HORNSBY GIRLS HIGH SCHOOL

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- **Reading Time- 5 minutes**
- **Working Time – 2 hours**
- **Write using a black or blue pen**
- **Approved calculators may be used**
- **A table of standard integrals is provided at the back of this paper**
- **All necessary working should be shown for every question**
- **Begin each question on a fresh sheet of paper**

Total marks (84)

- **Attempt Questions 1-7**
- **All questions are of equal value**

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Total Marks
Attempt Questions 1–7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your student number and question number at the top of the page. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Using the table of standard integrals, evaluate $\int \frac{1}{\sqrt{x^2+16}}\,dx$. **1**

(b) If $f(x)=\cot x$, find $f'(x)$ in simplest form. **2**

(c) Find the acute angle between the lines $y=2x+1$ and $3x+y-7=0$. **2**

(d) Write $P(x)=x^3-6x^2+3x+1$ in the form $P(x)=Q(x)\times T(x)+c$ where $T(x)=x-1$ and c is a constant. **2**

(e) Find the vertical and horizontal asymptotes of $y=\frac{3x^3}{x^3+8}$. **2**

(f) Evaluate $\int_0^1 \sqrt{1-x^2}\,dx$ using the substitution $x=\sin\theta$. **3**

Question 2 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The mathematics staff currently consists of 3 male and 6 female teachers. How many ways can a team of 5 be chosen from the mathematics staff which contains at least 3 female teachers? **2**

(b) The area bounded by the curve $y = e^x$, the x-axis and the ordinates at $x = 0$ and $x = 3$ is rotated about the x-axis. Use Simpson's rule with 3 function values to find an approximation of the volume so generated. **3**

(c) Determine the coordinates of the point Q if it divides the interval joining P (-3, 1) and R (5, -10) externally in the ratio $3:5$. **2**

(d) By sketching $y = \cos x + \sin x$ and $y = x$ for $0 \le x \le \frac{\pi}{2}$, find the number of solutions of $\cos x + \sin x - x = 0$ in the domain $0 \le x \le \frac{\pi}{2}$. **2**

(e) The probability that a vaccine for a new virus will not protect the patient from getting the virus is found to be 0.02%. If a group of 1000 people are vaccinated, find the probability, as a percentage, that:

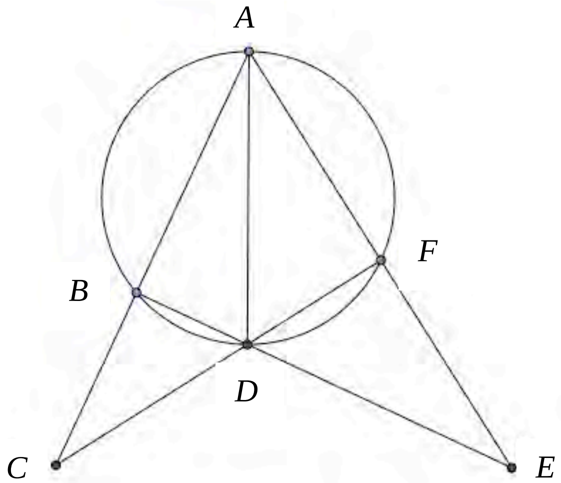
(i) all the group will be protected from getting the virus. **1**

(ii) at most 2 people will not be protected from getting the virus. **2**

Question 3 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Solve for x: $\frac{5}{x-1} < 3$ **2**

(b)



In the diagram above AE, BE, CF & CA are secants and $\angle ACD = \angle AED$

(i) Show that $\angle CBD = \angle EFD$ **1**

(ii) Prove that AD is a diameter of the circle $AFDB$. **2**

(c) Let T be the temperature in a room at time t and let A be the temperature of the room's surroundings. Newton's Law of Cooling states that the rate of change of temperature T is proportional to $(T - A)$.

(i) Verify that $T = A + Be^{kt}$, where B and k are constants, satisfies Newton's Law of Cooling. **1**

(ii) The temperature of a substance in a room of constant temperature $6°C$ is noted to be $29°C$. After 40 minutes the temperature of the substance is noted to be $14°C$. **3**

Find how long it takes the temperature of the substance to reach $9°C$. (Give your answer to the nearest minute.)

(d) Find the term independent of x in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{12}$. **3**

Question 4 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$. **2**
If $p + q = 8$ find the locus of *M*, the mid-point of *PQ*.

(b) Find all the values of θ in the domain $0 \le \theta \le 2\pi$ for which **3**

$$3\cos\theta - 2\sin\theta = 2.$$

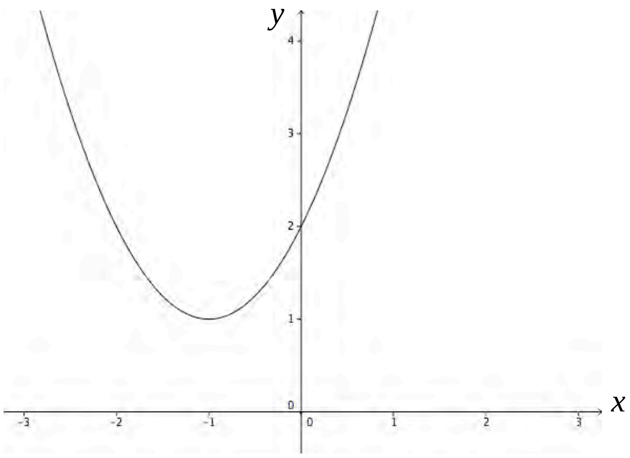
(c) (i) Differentiate $x\tan^{-1} x$ **2**

(ii) Hence evaluate $\int_0^1 \tan^{-1} x \, dx$ **2**

(d) If $f(x) = 3x + 5$ and $g(x) = \frac{x-5}{3}$, find $f(g(x) + g(f(x)))$ **3**

Question 5 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The graph of $f(x)=x^2+2x+2$ is shown on the diagram below.



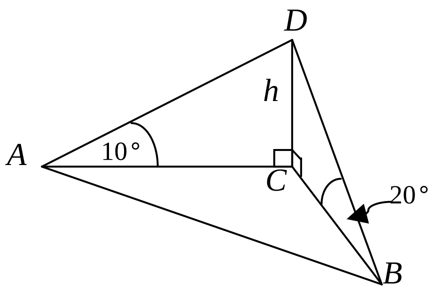
(i) The function $g(x)=x^2+2x+2$ is defined in the domain $x \geq -1$. Sketch the graph of *g(x)* and its inverse function $g^{-1}(x)$, showing clearly all intercepts with the coordinate axes. **2**

(ii) State the domain of $g^{-1}(x)$. **1**

(iii) Find an expression for $y=g^{-1}(x)$ in terms of x. **2**

(b) Use mathematical induction to prove that $4^n \geq 1+3n$, for integer $n \geq 1$. **3**

(c) A vertical flagpole *CD* of height h metres stands with its base *C* on horizontal ground. *A* and *B* are points on the ground where *A* is due West of *C* and *B* is on a bearing of $170°$ from *C*. From *A* and *B* the angles of elevation of the top *D* of the flagpole are $10°$ and $20°$ respectively.



(i) Express the length of AC in terms of h. **1**

(ii) Show that $\angle ACB$ is $100°$. **1**

(iii) If the distance between *A* and *B* is 40 metres, find the height of the tower *CD*. **2**

Question 6 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The acceleration $\ddot{x}$ m/s^2 at time t seconds of a particle moving in a straight line is given by $\ddot{x} = -4\cos 2t - 8\sin 2t$.
If the particle is at a distance of x metres from the origin at time t and if initially it is at $x=1$ with a velocity of 4 m/s,

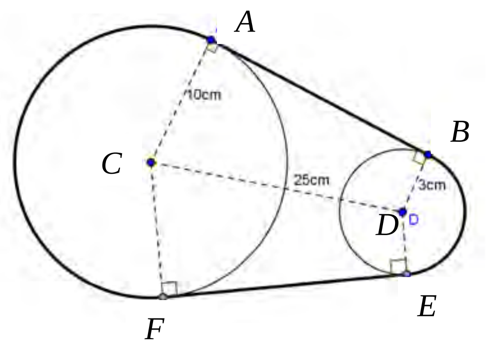
(i) Show that $\ddot{x} = -4x$. **3**

(ii) Show that the position of the particle after $\frac{\pi}{4}$ seconds is 2 metres to the right of the origin and the magnitude of its velocity is 2m/s at this time. **2**

(iii) Is the speed of the particle increasing or decreasing when $t=\frac{\pi}{4}$? Justify your answer. **1**

(iv) How long is it exactly between each time the particle passes the origin? Explain your answer. **1**

(b) A belt connects a driving pulley of radius 3cm to another pulley of radius 10cm, as shown in the diagram below:



(i) Show that $\angle ACD = 73^\circ 44'$ (Hint: Draw a line parallel to *AB* through *D)*. **1**

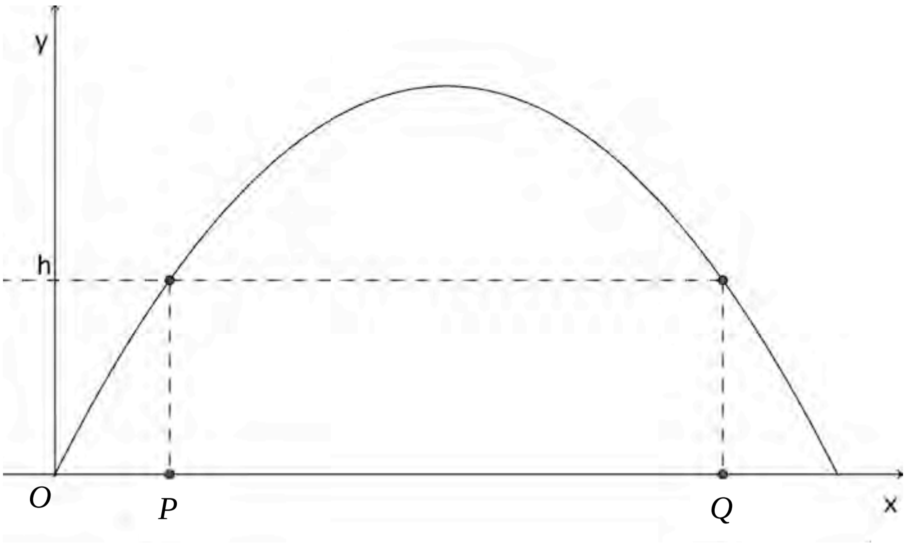
(ii) Find the length of *AB.* **1**

(iii) Find the total length of the belt. **3**

Question 7 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Find the value of n if the coefficient of x^6 is twice the coefficient of x^5 in the expansion of $(3+2x)^n$. **3**

(b)



A particle is projected with velocity $V\ ms^{-1}$ from a point O at an angle of elevation α. The particle just clears two vertical trees P and Q, of height h metres at horizontal distance of p metres and q metres from O respectively.
The acceleration due to gravity is taken as $10ms^{-2}$ and air resistance is ignored.

(i) Find the Cartesian equation of the flight of the particle. **1**

(ii) Show that $V^2 = \dfrac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h}$. **2**

(iii) Show that $\tan\alpha = \dfrac{h(p+q)}{pq}$. **3**

(iv) Another particle was projected with the same angle as the first particle but with velocity $W\ ms^{-1}$ so as to just clear the tree P and land half-way between the trees P and Q. **3**

This particle has equations for horizontal and vertical displacement to be $x = Wt\cos\alpha$ and $y = -5t^2 + Wt\sin\alpha$.

Show that $W^2 = \dfrac{5h(p+q)^2}{pq} + \dfrac{5pq}{h}$.

End of paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

HORNSBY GIRLS HIGH SCHOOL

Solution to Q1 Ext 1 2009 Trial

(a) $\int \frac{1}{\sqrt{x^2+16}}\,dx = \ln(x + \sqrt{x^2+16}) + C$

(b) $f(x) = \cot x$

$= \frac{\cos x}{\sin x}$

$f'(x) = \frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= \frac{-1}{\sin^2 x}$

$= -\operatorname{cosec}^2 x$

(c) $y = 2x+1$ $y = -3x+7$

$m_1 = 2$ $m_2 = -3$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{2 - -3}{1 - 6}\right|$

$\tan\theta = 1$

$\theta = 45°$

(d) $P(x) = x^3 - 6x^2 + 3x + 1$

$$\begin{array}{r} x^2 - 5x - 2 \\ x-1\,\big)\,\overline{x^3 - 6x^2 + 3x + 1} \\ x^3 - x^2 \\ \hline -5x^2 + 3x \\ -5x^2 + 5x \\ \hline -2x + 1 \\ +2 \\ \hline -1 \end{array}$$

$\therefore P(x) = (x-1)(x^2 - 5x - 2) - 1$

(e) $y = \frac{3x^3}{x^3+8}$

Horizontal asymptote:

$\lim_{x\to\infty} \frac{3x^3}{x^3+8} = \lim_{x\to\infty} \frac{\frac{3x^3}{x^3}}{\frac{x^3}{x^3} + \frac{8}{x^3}}$

$= \lim_{x\to\infty} \frac{3}{1 + \frac{8}{x^3}}$

$= 3$

$\therefore y = 3$ is horizontal asymptote.

Vertical asymptote:

Let $x^3 + 8 = 0$

$x = -2$

$\therefore$ Vertical asymptote is $x = -2$

(f)

$I = \int_0^1 \sqrt{1-x^2}\,dx$

Let $x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

When $x = 0$, $\theta = 0$

$x = 1$, $\theta = \frac{\pi}{2}$

$\therefore I = \int_0^{\pi/2} \sqrt{1 - \sin^2\theta}\,\cos\theta\, d\theta$

$= \int_0^{\pi/2} \cos^2\theta\, d\theta$

$= \frac{1}{2}\int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right) - \left(0 + \frac{1}{2}\sin 0\right)\right]$

$= \frac{\pi}{4}$.

Question 2

a) 2 Males at most $\therefore$ 5F 0M or 4F 1M or 3F 2M

No. group $= {}^6C_5 \times {}^3C_0 + {}^6C_4 \times {}^3C_1 + {}^6C_3 \times {}^3C_2$

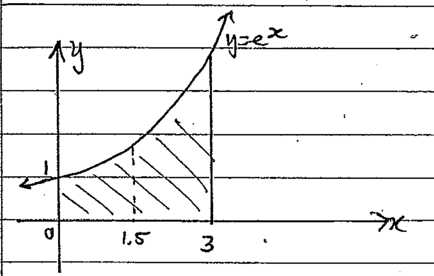
$= 6 + 45 + 60$

$= 111$

or ${}^9C_5 - {}^3C_3 \times {}^6C_2 = 126 - 15$

$= 111$

b)



$V \doteq \pi \int_0^3 (e^x)^2\, dx$

$\doteq \pi \frac{1}{2}\left\{e^0 + 4e^3 + e^6\right\}$

$\doteq 761.47641\ldots$

x	0	1.5	3
$e^{2x} = y^2$	1	e^3	e^6

Volume is approximately 761.5 units

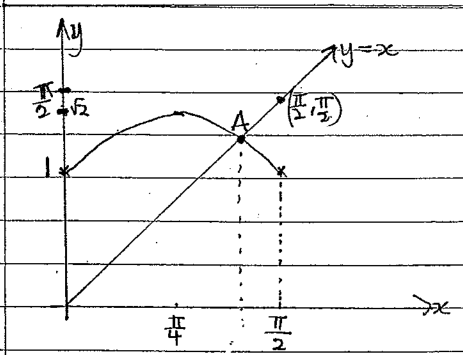
c) $P(-3, 1)$ $R(5, -10)$

$-3 : 5$ (or $3 : -5$)

Q is $\left(\frac{5 \times -3 + -3 \times 5}{5-3}, \frac{5 \times 1 + -3 \times -10}{5-3}\right)$

$= \left(-15, 17\frac{1}{2}\right)$

d)



A is where $y = \cos x + \sin x$ and $y = x$ intersect. (1 solution for $0 \le x \le \frac{\pi}{2}$)

ie. where $\cos x + \sin x = x$

$\cos x + \sin x - x = 0$.

Question 2 (cont'd)

e)(i) Let $p = 0.02\% = 0.0002$ (will not be protected from the virus).
$q = 99.98\% = 0.9998$ (will be protected from the virus).

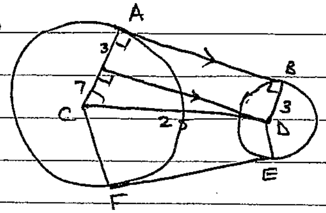
None will NOT be unprotected, all protected.

$^{1000}C_0 p^0 q^{1000} = {}^{1000}C_0 (0.9998)^{1000}$
$\doteqdot 0.818714376\ldots$
$= 81.8\%$ (to 1 decimal place)

(ii) At most 2 people will not be protected (ie 2 or 1 or none (all protected) unprotected.

$^{1000}C_2 p^2 q^{998} + {}^{1000}C_1 p^1 q^{999} + {}^{1000}C_0 p^0 q^{1000}$
$= {}^{1000}C_2 (0.0002)^2 (0.9998)^{998} + {}^{1000}C_1 (0.0002)(0.9998)^{999} + {}^{1000}C_0 (0.9998)^{1000}$
$\doteqdot 0.016364458\ldots + 0.16322563\ldots + 0.818714376\ldots$
$\doteqdot 0.998854465$
$= 99.9\%$ (correct to 1 decimal place).

Q6 b)



i) $\cos A\hat{C}D = \frac{7}{25}$
$\therefore A\hat{C}D = 73°44'$

ii) $AB^2 + 7^2 = 25^2$
$\therefore AB = \sqrt{25^2 - 7^2}$
$= 24$

iii) Total length = AB + arc BE + EF + arc FA
$= 24 + \frac{2\times 73°44'}{360°} \times 2\times\pi\times 3 + 24 + \frac{(360 - 2\times 73°44')}{360°} \times 2\pi \times 1\!\!\!1$
$\doteqdot 24 + 7.72 + 24 + 37.09..$
$\therefore$ Total length = 92.8 cm (to 1 d.p)

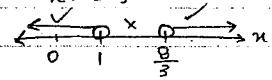
QUESTION 3.

a) $\frac{5}{x-1} < 3$, $x \neq 1$

Consider: $\frac{5}{x-1} = 3$

$5 = 3x - 3$
$8 = 3x$
$x = 2\frac{2}{3}$

Test



$\therefore x < 1$, $x > 2\frac{2}{3}$

b) i) $\angle ACD = \angle AED$ (given)
$\angle BDC = \angle FDE$ (vert. opp. ∠s =)
$\angle CBD = \angle EFD$ (∠ sum of Δ = 180°).

ii) $\angle CBD = \angle EFD$ (proven in (i))
$\angle ABD = \angle AFD$ (adj. supp. ∠s)
$\angle ABD + \angle AFD = 180°$ (opp. ∠s in cyclic quad suppl.)
$\therefore \angle ABD = \angle AFD = 90°$
$\therefore$ AD is the diameter (angle in semicircle = 90°)

c) i) $T = A + Be^{kt} \longrightarrow T - A = Be^{kt}$
$\frac{dT}{dt} = Bke^{kt}$
$= k(Be^{kt})$
$= k(T - A)$
$\therefore$ T satisfies Newton's Law of Cooling.

ii $29 = 6 + Be^0$
$\therefore B = 23$
$\therefore T = 6 + 23e^{kt}$
$14 = 6 + 23e^{40k}$
$\therefore k = \frac{1}{40}\ln\frac{8}{23}$
$k \doteqdot -0.0264$
$\therefore 9 = 6 + 23e^{kt}$
$\therefore t = \frac{\ln\frac{3}{23}}{k}$
$\therefore t \doteqdot 77.15$ min.
$t = 77$ min (to the nearest min)

d) $^{12}C_r (2x^3)^{12-r} \left(-\frac{1}{x}\right)^r$
$^{12}C_r (2^{12-r})(-1)^r (x^3)^{12-r} (x)^{-r}$
For term independent of x:
$x^{36-3r-r} = x^0$
$\therefore 36 = 4r$
$\therefore r = 9$
$\therefore$ Term independent of x is
$^{12}C_9\, 2^3 . (-1)^9$
$= -1760$.

QUESTION 4

a) $X = \dfrac{2ap + 2aq}{2}$

$= a(p+q)$

$\therefore X = 8a$

b) $3\cos\theta - 2\sin\theta = 2$

$\sqrt{3^2+2^2} = \sqrt{13}$

$\tan\alpha = 2/3$

$\therefore \sqrt{13}\cos(\theta + 0.588) = 2$

$\theta + 0.588 = \cos^{-1}(2/\sqrt{13})$

$\theta = 0.395,\ 4.712$

or $\theta = 22.6°,\ 270°$

OR $\dfrac{3(1-t^2)}{1+t^2} - 2\cdot\dfrac{2t}{1+t^2} = 2$

$3 - 3t^2 - 4t = 2 + 2t^2$

$5t^2 + 4t - 1 = 0$

$(5t-1)(t+1) = 0$

$\therefore \tan\theta/2 = \frac{1}{5} \quad \tan\theta/2 = -1$

$\theta = 0.395,\ \frac{3\pi}{2}$

c) i) $\dfrac{dy}{dx} = \tan^{-1}x + \dfrac{x}{1+x^2}$

ii) $\displaystyle\int_0^1 \tan^{-1}x\,dx = \left[x\tan^{-1}x\right]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$

$= \frac{\pi}{4} - \left[\frac{1}{2}\ln(1+x^2)\right]_0^1$

$= \pi/4 - \frac{1}{2}\ln 2$

$= 0.4388\ldots$

d) $g(f(x)) = \dfrac{3x+5-5}{3}$

$= x$

$g(x) + g(f(x)) = \dfrac{x-5}{3} + \dfrac{3x}{3}$

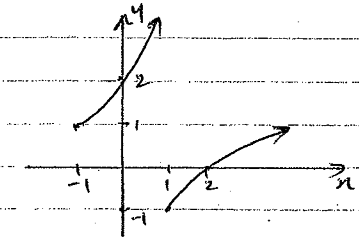
$= \dfrac{4x-5}{3}$

$\therefore f\left(\dfrac{4x-5}{3}\right) = 3\left(\dfrac{4x-5}{3}\right) + 5$

$= 4x$

QUESTION 5

a) i)



ii) $x \geq 1$

iii) $x = y^2 + 2y + 2$

$x = (y+1)^2 + 1$

$y + 1 = \sqrt{x-1}$

$y = \sqrt{x-1} - 1$

b) Prove true for $n = 1$

$4 \geq 1 + 3$ True

Assume true for $n = k$

$4^k \geq 1 + 3k$

Prove true for $n = k+1$

R.T.P. $4^{k+1} \geq 3k + 4$

now $4^k \geq 1 + 3k$

$4^{k+1} \geq 4 + 12k$

$\geq 3k + 4 + 9k$

$\geq 3k + 4$

$\therefore$ TRUE

etc...

c) i) $AC = \dfrac{h}{\tan 10°}$

ii) $270° - 170° = 100°$

iii) $BC = \dfrac{h}{\tan 20°}$

$\therefore 1600 = \dfrac{h^2}{\tan^2 10°} + \dfrac{h^2}{\tan^2 20°} - \dfrac{2h^2}{\tan 10° \tan 20°}\cos 100°$

$h^2 = \dfrac{1600\tan^2 10° \tan^2 20°}{\tan^2 20° + \tan^2 10° - 2\cos 100° \tan 10° \tan 20°}$

$\doteqdot 35.458$

$h = 5.955$

Q6 a) i) $\ddot{x} = -4\cos 2t - 8\sin 2t$

$\dot{x} = \int -4\cos 2t - 8\sin 2t \, dt$

$= -2\sin 2t + 4\cos 2t + C$

$t=0, \dot{x}=4$: $4 = -2(0) + 4(1) + C$

$\therefore C = 0$

$\dot{x} = -2\sin 2t + 4\cos 2t$ — ①

$x = \int -2\sin 2t + 4\cos 2t \, dt$

$= \cos 2t + 2\sin 2t + C_1$

$t=0, x=1 \therefore 1 = 1 + 2(0) + C_1$

$\therefore C_1 = 0$

$x = \cos 2t + 2\sin 2t$ — ②

$\ddot{x} = -4\cos 2t - 8\sin 2t$

$= -4(\cos 2t + 2\sin 2t)$

$= -4x$

ii) $t = \frac{\pi}{4}$, $x = 2\cos 2(\pi/4) + 2\sin 2(\pi/4)$

$= 2$ $\therefore$ 2 m to the right of the origin

$\dot{x} = -2\sin 2(\pi/4) + 4\cos 2(\pi/4)$

$= -2$ ie speed is 2 m/s

iii) when $t = \pi/4$, $\ddot{x} = -4(2) = -8$ m/s^2

$\therefore$ particle is increasing as accel & velocity have the same sign (ie moving in the same direction)

iv) passes origin when $x = 0$

$\cos 2t + 2\sin 2t = 0$

$2\sin 2t = -\cos 2t$

$\tan 2t = -1/2$

$\therefore 2t = n\pi + \tan^{-1}(-1/2)$

$t = \frac{n\pi}{2} + \frac{1}{2}\tan^{-1}(-1/2)$

$\therefore$ difference in time is $\frac{\pi}{2}$

[$n=0$, $t_0 = \frac{1}{2}\tan^{-1}(-1/2)$, $n=1$, $t_1 = \frac{\pi}{2} + \frac{1}{2}\tan(-1/2)$ $\therefore t_1 - t_0 = \frac{\pi}{2}$]

NB ⊛ Q6 b) see after Q2

Q7 Solutions to Question 7 Ext. 1 2009 Trial

(a) $(3+2x)^n = \sum_{k=0}^{n} {}^nC_k \, 3^{n-k}.(2x)^k$

$= \sum_{k=0}^{n} {}^nC_k . 3^{n-k}. 2^k x^k$

Coefficient of x^5 is ${}^nC_5 \, 3^{n-5} 2^5$

Coefficient of x^6 is ${}^nC_6 . 3^{n-6} 2^6$

Coefficient of x^6 = 2 × coefficient of x^5

${}^nC_6 \, 3^{n-6} 2^6 = 2.{}^nC_5 . 3^{n-5}. 2^5$

${}^nC_6 . \frac{3^{n-6}}{3^{n-5}} = {}^nC_5$

$\frac{n!}{(n-6)!\,6!} = \frac{3n!}{(n-5)!\,5!}$

$\frac{1}{6} = \frac{3}{n-5}$

$n - 5 = 18$

$n = 23$

(b)(i) $\ddot{x} = 0$ $\quad \ddot{y} = -10$

$\dot{x} = v\cos\alpha$ $\quad \dot{y} = -10t + v\sin\alpha$

$x = vt\cos\alpha$ $\quad y = -5t^2 + vt\sin\alpha$

$\Rightarrow t = \frac{x}{v}\sec\alpha$

$\therefore y = -5\left(\frac{x\sec\alpha}{v}\right)^2 + x\sin\alpha\sec\alpha$

$y = \frac{-5x^2(1+\tan^2\alpha)}{v^2} + x\tan\alpha$

(ii) Sub (p, h)

$h = \frac{-5p^2}{v^2}(1+\tan^2\alpha) + p\tan\alpha$

$h - p\tan\alpha = \frac{-5p^2(1+\tan^2\alpha)}{v^2}$

$p\tan\alpha - h = \frac{5p^2(1+\tan^2\alpha)}{v^2}$

$\therefore v^2 = \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h}$

(iii) $v^2 = \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h}$ .. ①

Similarly $v^2 = \frac{5q^2(1+\tan^2\alpha)}{q\tan\alpha - h}$ .. ②

$\therefore \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h} = \frac{5q^2(1+\tan^2\alpha)}{q\tan\alpha - h}$

$\frac{p^2}{p\tan\alpha - h} = \frac{q^2}{q\tan\alpha - h}$

$p^2 q\tan\alpha - p^2 h = q^2 p\tan\alpha - q^2 h$

$p^2 q\tan\alpha - q^2 p\tan\alpha = p^2 h - q^2 h$

$\tan\alpha\,(p^2 q - q^2 p) = p^2 h - q^2 h$

$\tan\alpha = \frac{h(p-q)(p+q)}{pq(p-q)}$

$\tan\alpha = \frac{h(p+q)}{pq}$

(iv) $v^2 = \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h}$

$\tan\alpha = \frac{h(p+q)}{pq}$

$\therefore v^2 = 5p^2\left[\left(1 + \frac{h^2(p+q)^2}{p^2q^2}\right) \div \left[\frac{ph(p+q)}{pq} - h\right]\right]$

$= 5p^2\left[\frac{p^2q^2 + h^2(p+q)^2}{p^2q^2} \times \frac{pq}{ph(p+q) - hpq}\right]$

$= 5\left[\frac{p^2q^2 + h^2(p+q)^2}{q} \times \frac{p}{ph(p+q-q)}\right]$

$= 5\left[\frac{p^2q^2 + h^2(p+q)^2}{pqh}\right]$

$= \frac{5p^2q^2}{pqh} + \frac{5h^2(p+q)^2}{pqh}$

$= \frac{5pq}{h} + \frac{5h(p+q)^2}{pq}$