hence show that 3

$$\binom{n}{0}+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+\cdots+\frac{1}{n+1}\binom{n}{n}=\frac{2^{n+1}-1}{n+1}$$

b) A particle moves according to the equation

$$x=6\sin^2\left(4t+\frac{\pi}{3}\right)$$

i) Show that the acceleration can be expressed in the form 3

$$\ddot{x}=64(3-x)$$

ii) Find the amplitude of the motion 1

iii) Find the period of the motion 1

Question 14 Continues on Next Page

c) i) Simplify 1

$$\sin(2\sin^{-1}x)$$

ii) Show that 2

$$\sin 3\theta=3\sin\theta-4\sin^3\theta$$

iii) By using the substitution $\theta=\sin^{-1}x$ and the answers to the previous parts show that 3

$$\int \sin(2\sin^{-1}x)\,dx=\frac{1}{2}\int(\sin 3\theta+\sin\theta)d\theta$$

End of Exam

Ext 1 Trial 2017

1) $\cos 2x = 2\cos^2 x - 1$
$= 2 \times \left(\frac{2}{5}\right)^2 - 1$
$= \frac{8}{25} - 1$
$= \frac{-17}{25}$

C

2) $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma} = \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$
$= \frac{-b/a}{-d/a}$
$= b/d$

B

3) $\frac{8!}{2!2!2!} = 5040$

~~B~~ D

4) $x = \log_5(y+3) - 2$
$y + 3 = 5^{x+2}$
$y = 5^{x+2} - 3$

B

5) $\cos 2A = 1 - 2\sin^2 A$
$\sin^2 A = \frac{1}{2}(1 - \cos 2A$
$\int \sin^2 2x \, dx = \frac{1}{2}\int 1 - \cos 4x \, dx$
$= \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right]$

A

6) $\frac{d}{dx}\tan^{-1}(2x+1) = \frac{1}{1+(2x+1)^2} \times 2$
$= \frac{2}{1 + 4x^2 + 4x + 1}$
$= \frac{1}{2x^2 + 2x + 1}$

B

CBDBA | BCBCB

7) $2\sin^2 x - 5\sin x - 3 = 0$
$(2\sin x + 1)(\sin x - 3) = 0$
$\sin x = -\frac{1}{2}$ or $\sin x = 3$
no soln
$x = n\pi + (-1)^n \times \left(-\frac{\pi}{6}\right)$

C

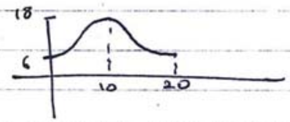
8) $V = \frac{4}{3}\pi r^3$
$\frac{dV}{dr} = 4\pi r^2$
$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$
$= 4\pi r^2 \times 2$
@ $r = 5$ $\frac{dV}{dt} = 200\pi$

B

9) $T = 20 + 80e^{kt}$
@ $t = 10$ $T = 70$
$70 = 20 + 80e^{10k}$
$k = \frac{1}{10}\ln\frac{5}{8}$
≈ -0.047

C

10)



$2a = 12$ $\quad 20 = \frac{2\pi}{n}$
$a = 6$ $\quad n = \frac{\pi}{10}$
$d = -6\cos\frac{\pi}{10}t$

B

11) a) $(-2, 8) \quad (4, -7)$

$5 : 2$

$$P = \left(\frac{-2\times 2 + 4\times 5}{5+2}, \frac{8\times 2 + -7\times 5}{5+2} \right)$$

$$= \left(\frac{16}{7}, \frac{-19}{7} \right)$$

b) $$\int_{-1}^{\sqrt{2}} \frac{dx}{\sqrt{2-x^2}} = \left[\sin^{-1} \frac{x}{\sqrt{2}} \right]_{-1}^{\sqrt{2}}$$

$$= \sin^{-1} 1 - \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{2} + \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

c) $$LHS = \sec x + \tan x$$

$$= \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$$

$$= \frac{1+2t+t^2}{1-t^2}$$

$$= \frac{(1+t)^2}{(1+t)(1-t)}$$

$$= \frac{1+t}{1-t}$$

$$= \frac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{x}{2}}$$

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$= RHS.$$

d) $$T_{k+1} = {}^{12}C_k\, x^{12-k} \left(-3x^{-2}\right)^k$$

$$= {}^{12}C_k\, x^{12-k} (-3)^k x^{-2k}$$

$$= {}^{12}C_k (-3)^k x^{12-3k}$$

for constant $12 - 3k = 0$

$k = 4$

$\therefore$ term is ${}^{12}C_4 (-3)^4 = 40\ 095$

e) i) $\angle ACB = 26°$ ($\angle$ between tangent & chord equals $\angle$ in alternate segment)

ii) $\angle AOB = 52°$ ($\angle$ @ the centre is twice $\angle$ standing on same arc @ the circumference)

iii) $\angle OAD = 26°$ (alternate $\angle$'s on parralel lines)

$\angle ODA = 102°$ ($\angle$ sum of Δ is $180°$)

$\angle BDC = 102$ (vertically opposite)

12 a) $2x - y = 0$

$y = 2x$

$m_1 = 2$

$x + 3y + 5 = 0$

$y = -\frac{1}{3}x - \frac{5}{3}$

$m_2 = -\frac{1}{3}$

$\tan\theta = \left|\frac{2 - -\frac{1}{3}}{1 + 2\times -\frac{1}{3}}\right|$

$= \left|\frac{7/3}{1/3}\right|$

$= 7$

$\theta = \tan^{-1} 7$

$= 81.86\ldots$

$= 82^\circ$

b) i) $-y = \sin^{-1}(x+2)$

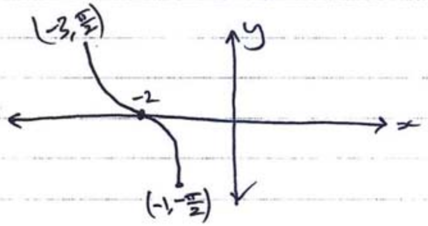
Domain: $-1 \le x+2 \le 1$

$-3 \le x \le -1$

Range: $-\frac{\pi}{2} \le -y \le \frac{\pi}{2}$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

ii)



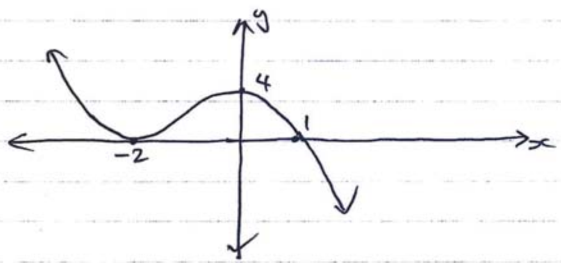
c) $P(2) = 2^3 + 2\times 2^2 - 5\times 2 - 6$

$= 8 + 8 - 10 - 6$

$= 0$

$\therefore$ $x - 2$ is a factor

d)



e) $u = x + 2$

$du = dx$

$x = u - 2$

$\int x\sqrt{x+2}\,dx = \int (u-2)\,u^{1/2}\,du$

$= \int u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\,du$

$= \frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{3/2} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

f) i) eqn of normal from ref. sheet

$x + py = ap^3 + 2ap$

A is @ $x = 0$

$py = ap^3 + 2ap$

$y = ap^2 + 2a$

$A = (0,\ 2a + ap^2)$

$A = \left(\frac{x_m + 2ap}{2}, \frac{y_m + ap^2}{2}\right)$

$\frac{x_m + 2ap}{2} = 0$

$x_m = -2ap$

$\frac{y_m + ap^2}{2} = 2a + ap^2$

$y_m + ap^2 = 4a + 2ap^2$

$y_m = 4a + ap^2$

ii) $p = \frac{x}{-2a}$

$y = 4a + ap^2$

$= 4a + a\left(\frac{x}{-2a}\right)^2$

$= 4a + \frac{x^2}{4a}$

$4ay = 16a^2 + x^2$

$x^2 = 4ay - 16a^2$

$= 4a(y - 4a)$

13) a) Prove true for $n=1$

$LHS = 3^1$ $\quad RHS = 1^2$

$= 3$ $\quad = 1$

$\therefore$ true for $n=1$

Assume true for $n=k$

$3^k > k^2$

Prove true for $n=k+1$

ie $3^{k+1} > (k+1)^2$

$LHS = 3^{k+1}$

$= 3 \times 3^k$

$> 3k^2$ by assumption

$= k^2 + k^2 + k^2$

$\geq k^2 + 2k + k^2$ since $k>$

$\geq k^2 + 2k + 1$ since $k \geq 1$

$= (k+1)^2$

$= RHS$

$\therefore$ the result is proven by the principal of mathematical induction.

b) i) $f(\tfrac{1}{2}) = (\tfrac{1}{2})^2 - \sin \tfrac{1}{2}$ $\qquad f(1) = 1^2 - \sin(1)$

$= -0.229\ldots$ $\qquad = 0.158\ldots$

$\therefore$ since $f(\tfrac{1}{2})$ & $f(1)$ are opposite in sign & $f(x)$ is continuous there must be a root on $\tfrac{1}{2} \leq x \leq 1$

ii) $f'(x) = 2x - \cos x$

$f'(0.8) = 2 \times 0.8 - \cos 0.8$

$= 0.903\ldots$

$f(0.8) = (0.8)^2 - \sin 0.8$

$= -0.077\ldots$

$x_2 = 0.8 - \dfrac{f(0.8)}{f'(0.8)}$

$= 0.8856\ldots$

$= 0.89$

c) i) $\dfrac{dx}{dt} = \dfrac{8-x^2}{x}$

$\dfrac{dt}{dx} = -\dfrac{1}{2} \cdot \dfrac{-2x}{8-x^2}$

$t = -\tfrac{1}{2} \ln(8-x^2) + c$

@ $t=0$ $x=1$

$0 = -\tfrac{1}{2} \ln 7 + c$

$c = \tfrac{1}{2} \ln 7$

$t = -\tfrac{1}{2} \ln 8-x^2 + \tfrac{1}{2} \ln 7$

$-2t = \ln\left(\dfrac{8-x^2}{7}\right)$

$8 - x^2 = 7e^{-2t}$

$x^2 = 8 - 7e^{-2t}$

$x = \pm\sqrt{8-7e^{-2t}}$

since @ $t=0$ $x=1$ it must be the +

$x = \sqrt{8-7e^{-2t}}$

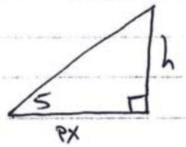
ii) as $t \to \infty$ $e^{-2t} \to 0$

$x \to \sqrt{8 - 7 \times 0}$

$= \sqrt{8} = 2\sqrt{2}$

iii) the particle is travelling right towards $x = 2\sqrt{2}$ with the acceleration left (−ve).

d) i)



$$\tan 5 = \frac{h}{PX}$$
$$PX = \frac{h}{\tan 5}$$
$$= h\cot 5$$

ii)
$$\tan 13 = \frac{h}{PY}$$
$$PY = \frac{h}{\tan 13}$$
$$= h\cot 13$$

$\angle YPX = 107^\circ$ & $XY = 8000\,m$

$$8000^2 = h^2\cot^2 5 + h^2\cot^2 13 - 2\times h\cot 5\times h\cot 13\times\cos 107$$
$$= h^2(\tan^2 85 + \tan^2 77 - 2\tan 85\tan 77\cos(107))$$
$$h = \frac{8000}{\sqrt{\tan^2 85 + \tan^2 77 - 2\tan 5\tan 77\cos 107}}$$
$$= 278.20\ldots$$
$$= 278\,m$$

14) a) i) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$

ii) integrating both sides

$$C + \frac{(1+x)^{n+1}}{(n+1)\times 1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \ldots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$$

sub $x = 0$

$$C + \frac{1}{n+1} = 0$$
$$C = -\frac{1}{n+1}$$
$$\frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \ldots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$$

sub $x = 1$

$$\frac{2^{n+1}-1}{n+1} = \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \ldots + \frac{1}{n+1}\binom{n}{n}$$

b) i)
$$x = 6\sin^2\left(4t+\frac{\pi}{3}\right)$$
$$\dot{x} = 12\sin\left(4t+\frac{\pi}{3}\right)\times\cos\left(4t+\frac{\pi}{3}\right)\times 4$$
$$= 48\sin\left(4t+\frac{\pi}{3}\right)\cos\left(4t+\frac{\pi}{3}\right)$$
$$\ddot{x} = 48\sin\left(4t+\frac{\pi}{3}\right)\times -\sin\left(4t+\frac{\pi}{3}\right)\times 4 + 48\cos\left(4t+\frac{\pi}{3}\right)\times\cos\left(4t+\frac{\pi}{3}\right),$$
$$= 192\cos^2\left(4t+\frac{\pi}{3}\right) - 192\sin^2\left(4t+\frac{\pi}{3}\right)$$
$$= 192\left(1 - 2\sin^2\left(4t+\frac{\pi}{3}\right)\right)$$
$$= 64\left(3 - 6\sin^2 4t+\frac{\pi}{3}\right)$$
$$= 64(3-x)$$

ii)
$$\ddot{x} = -64(x-3)$$

$\therefore$ the particle is in SHM about $x = 3$
since acceleration is proportional to distance from 3
and in the opposite direction.

iii) $x = 6\sin^2\left(4t + \frac{\pi}{3}\right)$

$= 3 \times 2\sin^2\left(4t + \frac{\pi}{3}\right)$

$= 3\left(1 - \cos\left(8t + \frac{2\pi}{3}\right)\right)$

$= 3 - 3\cos\left(8t + \frac{2\pi}{3}\right)$

$\therefore$ amplitude is 3

$\cos 2A = 1 - 2\sin^2 A$

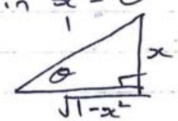
$2\sin^2 A = 1 - \cos 2A$

iv) $T = \frac{2\pi}{8}$

$= \frac{\pi}{4}$

c) i) $\sin(2\sin^{-1}x)$

let $\sin^{-1}x = \theta$



$= \sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2x\sqrt{1-x^2}$

ii) $\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta(1 - 2\sin^2\theta)$

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

iii) $\int \sin(2\sin^{-1}x)\,dx$

$\theta = \sin^{-1}x$

$\frac{d\theta}{dx} = \frac{1}{\sqrt{1-x^2}}$

$x = \sin\theta$

$= \int 2x\sqrt{1-x^2}\,dx$

$= \int \frac{2x(1-x^2)}{\sqrt{1-x^2}}\,dx$

$= \int 2\sin\theta(1-\sin^2\theta) \times d\theta$

$= \int 2\sin\theta - 2\sin^3\theta\,d\theta$

$= \frac{1}{2}\int 4\sin\theta - 4\sin^3\theta\,d\theta$

$= \frac{1}{2}\int 3\sin\theta - 4\sin^3\theta + \sin\theta\,d\theta$

$= \frac{1}{2}\int \sin 3\theta + \sin\theta\,d\theta$

iii) Alternate solution

$$x = \sin\theta$$
$$dx = \cos\theta \, d\theta$$

$$\int \sin 2\theta \times \cos\theta \, d\theta = \int 2\sin\theta\cos^2\theta \, d\theta$$
$$= \int 2\sin\theta(1-\sin^2\theta)\, d\theta$$
$$= \frac{1}{2}\int 4\sin\theta - 4\sin^3\theta \, d\theta$$
$$= \frac{1}{2}\int 3\sin\theta - 4\sin^3\theta + \sin\theta \, d\theta$$
$$= \frac{1}{2}\int \sin 3\theta + \sin\theta \, d\theta$$

Alt soln. $\int \sin(2\sin^{-1}x)\,dx = -\int -2x\sqrt{1-x^2}\,dx$

$$= -\frac{2}{3}(1-x^2)^{3/2} + C$$
$$= -\frac{2}{3}(1-\sin^2\theta)^{3/2} + C$$
$$= -\frac{2}{3}(\cos^2\theta)^{3/2}$$
$$= -\frac{2}{3}\cos^3\theta$$

diff

$$= -2\cos^2\theta \times -\sin\theta$$
$$= 2\sin\theta\cos^2\theta$$