

# HORNSBY GIRLS HIGH SCHOOL



# Mathematics Extension I

Year 12 Higher School Certificate  
Trial Examination Term 3 2013

STUDENT NUMBER: \_\_\_\_\_

### General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen  
Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

**Total marks – 70**

**Section I** Pages 3 – 6

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

**Section II** Pages 7 – 11

60 marks

Attempt Questions 11 – 14.

Start each question in a new writing booklet.

Write your student number on every writing booklet.

<i>Question</i>	<i>1-10</i>	<i>11</i>	<i>12</i>	<i>13</i>	<i>14</i>	<i>Total</i>
<i>Total</i>	/10	/15	/15	/15	/15	/70

*This assessment task constitutes 45% of the Higher School Certificate Course School Assessment*

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## Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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- 1 A polynomial equation has roots  $\alpha$ ,  $\beta$  and  $\gamma$ , where:  
 $\alpha + \beta + \gamma = -3$ ,  $\alpha\beta + \alpha\gamma + \beta\gamma = -2$  and  $\alpha\beta\gamma = 4$ .  
Which polynomial equation has the roots  $\alpha$ ,  $\beta$  and  $\gamma$ ?

- (A)  $x^3 - 3x^2 - 2x + 4 = 0$   
(B)  $x^3 + 3x^2 - 2x - 4 = 0$   
(C)  $x^3 + 3x^2 + 2x + 4 = 0$   
(D)  $x^3 - 3x^2 + 2x - 4 = 0$

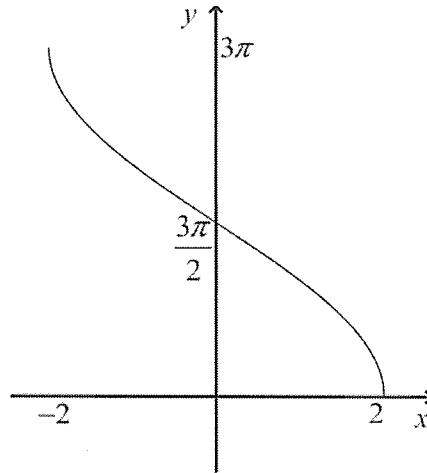
- 2 The solution to  $\frac{4}{x-3} \leq 2$  is:

- (A)  $3 \leq x \leq 5$   
(B)  $3 < x \leq 5$   
(C)  $x < 3$  or  $x \geq 5$   
(D)  $x \leq 3$  or  $x \geq 5$

- 3  $\int \cos^2 4x \, dx =$

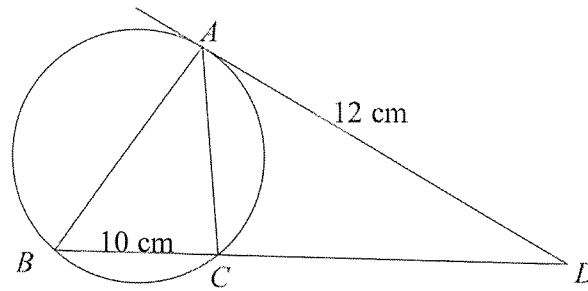
- (A)  $\frac{1}{2}x + \frac{1}{16}\sin 8x + C$   
(B)  $\frac{1}{2}x - \frac{1}{16}\sin 8x + C$   
(C)  $\frac{1}{2}x + \frac{1}{8}\sin 8x + C$   
(D)  $\frac{1}{2}x - \frac{1}{8}\sin 8x + C$

- 4 If  $P(x) = (x+2)(x+k)$  and if the remainder when  $P(x)$  is divided by  $(x-1)$  is 12, then:
- (A)  $k = 2$   
 (B)  $k = 3$   
 (C)  $k = 6$   
 (D)  $k = 11$
- 5 Which function best describes the graph below?



- (A)  $y = 2 \cos^{-1} 3x$   
 (B)  $y = 2 \cos^{-1} \frac{3x}{2}$   
 (C)  $y = 3 \cos^{-1} 2x$   
 (D)  $y = 3 \cos^{-1} \frac{x}{2}$
- 6 If the function  $f$  is defined by  $f(x) = x^5 - 1$ , then the inverse function of  $f$ , is defined by  $f^{-1}(x) =$
- (A)  $\sqrt[5]{x} - 1$   
 (B)  $\sqrt[5]{x-1}$   
 (C)  $\sqrt[5]{x} + 1$   
 (D)  $\sqrt[5]{x+1}$

- 7  $ABC$  is a triangle inscribed in a circle. The tangent to the circle at  $A$  meets  $BC$  produced at  $D$  where  $BC = 10$  and  $AD = 12$ . What is the length of  $CD$ ?



NOT TO  
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- (A) 6 cm  
(B) 7 cm  
(C) 8 cm  
(D) 9 cm

8  $\int \frac{x^2}{e^{x^3}} dx =$

- (A)  $-\frac{1}{3e^{x^3}} + C$   
(B)  $-\frac{1}{3}e^{x^3} + C$   
(C)  $-\frac{1}{3}\ln e^{x^3} + C$   
(D)  $\frac{1}{3}\ln e^{x^3} + C$

- 9 Consider the curve defined by the parametric equations  $x = \frac{1}{t}$  and  $y = \frac{t}{t+1}$ .

The graph of  $y = f(x)$  would have asymptotes:

- (A)  $x = 0$  only  
(B)  $x = 1, y = -1$   
(C)  $x = -1$  only  
(D)  $x = -1, y = 0$

- 10 The velocity,  $v$  metres per second, of a particle moving in simple harmonic motion along the  $x$  axis is given by the equation  $v^2 = 36 - 4x^2$ .

What is the amplitude, in metres of the motion of the particle?

- (A) 3
- (B) 2
- (C) 6
- (D) 4

**End of Section I**

## Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations

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**Question 11** (15 marks)      Start a new writing booklet

(a) Evaluate  $\int_0^2 \frac{dx}{\sqrt{16-x^2}}$ . 2

(b) Differentiate  $3x^2 \ln x$ , for  $x > 0$ . 2

(c) Find the acute angle between the lines  $x + 2y - 5 = 0$  and  $y = 4x + 5$ , giving your answer 3  
correct to the nearest minute.

(d) Use the substitution  $u = e^x$  to find  $\int \frac{e^x}{1+e^{2x}} dx$ . 3

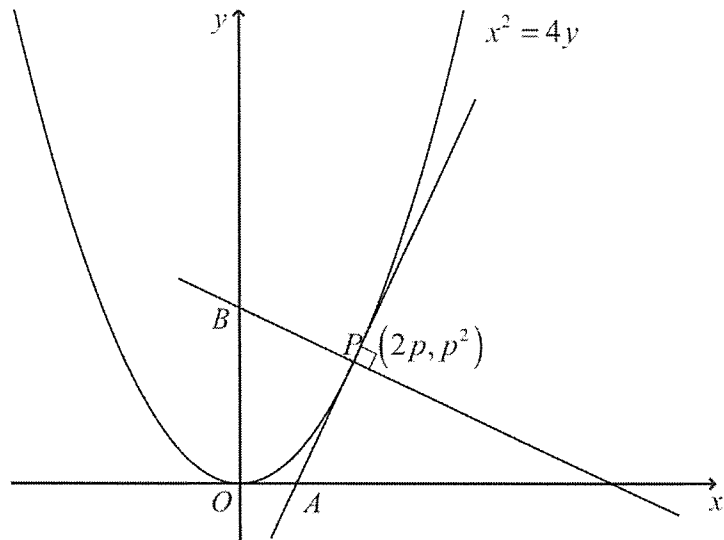
(e) The staff in a school office consists of 5 males and 8 females. 2  
How many committees of 5 staff can be chosen that contain exactly 3 females?

(f) Use the binomial theorem to find the term independent of  $x$  in the expansion of 3  
 $\left(2x - \frac{1}{x^2}\right)^{12}$ .

**Question 12** (15 marks)      Start a new writing booklet

(a) Use mathematical induction to prove that  $n! > 2^n$  for integer  $n \geq 4$ . 3

(b) The diagram below shows the graph of the parabola  $x^2 = 4y$ .  
 The tangent cuts the parabola at  $P(2p, p^2)$ ,  $p > 0$ , cuts the  $x$  axis at  $A$ .  
 The normal to the parabola at  $P$  cuts the  $y$  axis at  $B$ .



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(i) Show that the equation of the tangent  $AP$  is  $y = px - p^2$ . 2

(ii) Show that the equation of the normal  $PB$  is  $x + py = p^3 + 2p$ . 1

(iii) Find the coordinates of  $A$  and  $B$ . 2

(iv) Let  $C$  divide the interval  $AB$  in the ratio 2:1. 3

Find the Cartesian equation of the locus of  $C$ , giving any domain restrictions.

(c) Consider the function  $f(x) = 1 + \cos^{-1}(2x-1) - 2\cos^{-1}\sqrt{x}$  for  $0 \leq x \leq 1$ .

(i) Show that  $f'(x) = 0$  for  $0 \leq x \leq 1$ . 3

(ii) Sketch the graph of  $y = f(x)$  for  $0 \leq x \leq 1$ . 1



**Question 13** (15 marks)      Start a new writing booklet

- (a) A particle is moving in simple harmonic motion has its acceleration given by

$$\frac{d^2x}{dt^2} = -25x, \text{ where } x \text{ metres is the displacement of the particle after } t \text{ seconds.}$$

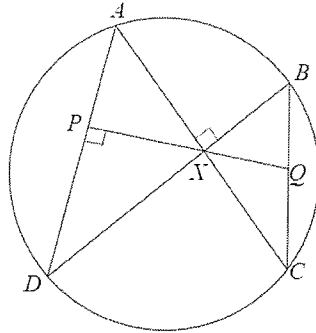
Initially, the particle's acceleration is  $50 \text{ ms}^{-2}$  and after  $\frac{\pi}{6}$  seconds, the particle's velocity is  $-10 \text{ ms}^{-1}$ .

- (i) Find the period of the motion. 1
- (ii) Show that  $x = a \sin(5t - \alpha)$  is a possible equation of motion for this particle, 2  
where  $a$  and  $\alpha$  are positive constants and  $\alpha$  is acute.
- (iii) Show that the amplitude of the motion is 4 metres. 2
- (iv) Find the value of  $\alpha$ . 1
- (v) Find the greatest speed of the particle and where the particle reaches this speed. 2
- (vi) How many times does the particle change direction in the first 2 seconds? 2
- (b) Let  $(2 + 3x)^7 = \sum_{k=0}^7 t_k x^k$
- (i) Write down an expression for  $t_k$ . 1
- (ii) Hence show that  $\frac{t_{k+1}}{t_k} = \frac{21-3k}{2k+2}$  where  $0 < k < 7$ . 2
- (iii) Hence, or otherwise, find the greatest coefficient in the expansion of  $(2+3x)^7$ . 2

**Question 14** (15 marks)      Start a new writing booklet

- (a) The diagram below shows points  $A, B, C$  and  $D$  on a circle. The lines  $AC$  and  $BD$  are perpendicular and meet at  $X$ .

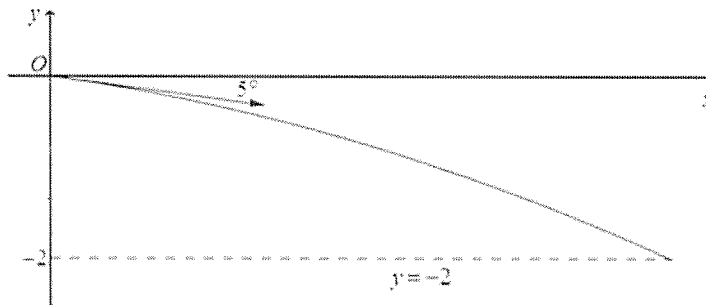
The perpendicular to  $AD$  through  $X$  meets  $AD$  at  $P$  and  $BC$  at  $Q$ .



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**Copy or trace this diagram into your writing booklet.**

- (i) Prove that  $\angle QXB = \angle QBX$ . 3
- (ii) Prove that  $PQ$  bisects  $BC$ . 2
- (b) A cricket ball leaves a bowler's hand 2 metres above the ground with a velocity of  $30 \text{ ms}^{-1}$  at an angle of projection of  $5^\circ$  **below** the horizontal, as shown below.



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Using the origin as the point where the ball leaves the bowlers hand, the coordinates of the ball at time  $t$  are given by:

$$x = 30t \cos 5^\circ$$

$$y = -30t \sin 5^\circ - 5t^2$$

**(Do not prove these results)**

- (i) Find the time it takes for the ball to strike the ground. 2
- (ii) Calculate the angle at which the ball strikes the ground. 2
- (iii) Show the motion of the ball is parabolic, even though it is projected at an angle 2  
below the horizontal.

**Question 14 continues on page 11**

Question 14 (continued)

(c) A television satellite tower stands on a large area of flat ground.

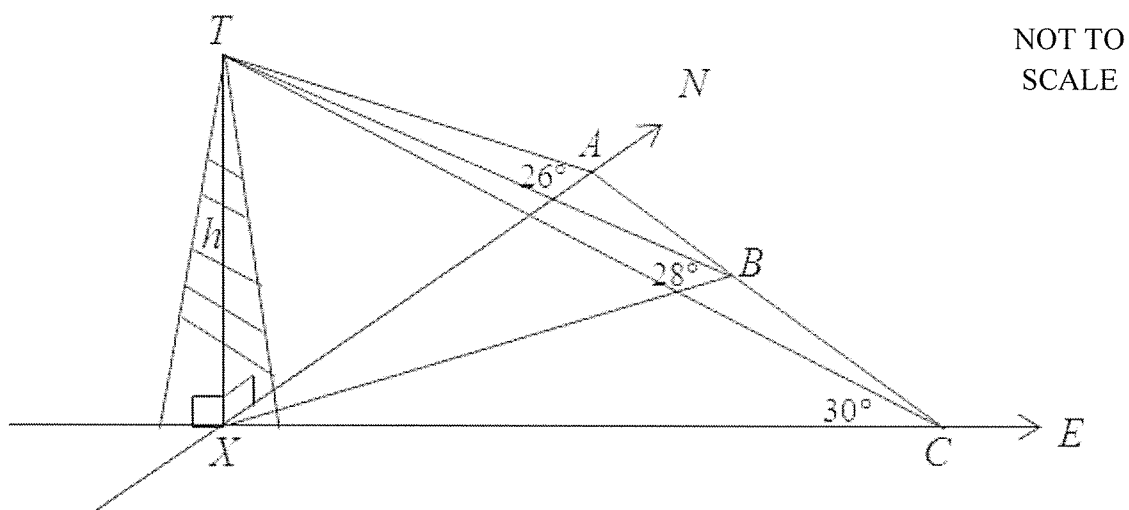
Three maintenance workers, A, B and C, are observing the tower.

Worker A is due north of the tower.

Worker C is due East of the tower.

Worker B is on the line of sight-from A to C (A, B and C are collinear).

The angles of elevation of the top of the tower from A, B and C are  $26^\circ$ ,  $28^\circ$  and  $30^\circ$  respectively.



- |       |  |          |
|-------|--|----------|
| (i)   | Find $\angle XAC$ , correct to one decimal place.  | <b>1</b> |
| (ii)  | Find $\angle ABX$ , correct to the nearest degree.   | <b>2</b> |
| (iii) | Hence, find the bearing of Worker B from the base of the tower X, correct to the nearest degree. | <b>1</b> |

**End of Paper**

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Multiple Choice

1.  $-\frac{b}{a} = -3$   $\frac{c}{a} = -2$   $-\frac{d}{a} = 4$

$\therefore a=1$   $b=3$   $c=-2$   $d=-4$  (B)

2.  $\frac{4}{x-3} \leq 2$

$4(x-3) \leq 2(x-3)^2$

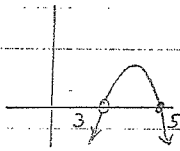
$4(x-3) - 2(x-3)^2 \leq 0$

$(x-3)[4 - 2(x-3)] \leq 0$

$(x-3)(10-2x) \leq 0$

$2(x-3)(5-x)$

$x < 3, x \geq 5$  (C)



3.  $\int \cos^2 4x \, dx = \frac{1}{2} \int [\cos 8x + 1] \, dx$

$= \frac{1}{2} \left[ \frac{\sin 8x}{8} + x \right]$

$= \frac{x}{2} + \frac{\sin 8x}{16} + c$

(A)

4.  $p(1) = 12$

$\therefore (1+2)(1+k) = 12$

$3+3k = 12$

$3k = 9$

$k = 3$

(B)

5.  $-1 \leq \frac{x}{2} \leq 1$

$-2 \leq x \leq 2$

(D) is only option

6.  $f(x) = x^5 - 1$

$\therefore$  for inverse

$x = f(x)^5 - 1$

$\therefore f(x)^5 = x + 1$

$\therefore f(x) = \sqrt[5]{x+1}$  (D)

7.  $AD^2 = BD \times CD$

$12^2 = (10+x)x$

$144 = 10x + x^2$

$0 = x^2 + 10x - 144$

$0 = (x+18)(x-8)$  (C)

$\therefore x = -18, 8$

$x = 8$  as  $x > 0$

8.  $\int \frac{x^2}{e^{2x}} \, dx = \int x^2 e^{-2x} \, dx$

$= -\frac{1}{3} e^{-2x} + c$

$= -\frac{1}{3e^{2x}} + c$  (A)

9.  $x = \frac{1}{t}$  then  $y = \frac{1}{x} = \frac{1}{\frac{1}{t} + 1}$

$= \frac{1}{1+t}$

$\therefore x \neq -1$  and  $y = 0$  (D)

10.  $v^2 = 36 - 4x^2$

$= 4(9 - x^2)$

$= n^2(a^2 - x^2)$  (A)

where  $n=2$  and  $a=3$

Question 11

(a)  $\int_0^2 \frac{dx}{\sqrt{16-x^2}} = \left[ \sin^{-1} \frac{x}{4} \right]_0^2$

$= \sin^{-1} \frac{1}{2} - \sin^{-1} 0$

$= \frac{\pi}{6} - 0$

$= \frac{\pi}{6}$

(b)  $\frac{d}{dx} (3x^2 \ln x) = 3x^2 \times \frac{1}{x} + 6x \times \ln x$

$= 3x + 6x \ln x$

OR

$= 3x(1 + 2 \ln x)$

(c)  $x + 2y - 5 = 0$  and  $4x - y + 5 = 0$

$m_1 = -\frac{1}{2}$

$m_2 = 4$

$\therefore \tan \alpha = \left| \frac{4 - (-\frac{1}{2})}{1 + 4(-\frac{1}{2})} \right|$

$= \left| \frac{\frac{9}{2}}{-1} \right|$

$= \frac{9}{2}$

$\alpha = 77.28^\circ$

(d)  $\int \frac{e^x}{1+e^{2x}} \, dx = \int \frac{u \, du}{1+u^2} \cdot \frac{1}{u}$

$u = e^x$

$\therefore \frac{du}{dx} = e^x$

$= \tan^{-1} u + c$

$= \tan^{-1} e^x + c$

$dx = \frac{du}{e^x}$

$= \frac{du}{u}$

$$(e) {}^8C_3 \times {}^5C_2 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1}$$

$$= 560$$

$\therefore$  560 different committees of 5 containing exactly 3 females could be chosen.

$$(f) \left(2x - \frac{1}{x^2}\right)^{12} \text{ has general term } {}^{12}C_r (2x)^{12-r} \left(-\frac{1}{x^2}\right)^r$$

$$\therefore \text{power of } x \text{ function} = 12 - r - 2r$$

$$= 12 - 3r$$

$\therefore$  constant term has power of zero

$$\therefore 12 - 3r = 0$$

$$\therefore r = 4$$

$$\text{If } r = 4 \quad {}^{12}C_4 (2x)^{12-4} \left(-\frac{1}{x^2}\right)^4 = 495 \times 2^8 \times x^8 \times -\frac{1}{x^8}$$

$$= -126720$$

### Question 12

(a) RTP  $n! > 2^n$  for  $n \geq 4$

Step 1 Prove for  $n = 4$

$$\text{LHS} = 4! = 24 \quad \text{RHS} = 2^4 = 16 \quad \therefore \text{true for } n = 4$$

Step 2 If result is true for  $n = k$  then  $k! > 2^k$

Step 3 RTP for  $n = k + 1$  i.e.  $(k+1)! > 2^{k+1}$

$$\text{LHS} = (k+1)!$$

$$= (k+1)k!$$

$$> (k+1)2^k$$

$$= k \cdot 2^k + 2^k \quad \text{for } k > 4$$

③

$$> 2^k + 2^k$$

$$= 2 \cdot 2^k$$

$$\therefore (k+1)! > 2^{k+1} \quad \text{for } k > 4$$

Step 4 Since the result is true for  $n = 4$ , it is therefore true for  $n = 5$ . Hence since it is true for  $n = 5$  it is true for  $n = 6$  and so on.

Therefore by the process of Mathematical Induction it is true for all integer  $k$  values greater than 4.

$$(b) x^2 = 4y \quad x = 2p, y = p^2$$

$$(i) \frac{dx}{dp} = 2 \quad \frac{dy}{dp} = 2p \quad \therefore \frac{dy}{dx} = \frac{2p}{2} = p$$

$$\therefore \text{Tangent } y - p^2 = p(x - 2p)$$

$$y - p^2 = px - 2p^2$$

$$y = px - p^2 \quad \text{as required}$$

$$(ii) \text{Normal } y - p^2 = -\frac{1}{p}(x - 2p)$$

$$py - p^3 = -x + 2p$$

$$x + py = p^3 + 2p \quad \text{as required}$$

$$(iii) A (y=0) \text{ on tangent } 0 = px - p^2$$

$$px = p^2$$

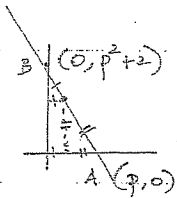
$$x = p \quad \therefore A(p, 0)$$

$$B (x=0) \text{ on normal } py = p^3 + 2p$$

$$y = p^2 + 2 \quad \therefore B(0, p^2 + 2)$$

④

(iv)  $A(p, 0)$ ,  $B(0, p^2+2)$



$\therefore C\left(\frac{p}{3}, \frac{2p^2+4}{3}\right)$  By the geometry

$\therefore x = \frac{p}{3}$  into  $y = \frac{2(3x)^2 + 4}{3}$

$= \frac{18x^2 + 4}{3}$

$y = 6x^2 + \frac{4}{3}$

$\therefore C$  is a parabola with Domain  $x > 0$  (Since  $p > 0$ ) and range  $y \geq \frac{4}{3}$ , Vertex  $(0, \frac{4}{3})$

(c) (i)  $f(x) = \cos^{-1}(2x-1) - 2\cos^{-1}\sqrt{x} + 1$   $0 \leq x \leq 1$

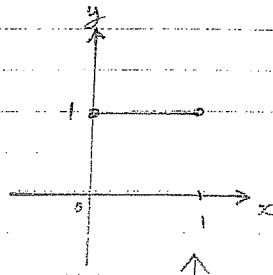
$f'(x) = \frac{-1}{\sqrt{1-(2x-1)^2}} \cdot x^2 - \frac{2x-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{-2}{\sqrt{1-(4x^2-4x+1)}} + \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{\sqrt{x}}$

$= \frac{-2}{\sqrt{4x-4x^2}} + \frac{1}{\sqrt{x}\sqrt{1-x^2}}$

$= \frac{-2}{2\sqrt{x}\sqrt{1-x^2}} + \frac{1}{\sqrt{x}\sqrt{1-x^2}}$

$= 0$  as required



(ii)  $f(x)$  is horizontal since  $f'(x) = 0$  for  $0 \leq x \leq 1$

$f(0) = \cos^{-1}(-1) - 2\cos^{-1}(0) + 1$   
 $= \pi - 2 \cdot \frac{\pi}{2} + 1 = 1$  (5)

Question 13

(a)  $\ddot{x} = -25x$

(i) period =  $\frac{2\pi}{\omega} = \frac{2\pi}{5}$  seconds

(ii)  $x = a \sin(5t - \alpha)$

$\dot{x} = 5a \cos(5t - \alpha)$

$\ddot{x} = -25a \sin(5t - \alpha)$

$= -5^2(\sin(5t - \alpha))$

$= -5^2 x$

(i)  $\ddot{x} = -25a \sin(-\alpha) = 50$

$\sin(-\alpha) = \frac{-2}{a}$

$\sin \alpha = \frac{2}{a}$  (1)

$\ddot{x} = 50a \cos(5\pi/6 - \alpha) = -10$

$a \cos(5\pi/6 - \alpha) = -2$  (2)

$a[\cos 5\pi/6 \cos \alpha + \sin 5\pi/6 \sin \alpha] = -2$

$a\left[-\frac{\sqrt{3}}{2} \times \frac{\sqrt{a^2-4}}{a} + \frac{1}{2} \times \frac{2}{a}\right] = -2$

$-\frac{\sqrt{3}\sqrt{a^2-4}}{2a} + \frac{1}{a} = \frac{-2}{a}$

$-\frac{\sqrt{3}\sqrt{a^2-4}}{2} + 1 = -2$

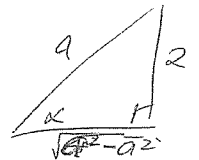
$\sqrt{a^2-4} = \frac{-6}{-\sqrt{3}}$

$a^2 - 4 = \frac{36}{3}$

$a^2 - 4 = 12$

$a^2 = 16$

$a = 4$  ( $a > 0$ )



From (1)

$$\sin \alpha = \frac{2}{9}$$

$$\sin \alpha = \frac{2}{4}$$

$$\sin \alpha = \frac{1}{2}$$

$$\alpha = \pi/6$$

max speed at centre of motion.

$$\ddot{x} = 20 \cos(5t - \pi/6)$$

$$-1 \leq \cos(5t - \pi/6) \leq 1$$

$$-20 \leq 20 \cos(5t - \pi/6) \leq 20$$

∴ max speed is  $20 \text{ ms}^{-1}$  and happens at  $x=0$ .

Changes direction when  $\ddot{x} = 0$

$$5t - \pi/6 = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2, \dots$$

$$5t = \frac{4\pi}{6}, \frac{10\pi}{6}, \frac{16\pi}{6}, \frac{22\pi}{6}, \dots$$

$$t = \frac{2\pi}{15}, \frac{5\pi}{15}, \frac{8\pi}{15}, \frac{11\pi}{15}, \dots$$

$$\approx \underline{0.4}, \underline{1.04}, \underline{1.67}, \dots$$

∴ Changes direction three times in first two seconds

$$(b) (i) (2 + 3x)^7 = \sum_{k=0}^7 {}^7C_k (2)^{7-k} (3x)^k.$$

$$\therefore t_k = {}^7C_k 2^{7-k} 3^k.$$

$$(ii) t_{k+1} = {}^7C_{k+1} 2^{6-k} 3^{k+1}.$$

$$\frac{t_{k+1}}{t_k} = \frac{7! \times 2^{6-k} \cdot 3^{k+1}}{(6-k)! (k+1)!} \times \frac{(7-k)! k!}{7! 2^{7-k} 3^k}.$$

$$= \frac{3 \times (7-k)}{2 \times (k+1)}$$

$$= \frac{21 - 3k}{2k + 2}.$$

$$(ii) \frac{21 - 3k}{2k + 2} > 1$$

$$21 - 3k > 2k + 2 \quad (2k + 2 > 0)$$

$$5k \leq 19$$

$$k \leq 3^{4/5}$$

∴  $k=3$  gives largest coefficient

$$t_{3+1} = t_4 = {}^7C_4 \cdot 2^3 \cdot 3^4$$

$$= 22680.$$



QUESTION 14

i) Let  $\angle QAP = x^\circ$

$\angle CAD = x^\circ$  (L's in same segment)

$\angle AYA + 90^\circ + x^\circ = 180^\circ$  (L sum  $\triangle AYD$ )

$\angle AXP = 90^\circ - x^\circ$

$\angle AXP + \angle AXP + \angle BXP = 180$  (straight line  $APQ$ )

$90 - x + 90 + \angle BXP = 180$

$\angle BXP = x$

$\therefore \angle BXP = \angle QBP$  (both equal  $x$ )

ii)  $\angle DAX + \angle AXP + \angle XDA = 180$  (L sum  $\triangle AXP$ )

$x + 90 + \angle XDA = 180$

$\angle XDA = 90 - x$

$\angle XDA = \angle XCB = 90 - x$  (L's in same segment)

$\angle AXP = \angle CXQ = 90 - x$  (vertically opposite L's)

$\therefore \angle CXQ = \angle XCB$  (both equal  $90 - x$ )

$\therefore \triangle QCX$  is isosceles

$\triangle BCX$  is isosceles

$\therefore BQ = XQ$  (L's opposite equal sides)

$XQ = CQ$  ( " " " " )

$\therefore BQ = CQ$  (both equal  $XQ$ )

$\therefore PQ$  bisects  $BC$

b) i) when  $y = -2$

$-2 = -30t \sin 5 - 5t^2$

$5t^2 + 30t \sin 5 - 2 = 0$

$t = \frac{-30 \sin 5 \pm \sqrt{900 \sin^2 5 + 40}}{10}$

$= 0.423$

$= 0.4$

ii)  $\dot{x} = 30 \cos 5$

$\dot{y} = -30 \sin 5 - 10t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\tan \theta = \frac{-30 \sin 5 - 10t}{30 \cos 5}$

$= -0.2213$

$\theta = -12.48^\circ$

$\therefore \theta = 12.48^\circ$  or  $167.52^\circ$

iii)  $x = 30t \cos 5$  — ①

$y = -30t \sin 5 - 5t^2$  — ②

from ①  $\Rightarrow t = \frac{x}{30 \cos 5}$  — ③

subst ③ into ②

$y = \frac{-30x \sin 5}{30 \cos 5} - \frac{5x^2}{900 \cos^2 5}$

$= \frac{-x^2}{180 \cos^2 5} - x \tan 5$

of the form  $y = ax^2 + bx + c$

$\therefore$  parabolic

c)  $\tan 26 = \frac{h}{XA}$

$XA = \frac{h}{\tan 26}$

$\tan 30 = \frac{h}{XC}$

$XC = \frac{h}{\tan 30}$

$\tan 28 = \frac{h}{XB}$

$XB = \frac{h}{\tan 28}$

i)  $\tan \angle XAC = \frac{XC}{XA}$

$= \frac{h}{\tan 30} \times \frac{\tan 26}{h}$

$= 0.84478$

$\angle XAC = 40.2^\circ$

ii)  $\frac{\sin \angle ABX}{XA} = \frac{\sin 40.2}{XB}$

$\sin \angle ABX = \frac{\sin 40.2 \times \tan 28}{\tan 26}$

$= 0.7037$

$\therefore \angle ABX = 45^\circ, 135^\circ$

$\therefore \angle ABX = 135^\circ$  or  $\angle ABX > 49.8^\circ$

iii)  $180 - 135 - 40 = 05^\circ T$  or  $N5^\circ E$