Gosford High School

2019 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I - 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 11)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

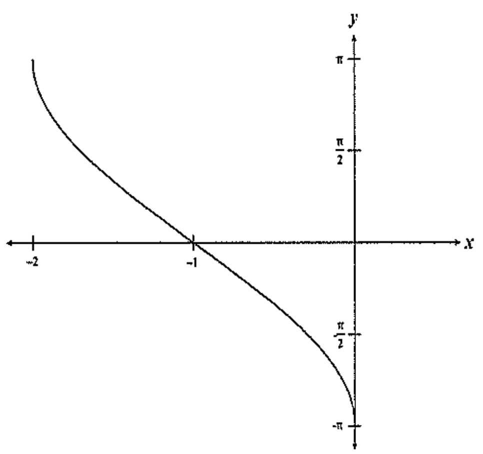
Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

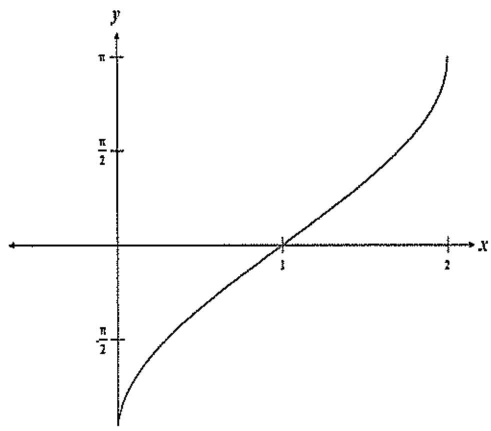
Use the multiple-choice answer sheet for Questions 1 – 10.

1 Which of the following represents the graph of $y = 2\sin^{-1}(1-x)$?

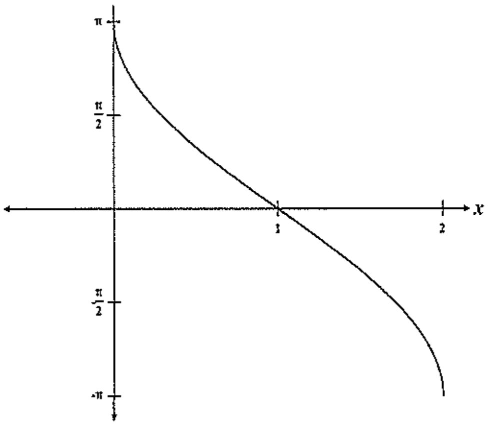
(A)



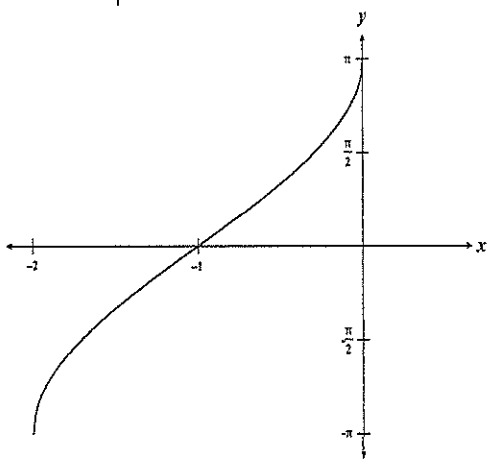
(B)



(C)



(D)



2. Find the derivative of $x^3 \ln x$ with respect to *x*.

(A) $3x^2 \ln x$

(B) $x^2 (3 \ln x + 1)$

(C) $3x$

(D) $x^2 (3 \ln x + 2x)$

3. Given $A=(2,-5)$ and $B=(-4,1)$, the co-ordinates of the point which divides AB internally in the ratio of $2:4$ are:

(A) $(\frac{-14}{3}, \frac{5}{3})$

(B) $(\frac{-14}{3}, -3)$

(C) $(0, -3)$

(D) $(-8, 11)$

4. Find the Cartesian equation of the curve defined by the parametric equations:

$x = \sin\theta$

$y = \cos^2\theta - 3$

(A) $y = -2 - x^2$

(B) $y = \sin^2 x - 3$

(C) $y = -3 + 3x^2$

(D) $y = \sin 2x + 3\cos^2 x$

5. The polynomial $P(x) = 8x^3 + ax^2 - 4x + 1$ has a factor of $2x + 1$. What is the value of a?

(A) −8

(B) 0

(C) 3

(D) 8

6. Given that $t = \tan\theta$ which of the following is equivalent to $\sec 2\theta$

(A) $\dfrac{1+4t^2}{1-4t^2}$

(B) $2\left(\dfrac{1+t^2}{1-t^2}\right)$

(C) $\dfrac{1+(\frac{t}{2})^2}{1-(\frac{t}{2})^2}$

(D) $\dfrac{1+t^2}{1-t^2}$

7. A particle is moving in simple harmonic motion and its displacement, x units, at time t seconds is given by the equation $x = A\cos(nt) + 2$

The period of the motion is 4π seconds and the particle is initially at rest, 12 units to the right of the origin. What are the values of A and n?

(A) $A = 10,\ n = \frac{1}{2}$.

(B) $A = 10,\ n = 2$.

(C) $A = 12,\ n = \frac{1}{2}$.

(D) $A = 12,\ n = 2$.

8. What is the derivative of $\tan^{-1}(2x-1)$?

(A) $\frac{2x+1}{4x^2-4x+2}$

(B) $\frac{2}{4x^2-4x+1}$

(C) $\frac{1}{2x^2-2x}$

(D) $\frac{1}{2x^2-2x+1}$

9. Which of the following gives a general solution of the equation $\cos 2x = \frac{1}{2}$?

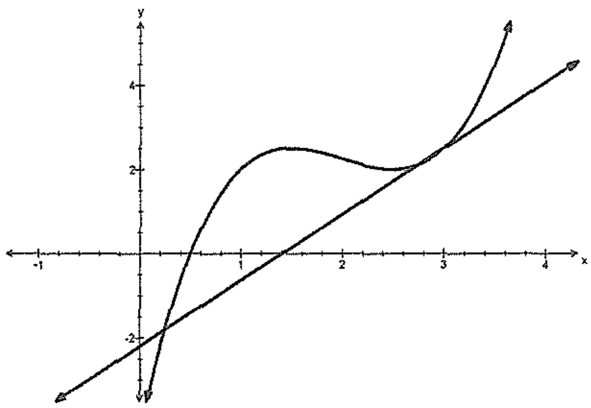
(A) $x = n\pi \pm \frac{\pi}{6}$ (where n is an integer)

(B) $x = n\pi \pm \frac{\pi}{3}$ (where n is an integer)

(C) $x = 2n\pi \pm \frac{\pi}{6}$ (where n is an integer)

(D) $x = 2n\pi \pm \frac{\pi}{3}$ (where n is an integer)

10.



The diagram shows the curve $y = f(x)$. The tangent to the curve at the point $x = 3$ cuts the x-axis at $x = \frac{7}{5}$. Which of the following is the value of $\frac{f(3)}{f'(3)}$?

(A) $-\frac{8}{5}$

(B) $-\frac{5}{8}$

(C) $\frac{5}{8}$

(D) $\frac{8}{5}$

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a new writing writing booklet.

(a) Find the exact value of $\int_0^{\pi} \cos^2 3x \, dx$ **2**

(b) Ten men and nine women (all of different ages) are called for jury selection for a trial. A jury of twelve people is to be chosen and the lawyers agree that there should be an equal number of women and men on the jury. How many juries containing the youngest woman and the youngest man can be formed? **2**

(c) (i) Show that the function

$f(x) = \log_e x - sinx + 1$ has a zero between 0.7 and 0.8 **2**

(ii) Hence use *halving-the-interval* method once to find the value of this zero ,correct to one decimal place. **2**

(d) Find, showing full working, $\lim_{x \to 0} \frac{x^2}{2 - 2\cos 2x}$ **2**

Question 11 continues on page 7

Question 11 (continued)

(e) (i) Find the point of intersection of $y = x^2$ and $y = \frac{1}{x}$ **1**

(ii) Find the acute angle in degrees to the nearest minute between these two curves at the point of intersection **1**

(f) Use the substitution $u = x - 3$ to evaluate $\int_3^4 x\sqrt{x-3}\,dx$ **3**

End of Question 11

Question 12 (15 marks) Use a new writing booklet.

(a) (i) Sketch the curve $y = 1 + \sin x$ for $0 \le x \le 2\pi$ **1**

(ii) Find the volume of the solid formed when the area bounded by the curve $y = 1 + \sin x$ and the x axis for $0 \le x \le 2\pi$ is rotated one revolution about the x axis. **2**

(b) (i) Show that $3\sin x \cos x = \frac{3}{2}\sin 2x$ **1**

(ii) Hence or otherwise find the exact value of $\int_0^{\frac{\pi}{2}} 9\sin^2 x\cos^2 x\, dx$ **2**

(c) Let $f(x) = x^2 - 6x + 2$

(i) By expressing $f(x)$ in the form $(x-h)^2 + k$ find the range of $f(x)$ **2**

(ii) The domain of $f(x)$ is restricted such that it has an inverse. The domain chosen contains $x = 5$. State this domain and find the equation of the inverse $f^{-1}(x)$ **2**

(iii) Find the point(s) of intersection of $y = f(x)$ and $y = f^{-1}(x)$ given that $f(x)$ maintains its restricted domain **2**

(d) Show that the constant term in the expansion $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9 = \frac{{}^9C_6}{6^3}$ **3**

End of Question 12

Question 13 (15 marks) Use a new writing booklet.

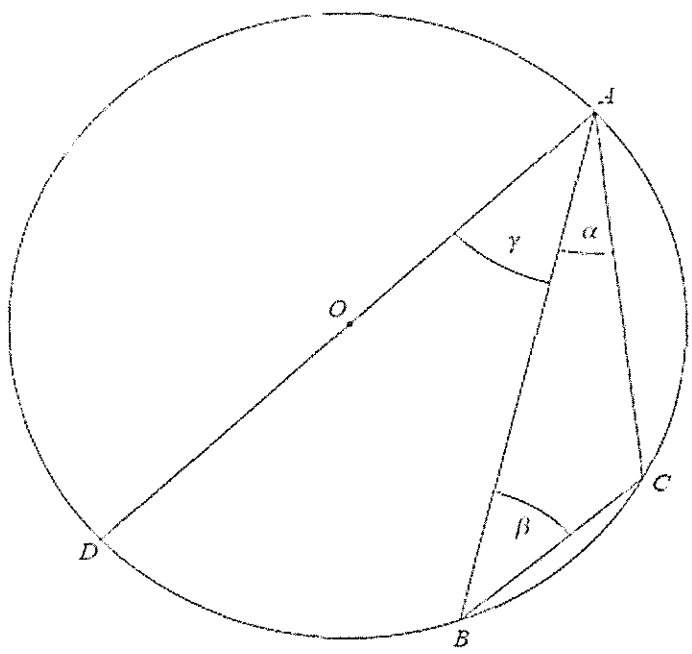
(a) By equating the coefficient of x^n on both sides of the identity 3

$$(1+x)^n(1+x)^n = (1+x)^{2n},$$

Show that $$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots \binom{n}{n-1}^2 + \binom{n}{n}^2 = \frac{(2n)!}{(n!)^2}$$

(b) Use the principal of Mathematical Induction to prove that $7^n - 3^n$ is a multiple of 4 for all positive integral values of n. 3

(c) The diagram shows points B and C on a semi-circle with centre O and diameter AD.



Given that $\angle BAC = \alpha$, $\angle ABC = \beta$ and $\angle OAB = \gamma$, find the value of $\alpha + \beta + \gamma$, giving reasons. 2

Question 13 continues on page 10

Question 13 continued

(d) Falling into cold water is particularly dangerous as the body loses body heat 25 times faster in cold water than in cold air. The normal body temperature is $37^{\circ}C$.

Shivering begins at an approximate body temperature of $36^{\circ}C$, and a person can fall into unconsciousness at $30^{\circ}C$

The body temperature of a person, $T^{\circ}C$ who has been in the water for *t* minutes can be modelled by the differential equation

$$\frac{dT}{dt} = k(T - T_w)$$ where T_w is the temperature of the water and k is a constant

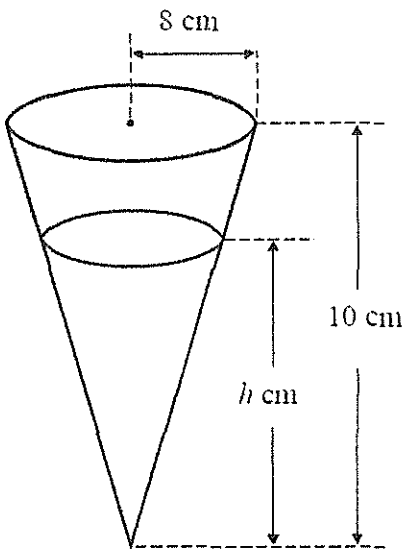
(i) Show that $T = T_w + Be^{-kt}$, where *B* is a constant, is a solution of this differential equation. **1**

(ii) If the water temperature is $2^{\circ}C$, a person is expected to start shivering after 5 minutes. **2**

Show that $T = 2 + 35e^{-0.0058\,t}$

(iii) Find how long a person can stay in the water before becoming unconscious. Give your answer in minutes, correct to 1 decimal place **2**

(e) **2**



The figure above shows an inverted conical cup with base radius 8 cm and height 10cm. Some water is poured into the cup at a constant rate of $\frac{2\pi}{5}\ cm^3$ per minute. Let the depth of the water be *h cm* at time *t minutes*.

Find the rate of change in the area of the water surface when *h=4 cm*

Question 14 (15 marks) Use a new writing booklet.

(a) The acceleration of a particle P is given by the equation

$$\ddot{x} = 18x^3 + 27x^2 + 9x$$

Initially $x = -2$ and the velocity, $v = -6$.

(i) Show that $v^2 = 9x^2(1 + x)^2$ **2**

(ii) Hence, or otherwise, show that **2**

$$\int \frac{1}{x(1+x)} dx = -3t + C, \text{ for some constant C}$$

(iii) Find the derivative of $\log_e(1 + \frac{1}{x})$ **1**

(iv) Using your result in part iii and the initial conditions, find x as a function of t. **2**

(b) (i) Show that $2\sin 2x - 3\cos 2x - 3\sin x + 3 = \sin x(6\sin x + 4\cos x - 3)$ **1**

(ii) Express $6\sin x + 4\cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **2**

(iii) Hence, solve $2\sin 2x - 3\cos 2x - 3\sin x + 3 = 0$ for $0 \leq x < \pi$ **2**
Answer in radians correct to 3 significant figures.

(c) It is given that $P(x) = (x-a)^3 + (x-b)^3$, where $a \neq b$ **3**
Prove that $P(x)$ has no stationary points

End of Paper

TRIAL SOLUTIONS (1)

1. C.

2. B

3. C

4. A $x^2 = \sin^2\theta$

$y = (1 - \sin^2\theta - 3)$

$= -2 - \sin^2\theta$

$= -2 - x^2$

5. A

6 D

7 A

8. D

9 A

10. D

9. $\theta = 2n\pi \pm \cos^{-1}\alpha$.

$\cos 2x = \frac{1}{2}$

$2x = 2n\pi \pm \frac{\pi}{3}$

$x = n\pi \pm \frac{\pi}{6}$

A.

6. D

$\sec 2\theta = \frac{1}{\cos 2\theta}$

$= \frac{1}{2\cos^2\theta - 1}$

$= \frac{1}{2\left(\frac{1}{1+t^2}\right) - 1}$

$= \frac{1+t^2}{1-t^2}$

7. A

8. $\frac{1}{(2x-1)^2+1} \cdot 2$

$= \frac{2}{4x^2-4x+2}$

$= \frac{2}{2(2x^2-2x+1)}$

$= \frac{1}{2x^2-2x+1}$

10. $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$\frac{7}{5} = 3 - \frac{f(3)}{f'(3)}$

$\therefore \frac{f(3)}{f'(3)} = 3 - \frac{7}{5}$

$= \frac{8}{5}$

D

QUESTION 11

a) $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\cos^2 3x = \frac{1+\cos 6x}{2}$

$\frac{1}{2}\int_0^{\pi} 1+\cos 6x \, dx = \frac{1}{2}\left[x + \frac{\sin 6x}{2}\right]_0^{\pi}$

$= \frac{\pi}{2}$

b) 10M 9W put youngest in

need 5 men, 5 w.

${}^9C_5 \times {}^8C_5 = 126 \times 56$

$= 7056$

c) $f(0{\cdot}7) = \ln 0{\cdot}7 - \sin 0{\cdot}7 + 1$

$= -00089$

< 0

$f(0{\cdot}8) = \ln(0{\cdot}8) - \sin 0{\cdot}8 + 1$

$= 0{\cdot}05950$

> 0

since $f(x)$ is continuous for $x > 0$ and there is a sign change between $x = 0{\cdot}7$ and $0{\cdot}8$ $f(x)$ has a zero here

ii) $f(0{\cdot}75) = \ln 0{\cdot}75 - \sin 0{\cdot}75 + 1$

$= 0{\cdot}030679$

> 0

hence a better value between $x = 0{\cdot}7$ and $0{\cdot}75$ to 1 dp zero at $x = 0{\cdot}7$

d) $\lim_{x\to 0} \frac{x^2}{2(1-\cos 2x)}$ (2)

$= \lim_{x\to 0} \frac{x^2}{2(2\sin^2 x)}$

$= \lim_{x\to 0} \frac{x^2}{4\sin^2 x}$

$= \frac{1}{4}$

e) $x^2 = \frac{1}{x}$

$x^3 = 1$

$x = 1 \quad y = 1$

ii) $y = x^2$

$\frac{dy}{dx} = 2x$

$m_1 = 2$

$y = x^{-1}$

$\frac{dy}{dx} = -x^{-2} = \frac{-1}{x^2}$

$m_2 = -1$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{2+1}{1-2}\right|$

$= 3$

$\theta = 71°34'$

f) $u = x - 3$ $\quad x = 4 \quad x = 3$

$\frac{du}{dx} = 1$ $\quad u = 1 \quad u = 0$

$\int_0^1 (u+3) \cdot u^{1/2} \, du$

$\int_0^1 u^{3/2} + 3u^{1/2} \, du$

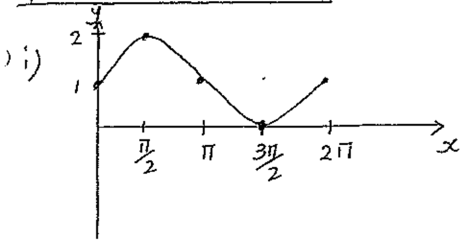
$\left[\frac{2}{5}u^{5/2} + 2u^{3/2}\right]_0^1$

$= \frac{2}{5} + 2$

$= 2^2/5$

QUESTION 12

a) i)



ii) $\pi\int_0^{2\pi} (1+\sin x)^2 \, dx$

$\pi\int_0^{2\pi} 1 + 2\sin x + \sin x^2 \, dx$

$= \pi\int_0^{2\pi} 1 + 2\sin x + \left(\frac{1-\cos 2x}{2}\right) dx$

$= \frac{\pi}{2}\int_0^{2\pi} 3 + 4\sin x - \cos 2x \, dx$

$= \frac{\pi}{2}\left[3x - 4\cos x - \frac{1}{2}\sin 2x\right]_0^{2\pi}$

$= \frac{\pi}{2}\left[(6\pi - 4) - (-4)\right]$

$= \frac{\pi}{2} \times 6\pi$

$= 3\pi^2$ units^3

b) (i) RHS $\frac{3}{2}\sin 2x = \frac{3}{2} \cdot 2\sin x \cos x$

$= 3\sin x \cos x$

$= LHS$

ii) $(3\sin x \cos x)^2 = 9\sin^2 x \cos^2 x$

$\therefore \int_0^{\frac{\pi}{2}} \left(\frac{3}{2}\sin^2 2x\right) dx$

$\frac{9}{4}\int_0^{\frac{\pi}{2}} \frac{1-\cos 4x}{2} \, dx$

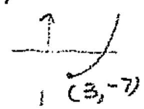
$\frac{9}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$

$= \frac{9}{8} \times \frac{\pi}{2} = \frac{9\pi}{16}$

c) $f(x) = x^2 - 6x + 2$

$= (x-3)^2 - 7$

range $y \geq -7$



ii)

Domain $x \geq 3$ of $f(x)$

inverse of $f(x)$

$y = (x-3)^2 - 7$

$x = (y-3)^2 - 7$

$x + 7 = (y-3)^2$

$\pm\sqrt{x+7} = y - 3$

$y = 3 \pm \sqrt{x+7}$

Range of inverse $y \geq 3$

so $y = 3 + \sqrt{x+7}$

solve with $y = x$.

iii) $y = x$ $\quad y = (x-3)^2 - 7$

$(x-3)^2 - 7 = x$

$x^2 - 7x + 2 = 0$

$x = \frac{7 \pm \sqrt{49-8}}{2}$

$x = \frac{7 \pm \sqrt{41}}{2}$ $\quad y = \frac{7 \pm \sqrt{41}}{2}$

but $x \geq 3$

$x = \frac{7+\sqrt{41}}{2}$ $\quad y = \frac{7+\sqrt{41}}{2}$

Q 12

d) $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$

${}^nC_k \, a^{n-k} \cdot b^k = T_{k+1}$

$= {}^9C_k \left(\frac{3x^2}{2}\right)^{9-k} \cdot \left(-\frac{1}{3x}\right)^k$

$= {}^9C_k \, \frac{3}{2}^{9-k} \cdot \left(\frac{-1}{3}\right)^k \cdot x^{-k} \cdot x^{18-2k}$

constant term when

$x^{18-3k} = x^0$

$k = 6$

T_7 is the constant term

$\therefore {}^9C_6 \cdot \left(\frac{3}{2}\right)^3 \cdot \left(\frac{-1}{3}\right)^6$

$= {}^9C_6 \cdot \frac{3^3}{2^3} \times \frac{1}{3^6}$

$= {}^9C_6 \, \frac{1}{2^3 \times 3^3} = \frac{{}^9C_6}{6^3}$

QUESTION 13.

$(1+x)^n$

$({}^nC_0 + {}^nC_1 x + \ldots {}^nC_k x^k \ldots {}^nC_n x^n)$

LHS $(1+x)^n(1+x)^n$

terms in x^n

${}^nC_0 \cdot {}^nC_n x^n + {}^nC_1 x \cdot {}^nC_{n-1} x^{n-1}$

$+ \therefore {}^nC_k x^k \cdot {}^nC_{n-k} \cdot x^{n-k} \ldots$

${}^nC_0 \cdot {}^nC_n + {}^nC_1 \cdot {}^nC_{n-1} \ldots {}^nC_k \cdot {}^nC_{n-k} + \ldots {}^nC_n \cdot {}^nC_0$

since ${}^nC_0 = {}^nC_1$

LHS $= {}^nC_0{}^2 + {}^nC_1{}^2 + \therefore {}^nC_k{}^2 + {}^nC_n{}^2$

$= \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots \binom{n}{k}^2 + \binom{n}{n}^2$

RHS $(1+x)^{2n}$ coeff of x^n.

$= {}^{2n}C_n = \frac{(2n)!}{(n)!(2n-n)!}$

$= \frac{(2n)!}{(n!)^2}$

LHS = RHS

b) Prove $7^n - 3^n$ is a multiple of 4 for $n \geq 1$

Prove true $n=1$

$7^1 - 3^1 = 4$

$= 4 \times 1$ multiple of 4

Assume true $n=k$ $\quad k$ an integer

$7^k - 3^k = 4a$ $\quad a$ an integer

Prove true for $n = k+1$

$7^{k+1} - 3^{k+1} = 7(7^k - 3^k) + 4.3^k$

$= 7^{k+1} - 7.3^k + 4.3^k$

$= 7(4a) + 4.3^k$

$= 4\left[7a + 3^k\right]$

$= 4q$ $\quad q$ an integer

since true for $n=1$ true for $n = 1+1 = 2$. since true for $n=k$ true for $n=k+1$. true for all positive integer n

$\longrightarrow$

13.

c) $\angle ACB = 180 - (\alpha+\beta)$

$\angle$ sum $\Delta = 180^\circ$

• $\angle ADB = \alpha + \beta$

opposite $\angle$s in cyclic quad ACBD are supplementary

• $\angle ABD = 90^\circ$

angle in semicircle $= 90^\circ$

$\therefore \angle DAB + \angle ADB = 90^\circ$

$\angle$ sum of $\Delta = 180^\circ$

$\therefore \gamma + \alpha + \beta = 90^\circ$

d) $T = T_w + Be^{-kt}$ (1)

$\frac{dT}{dt} = -kBe^{-kt}$

$= -k(T - T_w)$ from (1)

ii) $T = 2 + Be^{-kt}$ $T_w = 2$

$t = 0$ $T = 37^\circ$

$37 = 2 + Be^0$

$B = 35$

$T = 2 + 35e^{-kt}$

$t = 5$ $T = 36$

$36 = 2 + 35e^{-5k}$

$\frac{34}{35} = e^{-5k}$

$\ln\left(\frac{34}{35}\right) = -5k$

$k = 0.00579$

$\therefore T = 2 + 35e^{-0.0058t}$

iii) Unconscious at 30°C.

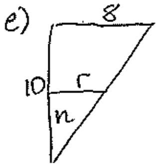
$30 = 2 + 35e^{-0.0058t}$

$\frac{28}{35} = e^{-0.0058t}$

$\ln\left(\frac{28}{35}\right) = -0.0058t$

$t = 38.5$ minutes

e)



$\frac{h}{10} = \frac{r}{8}$

$r = \frac{4h}{5}$

$\frac{dV}{dt} = \frac{2\pi}{5}$

$A = \pi r^2$

$= \pi \cdot \frac{16h^2}{25}$

$\frac{dA}{dh} = \frac{32\pi h}{25}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \cdot \frac{16h^2}{25} \cdot h$

$= \frac{16\pi h^3}{75}$

$\frac{dV}{dh} = \frac{16\pi h^2}{25}$

need $\frac{dA}{dt}$ when $h = 4$

now

$\frac{dA}{dt} = \frac{dA}{dh} \cdot \frac{dh}{dV} \cdot \frac{dV}{dt}$

$= \frac{32\pi h}{25} \cdot \frac{25}{16\pi h^2} \cdot \frac{2\pi}{5}$

$= \frac{4\pi}{5h}$

$= \frac{4\pi}{20}$ when $h = 4$

$= \frac{\pi}{5}$ cm/min

QUESTION 14.

a) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 18x^3 + 27x^2 + 9x$

$\frac{1}{2}v^2 = \frac{18x^4}{4} + 9x^3 + \frac{9x^2}{2} + k$

$v^2 = 9x^4 + 18x^3 + 9x^2 + k_2$

$= 9x^2(x^2 + 2x + 1) + k_2$

$= 9x^2(x+1)^2 + k_2$

when $x = -2$ $v = -6$

$36 = 9 \times 4 \times (-1)^2 + k_2$

$k_2 = 0$

$v^2 = 9x^2(x+1)^2$

ii. $v = \pm 3x(x+1)$ since moving to the left from initial values

$v = \frac{dx}{dt} = -3x(x+1)$

$\therefore \frac{dx}{x(x+1)} = -3$

$\therefore \int \frac{dx}{x(x+1)} = -3\int dt$

$= -3t + C.$

iii) $\frac{d}{dx} \ln\left(1 + \frac{1}{x}\right) = \frac{-1/x^2}{1 + 1/x}$

$= \frac{-1}{x^2 + x}$ or $\frac{-1}{x(x+1)}$

iv) from (iii) $\int \frac{dx}{x(1+x)} = -\ln\left(1 + \frac{1}{x}\right) + c_2$

from (ii) $-3t + c = -\ln\left(1 + \frac{1}{x}\right) + c_2$

$3t = \ln\left(1 + \frac{1}{x}\right) + c_3$

when $t = 0$ $x = -2$

$0 = \ln\left(\frac{1}{2}\right) + c_3$

$c_3 = -\ln\left(\frac{1}{2}\right)$ or $\ln 2.$

→

$\ldots \quad \ln\left(1+\frac{1}{x}\right) + \ln 2$

$3t = \ln\left[\left(1+\frac{1}{x}\right)\cdot 2\right]$

$e^{3t} = 2\left(1+\frac{1}{x}\right)$

$e^{3t} - 2 = \frac{2}{x}$

$x = \frac{2}{e^{3t}-2}$

b) LHS $= 2\sin 2x - 3\cos 2x - 3\sin x + 3$

$= 2(2\sin x\cos x) - 3(1-2\sin^2 x) - 3\sin x + 3$

$= 4\sin x\cos x - 3 + 6\sin^2 x - 3\sin x + 3$

$= \sin x(6\sin x + 4\cos x - 3)$

$=$ RHS

ii) $6\sin x + 4\cos x \equiv R\sin(x+\alpha)$

$R^2 = 6^2 + 4^2$

$R = \sqrt{52}$

$\frac{6}{\sqrt{52}}\sin x + \frac{4}{\sqrt{52}}\cos x = \sin x\cos\alpha + \cos x\sin\alpha$

$\cos\alpha = \frac{6}{\sqrt{52}}$

$\alpha = .5880$

$\therefore 6\sin x + 4\cos x = \sqrt{52}\sin(x + 0{\cdot}5880)$

iii) solve $\sin x\left(\sqrt{52}\sin(x+0{\cdot}5880) - 3\right) = 0$

$\sin x = 0$ or $\sin(x + 0{\cdot}5880) = \frac{3}{\sqrt{52}}$

$0, \ldots$

$x + 0{\cdot}5880 = 0{\cdot}429$

$x = -0{\cdot}1589$

or

$x + 0{\cdot}5880 = 2{\cdot}7125$

$x = 2{\cdot}12$

$\therefore x = 0$ or $x = 2{\cdot}12$

14 c) $P(x) = (x-a)^3 + (x-b)^3$

$P'(x) = 3(x-a)^2 + 3(x-b)^2$

stat point when $P'(x) = 0$

$\therefore 3(x-a)^2 + 3(x-b)^2 = 0$

$(x-a)^2 + (x-b)^2 = 0$

$x^2 - 2ax + a^2 + x^2 - 2bx + b^2 = 0$

$2x^2 - (2a+2b)x + (a^2+b^2) = 0$

$\Delta = b^2 - 4ac$

$= (2a+2b)^2 - 4\cdot 2(a^2+b^2)$

$= 4a^2 + 8ab + 4b^2 - 8a^2 - 8b^2$

$= -4a^2 + 8ab - 4b^2$

$= -4(a^2 - 2ab + b^2)$

$= -4(a-b)^2$

< 0 since $a \neq b$

$\therefore$ no roots, no solution

$\therefore P(x)$ has no stat pts