Gosford High School

2018 HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics Extension 1

- **General Instructions**
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 10

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

Attempt Questions 1 – 10.

Allow approximately 15 minutes for this section.

Answer on the multiple choice answer sheet provided.

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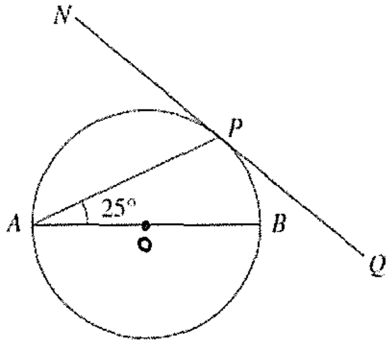
1. AB is a diameter of a circle and NQ is tangent to the circle at P, as shown.



Given that $\angle PAB = 25°$, what is the magnitude of $\angle NPA$?

(A) 25°

(B) 35°

(C) 55°

(D) 65°

2. When the polynomial $P(x) = 3x^3 - 2x^2 + cx - 5$ is divided by $(x + 1)$, the remainder is -14. What is the value of c?

(A) -10

(B) -8

(C) -4

(D) 4

3. Which expression is equivalent to $(sin2x + cos2x)^2$?

(A) 1

(B) $1 + sin2x$

(C) $1 + \frac{1}{2}sin4x$

(D) $1 + sin4x$

4. The acute angle between the lines $y = 3x$ and $y = mx$ is equal to $\frac{\pi}{4}$.
What is the value of m?

(A) -2 or $-\frac{1}{2}$

(B) -2 or $\frac{1}{2}$

(C) 2 or $-\frac{1}{2}$

(D) 2 or $\frac{1}{2}$

5. Which expression is equal to $\int sin^2 6x dx$?

(A) $\frac{x}{2} + \frac{1}{24}sin2x + c$

(B) $\frac{x}{2} - \frac{1}{24}sin12x + c$

(C) $\frac{1}{18}sin^3 6x + c$

(D) $-\frac{1}{18}cos^3 6x + c$

6. What is the general solution of the equation $tan4\theta = -\frac{1}{\sqrt{3}}$?

(A) $\theta = \frac{n\pi}{4} - \frac{\pi}{24}$

(B) $\theta = \frac{n\pi}{4} + \frac{\pi}{6}$

(C) $\theta = \frac{n\pi}{4} - \frac{\pi}{6}$

(D) $\theta = \frac{n\pi}{4} + \frac{\pi}{24}$

7. The displacement, x, of a particle at time t is given by $x = 3sin2t + 4cos2t$. What is the maximum velocity of the particle?

(A) 2

(B) 5

(C) 10

(D) 14

8. In how many ways can a committee of 2 men and 3 women be selected from a group of 6 men and 8 women?

(A) ${}^6P_2 \times {}^8P_3$

(B) ${}^6C_2 \times {}^8C_2$

(C) ${}^6P_3 \times {}^8C_2$

(D) ${}^6C_2 \times {}^8C_3$

9. Which expression represents the derivative of $\cos^{-1}(\frac{2}{x})$, $x > 0$?

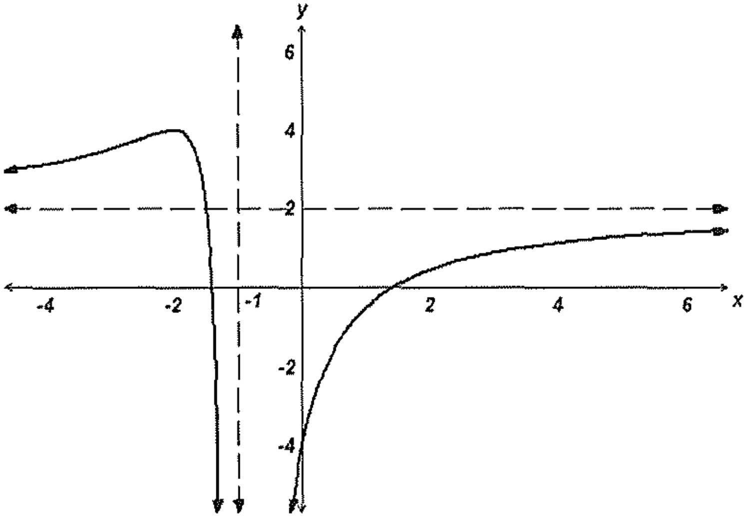
(A) $\frac{2}{\sqrt{x^2-4}}$

(B) $-\frac{2}{\sqrt{x^2-4}}$

(C) $\frac{2}{x\sqrt{x^2-4}}$

(D) $-\frac{2}{x\sqrt{x^2-4}}$

10. The graph shown below has equation of the form $y = \frac{ax^2-b}{(x+c)^2}$



The values of a, b and c are?

(A) $a = 2, b = 4, c = -1$

(B) $a = 2, b = 4, c = 1$

(C) $a = \frac{1}{2}, b = 4, c = 1$

(D) $a = 2, b = -4, c = 1$

Section II

60 marks.

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a separate writing booklet. Additional writing booklets are available.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Start a new writing booklet** **Marks**

a) State the domain and range of : **2**

$y = 2\cos^{-1} x - 1$

b) Find **2**

$\lim_{x \to 0} \frac{sinx + sin2x}{2x}$

c) Solve **2**

$\frac{4x-1}{x+2} \geq 1$

d)

(i) Show that the equation $xlog_e x - 1 = 0$ has a root, α, between 1.7 and 1.8. **2**

(ii) Use one application of Newton's method with an initial approximation of $\alpha_0 = 1.75$ to find the next approximation of this root.
Give the answer correct to 2 decimal places. **2**

e) The point $P(10,13)$ divides the interval joining $A(x, y)$ and $B(7,7)$ externally in the ratio 8: 3.
What are the coordinates of A? **2**

f) For the cubic equation $4x^3 - 6x + 10 = 0$ with roots α, β and γ, find the value of:

(i) $\alpha^2 + \beta^2 + \gamma^2$ **1**

(ii) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\alpha^2\gamma^2} + \frac{1}{\beta^2\gamma^2}$ **2**

Question 12 (15 marks) **Start a new writing booklet.**

a) Given the function:

$$f(x) = x - \frac{1}{2}x^2$$

(i) If the domain of $f(x)$ is restricted to $x \leq 1$, show that the inverse function, $f^{-1}(x)$, is given by: **3**

$$f^{-1}(x) = 1 - \sqrt{1 - 2x}$$

(ii) State the domain of this inverse function **1**

(iii) Draw a neat sketch of $f^{-1}(x)$ **2**

b) An oven which had been heated to 180° C was switched off when the baking was finished at 11.30 am. The oven was in a kitchen which was kept at a constant temperature of 22° C. **3**

After t minutes, the temperature, $T°\ C$, of the oven is given by

$$T = A + Be^{-kt}$$

After 10 minutes, the oven's temperature has dropped to 115° C.
At what time, to the nearest minute, will the oven's temperature drop to 23° C?

c) Use the substitution $u = x^2 + 1$ to evaluate **3**

$$\int_0^2 \frac{x}{(x^2 + 1)^3}\, dx$$

d) The volume of water in a tidal pool is given by **3**

$$V = 2cos\frac{3\pi}{x}$$

Where x is the depth of water in the pool in metres.

Find the exact rate at which the depth of the pool will be increasing when the volume of water is increasing at $12m^3/h$ and the depth is $1.2\ m$.

Question 13 (15 marks) Start a new writing booklet.

a) Show that **2**

$$cos\left(2sin^{-1}\left(\frac{3}{4}\right)\right)$$

can be written in the form $\frac{a}{b}$, where a and b are integers.

b) The depth of water, y metres, in a tidal creek is given by

$$4\frac{d^2y}{dt^2} = 5 - y$$

With the time, t , being measured in hours.

(i) Prove that the vertical motion of the water is simple harmonic and find the centre of motion. **2**

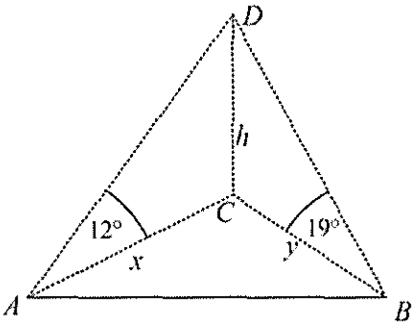
(ii) What is the period of the motion? **1**

(iii) Given that low tide is at 1 m and high tide is at 9 m, and that $y = b - acosnt$ is a solution to the equation $4\frac{d^2y}{dt^2} = 5 - y$, write down the values of a, b and n. **1**

(iv) If the low tide one day is at 1 pm, when is the next time that a boat requiring 3 m of water can enter the creek? Give your answer correct to the nearest minute. **2**

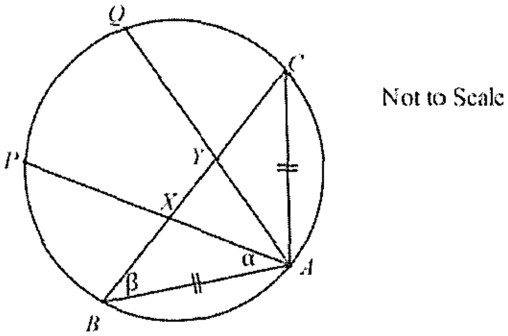
c) Jack is running due east along a path. At a point A along the path he notices a tower in the distance on a bearing of 048° with an angle of elevation of 12°. **3**

At point B, 170 m further along the path, the tower is now on a bearing of 352° with an angle of elevation of 19°.



Find the height of the tower to the nearest metre.

d) In the circle below, $AB = AC$. Let $\angle PAB = \alpha$ and $\angle ABC = \beta$.



(i) Copy the diagram into your answer booklet and give a reason why $\angle PQB = \alpha$. **1**

(ii) Prove $\angle AQB = \beta$. **1**

(iii) Prove that $XYQP$ is a cyclic quadrilateral. **2**

Question 14 (15 marks) Start a new writing booklet.

a) Use mathematical induction to prove that for integers $n \geq 1$, **3**

$$1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{1}{2}\right)^3 + \dots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}$$

b) Consider the expansion of $(1+x)^{n-1}$ for integers $n > 2$.

Show that **2**

$$n\binom{n-1}{1} + n\binom{n-1}{2} + \dots + n\binom{n-1}{n-2} = n(2^{n-1} - 2)$$

c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$, where $a > 0$ and $pq = 1$, are two points on the parabola with equation $x^2 = 4ay$. M is the midpoint of PQ.

(i) Show that as P and Q move on the parabola, the locus of M lies on the parabola $x^2 = 2a(y + a)$ **2**

(ii) Given that $\left|p + \frac{1}{p}\right| \geq 2$ for $p \neq 0$, find the domain and range of the locus of M. **2**

d) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} and acceleration a ms^{-2} given by $a = e^{-x} - e^{-2x}$.
The particle starts $log_e 2$ metres to the right of O moving towards O with speed $\frac{1}{2}$ ms^{-1}.

(i) Show that $v = e^{-x} - 1$ **3**

(ii) Show that $t = -\int \frac{e^x}{e^x - 1} dx$ **1**

(iii) Find x in terms of t and hence find the limiting position of the particle. **2**

Ext 1 Trial 2018

Multiple choice

1) $\angle PAB = 25°$

$\angle APB = 90°$

$\angle PBA = \angle NPA = 180 - 90 - 25$

$= 65$ Ⓓ

2) $P(x) = 3x^3 - 2x^2 + cx - 5$

$P(-1) = -14$

$\therefore -14 = -3 - 2 - c - 5$

$\therefore c = 4$ Ⓓ

3) $(\sin 2x + \cos 2x)^2$

$= \sin^2 2x + 2\sin 2x \cos 2x + \cos^2 2x$

$= 1 + \sin 4x$ Ⓓ

4) $y = 3x \quad y = mx$

$\tan\frac{\pi}{4} = \left|\frac{3-m}{1+3m}\right|$

$\frac{3-m}{1+3m} = \pm 1$

$\therefore 3 - m = 1 + 3m \qquad 3 - m = -1 - 3m$

$4m = 2 \qquad 2m = -4$

$m = \frac{1}{2} \qquad m = -2$

Ⓑ

5) $\int \sin^2 6x \, dx$

$\cos 12x = 1 - 2\sin^2 6x$

$I = \frac{1}{2}\int 1 - \cos 12x \, dx$

$= \frac{1}{2}\left[x - \frac{\sin 12x}{12}\right] + c$ Ⓑ

$= \frac{x}{2} - \frac{\sin 12x}{24} + c$

6) $\tan 4\theta = -\frac{1}{\sqrt{3}}$

$\therefore 4\theta = \frac{5\pi}{6} + k\pi = n\pi + \frac{5\pi}{6}$

$\theta = \frac{5\pi + 6k\pi}{24}$

$= \frac{n\pi}{4} - \frac{\pi}{24}$ Ⓐ

7) $x = 3\sin 2t + 4\cos 2t$

max v at $x = 0$

$R = \sqrt{9+16} = 5$

$\therefore 5\sin(2t + \theta) = 0$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{3} \quad \therefore \theta = 0.927$

$\therefore \sin(2t + \theta) = 0, \pi, 2\pi$

$\therefore 2t + 0.927 = 0, \pi, 2\pi$

$t \doteqdot 1.1$

$\therefore v = 6\cos 2t - 8\sin 2t$

$\doteqdot 10$ Ⓒ

8) 6m 8w

2m 3w

choices $= {}^6C_2 \times {}^8C_3$ Ⓓ

9) $\frac{d}{dx}\cos^{-1}\left(\frac{2}{x}\right)$

$y = \cos^{-1}\frac{2}{x} \qquad u = \frac{2}{x}$

$y = \cos^{-1} u \qquad \frac{du}{dx} = -2x^{-}$

$\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$

$\therefore \frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{4}{x^2}}} \times \frac{-2}{x^2}$

$= \frac{-x}{\sqrt{x^2-4}} \times \frac{-2}{x^2}$

$= \frac{2}{x\sqrt{x^2-4}}$ Ⓒ

10) $y=\frac{ax^2-b}{(x+c)^2}$

$\underline{\underline{c=+1}}$

$$\lim_{x\to\infty} \frac{\frac{ax^2}{x^2}-\frac{b}{x^2}}{\frac{x^2}{x^2}+\frac{2xc}{x^2}+\frac{c}{x^2}}$$

$= a$

$\therefore \underline{\underline{a=2}}$

at $x=0$

$y=\frac{-b}{c^2}$

$-4=\frac{-b}{1} \quad \therefore \underline{\underline{b=4}}$ Ⓑ

Q11

a) $y = 2\cos^{-1} x - 1$

$\frac{y+1}{2} = \cos^{-1} x$

d: $-1 \le x \le 1$

r: $0 \le \frac{y+1}{2} \le \pi$

$0 \le y+1 \le 2\pi$

$-1 \le y \le 2\pi - 1$

b) $\lim_{x \to 0} \frac{\sin x + \sin 2x}{2x}$

$= \lim_{x \to 0} \left(\frac{1}{2} \frac{\sin x}{x} + \frac{\sin 2x}{2x} \right)$

$= \frac{1}{2}(1) + (1)$

$= \frac{3}{2}$

c) $\frac{4x-1}{x+2} \ge 1$

c.v at

$\boxed{x = -2}$ $x \neq -2$

$4x - 1 = x + 2$

$3x = 3$

$x = 1$

✓ | ✗ | ✓
-2 1

$x = -3$ $\frac{-13}{-1} \ge 1$

$x = 0$ $\frac{-1}{2} \ge 1$

$x = 2$ $\frac{7}{4} \ge 1$

$x < -2, \ x \ge 1$

d) $x \log_e x - 1 = 0$

i) $f(x) = x \log_e x - 1$

$f(1.7) = 1.7 \ln 1.7 - 1$

$\doteqdot -0.09820$

$f(1.8) = 1.8 \ln 1.8 - 1$

$\doteqdot 0.05870$

$\therefore$ root lies between $x = 1.7$ and $x = 1.8$

ii) $d_0 = 1.75$

$d = 1.75 - \frac{f(1.75)}{f'(1.75)}$

$f(1.75) \doteqdot -0.02067$

$f'(x) = \log_e x + x \times \frac{1}{x}$

$= \log_e x + 1$

$f'(1.75) = 1.5596$

$\therefore d = 1.75 + \frac{0.02067}{1.5596}$

$\doteqdot 1.76$

e) $P(10,13)$

$A(x,y)$ $B(7,7)$

$8 : -3$

$10 = \frac{-3x+56}{5}$ $13 = \frac{-3y+56}{5}$

$50 = -3x + 56$ $65 = -3y + 56$

$x = 2$ $y = -3$

$A(2,-3)$

f) $4x^3 - 6x + 10 = 0$

roots α, β, γ

$\therefore \Sigma\alpha = 0$

$\Sigma\alpha\beta = \frac{-3}{2}$

$\alpha\beta\gamma = \frac{-5}{2}$

i) $\alpha^2+\beta^2+\gamma^2$

$(\alpha+\beta+\gamma)^2 = \alpha^2+\alpha\beta+\alpha\gamma$

$+\alpha\beta+\beta^2+\beta\gamma+\alpha\gamma+\beta\gamma+\gamma^2$

$= \Sigma\alpha^2 + 2\Sigma\alpha\beta$

$\therefore\ \alpha^2+\beta^2+\gamma^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta$

$= 0^2 - 2\left(-\frac{3}{2}\right)$

$= 3$

ii) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\alpha^2\gamma^2} + \frac{1}{\beta^2\gamma^2}$

$= \frac{\gamma^2+\beta^2+\alpha^2}{\alpha^2\beta^2\gamma^2}$

$= \frac{3}{\left(\frac{-5}{2}\right)^2}$

$= \frac{12}{25}$

Q12

a) $f(x) = x - \frac{1}{2}x^2$

i) $f^{-1}: \quad x = y - \frac{1}{2}y^2$

$2x = 2y - y^2$

$(y-1)^2 = 1 - 2x$

$y - 1 = \pm\sqrt{1-2x}$

$y = 1 \pm \sqrt{1-2x}$

now $y \leq 1$ (restricted)

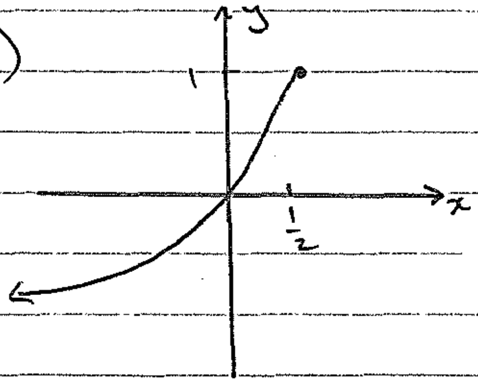
$\therefore f^{-1}(x) = 1 - \sqrt{1-2x}$

ii) $1 - 2x \geq 0$

$2x \leq 1$

$x \leq \frac{1}{2}$

iii)



b) $T = A + Be^{-kt}$

at $t=0 \quad T = 180 \quad A = 22, B = 158$

$t = 10 \quad T = 115$

find t at $T = 23$

$115 = 22 + 158e^{-10k}$

$k \doteqdot 0.053$

$23 = 22 + 158e^{-0.053 \times t}$

$t = 95.52$ mins

time = 1:06pm

c) $\displaystyle\int_0^2 \frac{x}{(x^2+1)^3}\,dx$

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$x = 2 \quad u = 5$

$x = 0 \quad u = 1$

$I = \displaystyle\int_1^5 \frac{1}{u^3} \cdot \frac{du}{2}$

$= \frac{1}{2}\left[\frac{u^{-2}}{-2}\right]_1^5$

$= -\frac{1}{4}\left(\frac{1}{25} - 1\right)$

$= \frac{6}{25}$

d) $V = 2\cos\frac{3\pi}{x} \quad = 2\cos(3\pi x^{-1}$

find $\frac{dx}{dt}$ at $\frac{dV}{dt} = 12$

and $x = 1.2$

$\frac{dx}{dt} = \frac{dx}{dV} \cdot \frac{dV}{dt}$

$\frac{dV}{dx} = -2\sin\left(\frac{3\pi}{x}\right) \times -3\pi x^{-2}$

$= \frac{6\pi}{x^2}\sin\left(\frac{3\pi}{x}\right)$

$\therefore \frac{dx}{dt} = \frac{x^2}{6\pi\sin\left(\frac{3\pi}{x}\right)} \cdot 12$

$= \frac{2x^2}{\pi\sin\left(\frac{3\pi}{x}\right)}$

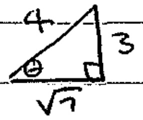
at $x = 1.2$

$\frac{dx}{dt} = \frac{2\left(\frac{6}{5}\right)^2}{\pi\sin\left(\frac{3\pi \times 5}{6}\right)}$

$= \frac{72}{25\pi\sin\left(\frac{15\pi}{6}\right)} = \frac{72}{25\pi}$

Q13

a) $\cos\left(2\sin^{-1}\frac{3}{4}\right)$



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \frac{7}{16} - \frac{9}{16}$

$= -\frac{1}{8}$

b) $4\frac{d^2y}{dt^2} = 5-y$

i) $y'' = \frac{5-y}{4}$

$= -\frac{1}{4}(y-5)$ ∴ in SHM

centre of motion $=5$

ii) $T = \frac{2\pi}{\frac{1}{2}}$

$= 4\pi$

iii) $y = b - a\cos nt$

ampl $= \frac{8m}{2} = 4m$

∴ $\underline{\underline{a=4}}$

$\underline{\underline{n=\frac{1}{2}}}$

at $y=1$ $\cos nt=1$

∴ $\underline{\underline{b=5}}$

iv) $3 = 5-4\cos\frac{t}{2}$

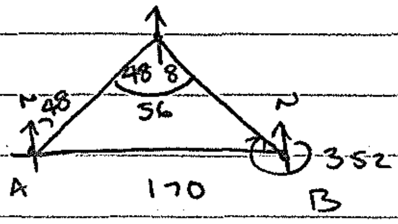
$\frac{1}{2} = \cos\frac{t}{2}$

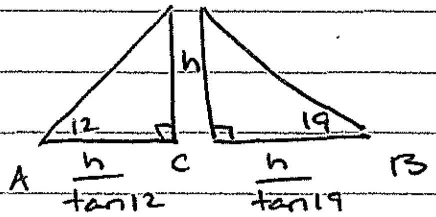
$\frac{t}{2} = \frac{\pi}{3}$

$t = \frac{2\pi}{3}$ (2h 6min)

∴ time is 3:06pm

c)





$170^2 = \frac{h^2}{\tan^2 12} + \frac{h^2}{\tan^2 19} - \frac{2h^2\cos}{\tan12\tan1}$

$= h^2(\cot^2 12 + \cot^2 19 - 2\cos 5$
$\cot 12\, c$

$= h^2(\tan^2 78 + \tan^2 71 - 2\cos 56$
$\times \tan 71 \tan 7$

$h \doteqdot 43m$

d) i) $\angle PQB = \angle PAB = \alpha$
(= ∠'s at circumf from same arc PB)

ii) AB = AC (given)
∴ $\angle ABC = \angle ACB = \beta$
(= ∠'s opposite = sides △)
$\angle ACB = \angle AQB = \beta$
(= ∠'s at circumf from same arc AB)

iii) $\angle PQY = \angle PQB + \angle YQB$

$= \alpha + \beta$ (from ii)

$\angle AXY = \angle ABX + \angle BAX$

$= \alpha + \beta$

(exterior $\angle$ of $\triangle$ = sum 2 opposite interior $\angle$'s)

$\therefore \angle AXY = \angle PQY$

$\therefore$ XYQP is cyclic quad

(exterior $\angle$ = opp interior $\angle$)

Q14

a) $1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 \ldots + n\left(\frac{1}{2}\right)^{n-1}$

$= 4 - \frac{n+2}{2^{n-1}}$

Prove for $n \geq 1$

① Prove $n=1$

LHS $= 1$ RHS $= 4 - \frac{1+2}{2^{1-1}}$

$= 4 - \frac{3}{1}$

$= 1$

$\therefore$ LHS = RHS

$\therefore$ true $n=1$

② Assume true $n=k$, $k \geq 1$ where k is a positive integer.

ie

$1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 \ldots + k\left(\frac{1}{2}\right)^{k-1}$

$= 4 - \frac{k+2}{2^{k-1}}$

③ Prove $n=k+1$

RTP

$1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 \ldots + k\left(\frac{1}{2}\right)^{k-1}$

$+ (k+1)\left(\frac{1}{2}\right)^{k+1-1}$

$= 4 - \frac{(k+1)+2}{2^{k+1-1}}$

LHS $= 1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 \ldots + k\left(\frac{1}{2}\right)^{k-1}$

$+ (k+1)\left(\frac{1}{2}\right)^{k+1-1}$

$= 4 - \frac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$

$= 4 - \frac{k+2}{2^{k-1}} + \frac{k+1}{2^k}$

$= 4 + \frac{-2(k+2) + k+1}{2^k}$

$= 4 + \frac{-2k - 4 + k + 1}{2^k}$

$= 4 + \frac{-k-3}{2^k}$

$= 4 - \frac{k+3}{2^k}$

$= 4 - \frac{(k+1)+2}{2^{k+1-1}}$

$\therefore$ true $n=k+1$

b) $(1+x)^{n-1}$ $n > 2$

$= \binom{n-1}{0} + \binom{n-1}{1}x + \binom{n-2}{2}x^2 \ldots + \binom{n-1}{n-1}x$

let $x=1$

$\therefore 2^{n-1}$

$= \binom{n-1}{0} + \binom{n-1}{1} + \binom{n-2}{2} \ldots + \binom{n-1}{n-1}$

$\therefore 2^{n-1} - 2 = \binom{n-1}{1} + \binom{n-2}{2} \ldots + \binom{n-1}{n-2}$

$(\times n)$

$n(2^{n-1} - 2) = n\binom{n-1}{1} + \binom{n-2}{2}n \ldots + \binom{n-}{n-}$

c) $P(2ap, ap^2)$ $Q(2aq, aq^2)$

i) $M = \left(\frac{2a(p+q)}{2}, \frac{a(p^2+q^2)}{2}\right)$

$\therefore x = a(p+q)$ $y = \frac{a(p^2+q^2)}{2}$

$p+q = \frac{x}{a}$

$\frac{2y}{a} = (p+q)^2 - 2($

$\therefore \frac{2y}{a} = \left(\frac{x}{a}\right)^2 - 2(1)$

$\frac{2y}{a} = \frac{x^2}{a^2} - 2$

$\therefore x^2 = 2ay + 2a^2$

$x^2 = 2a(y+a)$

ii) $\left|p + \frac{1}{p}\right| \geq 2$

$pq = 1$

$\therefore q = \frac{1}{p}$

$\therefore |p+q| \geq 2$

& $x = 2a(p+q)$

$\therefore \frac{x}{2a} = p+q$

$\left|\frac{x}{2a}\right| \geq 2$

$|x| \geq 4a$ $\circledast$

d: $x \leq -4a$, $x \geq 4a$

$y = \frac{a}{2}\{(p+q)^2 - 2pq\}$

$= \frac{a}{2}\{(p+q)^2 - 2\}$

$\frac{2y}{a} + 2 = (p+q)^2$

$\therefore (p+q)^2 \geq 4$

$\frac{2y}{a} + 2 \geq 4$

$\frac{2y}{a} \geq 2$

r: $y \geq a$

Since $y = \frac{a}{2}\left[(p+q)^2 - 2pq\right]$

$\therefore y \geq \frac{a}{2}[4-2]$

$y \geq a.$

d) $a = e^{-x} - e^{-2x}$

at $t=0$ $x = \log_e 2$, $v = -\frac{1}{2}$

i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = e^{-x} - e^{-2x}$

$\therefore \frac{1}{2}v^2 = -e^{-x} + \frac{e^{-2x}}{2} + c_1$

$v^2 = -2e^{-x} + e^{-2x} + c$

at $x = \ln 2$ $v = -\frac{1}{2}$

$\therefore \frac{1}{4} = -2e^{-\ln 2} + e^{-2\ln 2} + c$

$\frac{1}{4} = -2\left(\frac{1}{2}\right) + \frac{1}{4} + c$

$\therefore c = 1$

$\therefore v^2 = -2e^{-x} + e^{-2x} + 1$

$= (e^{-x})^2 - 2(e^{-x}) + 1$

$= (e^{-x} - 1)^2$

$\therefore v = e^{-x} - 1$

ii) $\therefore \frac{dx}{dt} = e^{-x} - 1$

$= \frac{1}{e^x} - 1$

$= \frac{1 - e^x}{e^x}$

$\therefore \frac{dt}{dx} = \frac{e^x}{1 - e^x}$

$\therefore t = \int \frac{e^x}{1-e^x}\,dx$

$= -\int \frac{e^x}{e^x - 1}\,dx$

iii) $t = -\ln(e^x - 1) + c$

at $t = 0$ $x = \ln 2$

$\therefore 0 = -\ln(e^{\ln 2} - 1) + c$

$= -\ln(2 - 1) + c$

$0 = c$

$\therefore t = -\ln(e^x - 1)$

$\therefore e^{-t} = e^x - 1$

$e^{-t} + 1 = e^x$

$x = \ln(e^{-t} + 1)$

$= \ln\left(\frac{1}{e^t} + 1\right)$

as $t \to \infty$ $\frac{1}{e^t} \to 0$

$\ln\left(\frac{1}{e^t} + 1\right) \to \ln 1$

$\to 0$

$\therefore x \to 0$