HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate
Trial Examination Term 3 2016

STUDENT NUMBER: ____

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators and drawing templates may be used
- A reference sheet is provided separately
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 70

Section I Pages 3 – 5

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 6 – 11

60 marks

Attempt Questions 11 – 14

Start each question in a new writing booklet

Write your student number on every writing booklet

Question	***1-10***	***11***	***12***	***13***	***14***	***Total***
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/70*

This assessment task constitutes 45% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 The cubic equation $2x^3 + x^2 - 32x - 16 = 0$ has roots α, $-\alpha$ and β.
The value of β is?

(A) $\frac{-1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 8

2 The point P divides the interval AB externally in the ratio $3:2$, where $A = (-5, 6)$ and $B = (-2, 3)$.
The x-value of P is:

(A) 1

(B) 2

(C) 4

(D) 5

3 The exact value of $\int_0^{\pi} \cos^2(3x)\,dx$ is

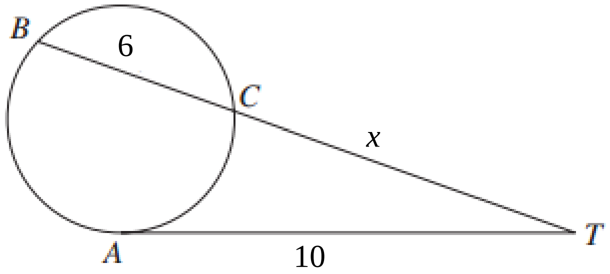
(A) $-\frac{\pi}{2}$

(B) $\frac{-2}{3}$

(C) 0

(D) $\frac{\pi}{2}$

4 The line AT is a tangent to the circle at A and BT is a secant meeting the circle at B and C. Given that $AT = 10$ cm and $BC = 6$, the value of x is closest to?



NOT TO SCALE

(A) 1.67

(B) 4

(C) 7.44

(D) 16.67

5 The domain of $y = 2 + \sin^{-1}\dfrac{2x}{3}$ is:

(A) $-\dfrac{3\pi}{2} \le x \le \dfrac{3\pi}{2}$

(B) $-\dfrac{2\pi}{3} \le x \le \dfrac{2\pi}{3}$

(C) $-\dfrac{2}{3} \le x \le \dfrac{2}{3}$

(D) $-\dfrac{3}{2} \le x \le \dfrac{3}{2}$

6 What is the value of $\lim\limits_{x \to \pi} \dfrac{\sin(x-\pi)\cos(2\pi - x)}{(x-\pi)}$?

(A) -1

(B) 0

(C) $\dfrac{1}{\pi}$

(D) 1

7 A particle with displacement x m from the origin has acceleration $\ddot{x} = -4x$. If initially the particle is stationary at $x = 2$, the expression for v^2 is

(A) $16 - 4x^2$

(B) $-4x^2$

(C) $-2x^2$

(D) $8 - 2x^2$

8 The solutions to $|2x+1|<|4-x|$ are:

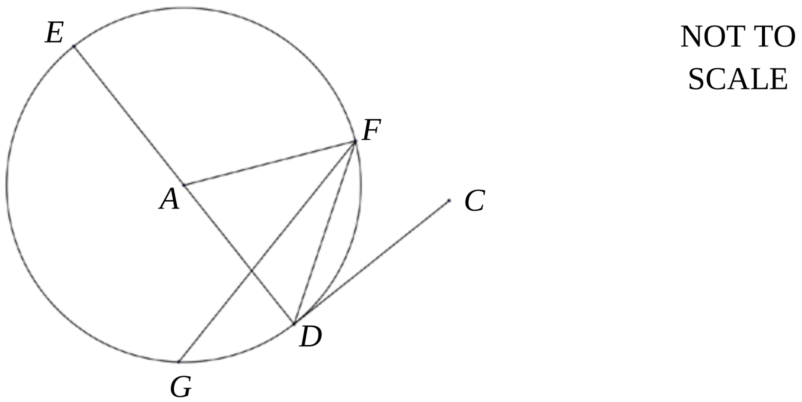
(A) $-5<x<1$

(B) $x>5$ or $x>1$

(C) $x>-1$

(D) *all real x*

9 In the diagram below, a tangent is drawn from C to meet the circle with centre A at the point D, where $\angle CDF=39°$. ED is a diameter of the circle.



The size of $\angle FAD$ is?

(A) $39°$

(B) $78°$

(C) $98°$

(D) $117°$

10 The primitive of $\dfrac{1}{\sqrt{4-9x^2}}$ is

(A) $\dfrac{1}{2}\sin^{-1}2x+C$

(B) $\dfrac{1}{2}\sin^{-1}3x+C$

(C) $\dfrac{1}{3}\sin^{-1}\dfrac{x}{2}+C$

(D) $\dfrac{1}{3}\sin^{-1}\dfrac{3x}{2}+C$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

(a) In how many ways can five different books be arranged in a row? **1**

(b) Solve the inequality $\dfrac{x-1}{x+1} \geq 2$. **3**

(c) The remainder when $P(x) = (ax+b)^3$ is divided by $(x-2)$ is 8 and the remainder when it is divided by $(x+3)$ is -27. Find the values of a and b **3**

(d) (i) Express $\cos\left(x+\dfrac{\pi}{3}\right)$ in the form $a\cos x - b\sin x$. **1**

(ii) Hence, or otherwise, solve $\cos\left(x+\dfrac{\pi}{3}\right) = \cos\left(x-\dfrac{\pi}{3}\right)$ for $-2\pi \leq x \leq 2\pi$. **2**

(e) Find the acute angle between the lines $3x-4y=3$ and $x-2y=11$, correct to the nearest minute. **2**

(f) Using the substitution $u = \dfrac{1}{2}x - 1$, show that $\displaystyle\int_4^6 \frac{x}{\sqrt{\frac{1}{2}x-1}}\,dx = \frac{8}{3}\left(5\sqrt{2}-4\right)$. **3**

Question 12 (15 marks) Start a new writing booklet

(a) Show that $\tan^{-1}\left(\frac{3}{4}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\frac{\pi}{2}$. **2**

(b) Let $f(x)=x^2-6x+2$.

(i) By expressing $f(x)$ in the form $(x-h)^2+k$, or otherwise, find the range of $f(x)$. **1**

(ii) The domain of $f(x)$ is restricted to the largest domain such that it has an inverse. **2**

The domain contains $x=5$. Find the equation of the inverse function $f^{-1}(x)$.

(iii) Find the points of intersection of $y=f(x)$ and $y=f^{-1}(x)$, given that $f(x)$ maintains **2**

its restricted domain.

(c) A circular stain is spreading so that the rate of increase of the radius is inversely proportional to the square root of the radius, $\frac{dr}{dt}=\frac{k}{\sqrt{r}}$. Initially the radius of the stain is 4 cm and it is increasing at a rate of $2\ \text{cms}^{-1}$.

(i) Find the value of k. **1**

(ii) Find the exact length of radius of the stain at the time when the area is increasing at a rate of $115\ \text{cm}^2\text{s}^{-1}$. **3**

(d) A secant PQ cuts the parabola $x^2=4ay$ at $P\left(2ap,ap^2\right)$ and $Q\left(2aq,aq^2\right)$ and makes an angle of $45°$ with the positive direction of the x-axis. M is the midpoint of PQ.

(i) Show that $p+q=2$. **2**

(ii) Find the Cartesian equation and give a geometrical description of the locus of M. **2**

Question 13 (15 marks) Start a new writing booklet

(a) A seafood salad is removed from the refrigerator and placed in a room. The salad is initially $4°C$ when removed from the refrigerator and the room has constant temperature $20°C$.

The rate of change of the temperature of the salad is proportional to the difference between the temperature, T, of the salad and the temperature of the room.

That is T satisfies the equation $\frac{dT}{dt} = k(T-20)$ where t is the number of minutes after the salad was removed from the refrigerator.

(i) Show that $T = 20 + Ae^{kt}$ is a solution to the differential equation. **1**

(ii) The temperature of the salad is $7°C$ after 15 minutes. **2**

Show that the value of k is $\frac{1}{15}\log_e\left(\frac{13}{16}\right)$.

(iii) The salad is no longer safe to eat if it spends 60 minutes over the temperature of $15°C$. **2**
How many minutes after the salad is removed from the refrigerator is it no longer safe to eat? Answer correct to the nearest minute.

(b) The rise and fall of the tide is assumed to be simple harmonic motion, with the time between successive high tides being 12 hours.

A ship is to sail from a wharf to a harbour entrance and then out to sea. On the morning the ship is to sail, high tide at the wharf occurs at 4 am. The water depths at the wharf at high tide and low tide are 8 metres and 2 metres respectively.

(i) The water depth, x metres, at time t hours after 4 am at the wharf is given by $x = b + a\cos nt$. **3**

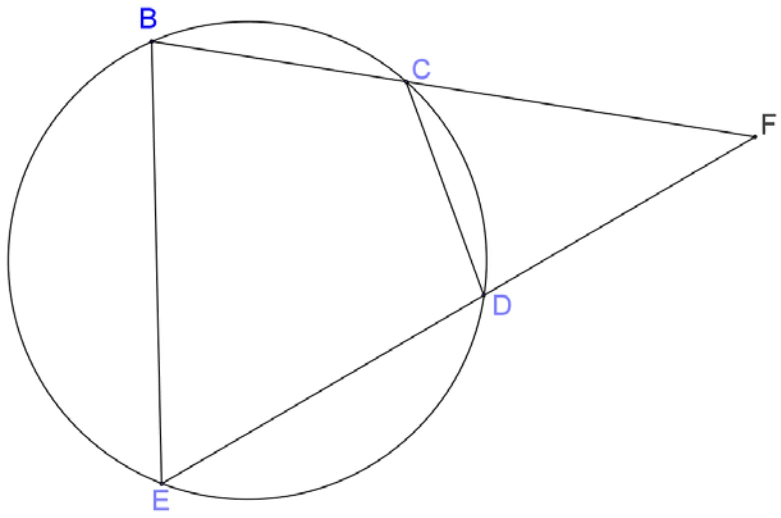
Show that $b = 5$, $n = \frac{\pi}{6}$ and $a = 3$.

(ii) An overhead fuel line obstructs the ship's exit from the wharf. The ship can only leave if the water depth at the wharf is 7 metres or less. It is also too shallow for the ship to safely leave the wharf if the water level is below 3 metres. **3**

Between what times can the ship first leave the wharf, correct to the nearest minute?

Question 13 continues on page 9

(c) The diagram below shows the points B, C, D E lying on the circumference of a circle. The secant through BC meets the secant through ED at the point F.



NOT TO SCALE

(i) Prove ΔFCD is similar to ΔFEB. **2**

(ii) If $BE:CD = 2:1$ and the area of ΔFCD is 6 square units, find the area of the cyclic quadrilateral $BCDE$. **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet

(a) Prove, by mathematical induction, that $n^3 + 5n$ is divisible by 3, for all positive integer n. **3**

(b) In a doctor's waiting room, there are 14 seats in a row. Eight people are waiting to be seen. Three of the people are in the same family and they must sit in three seats that are together. In how many ways can the eight people be seated? **2**

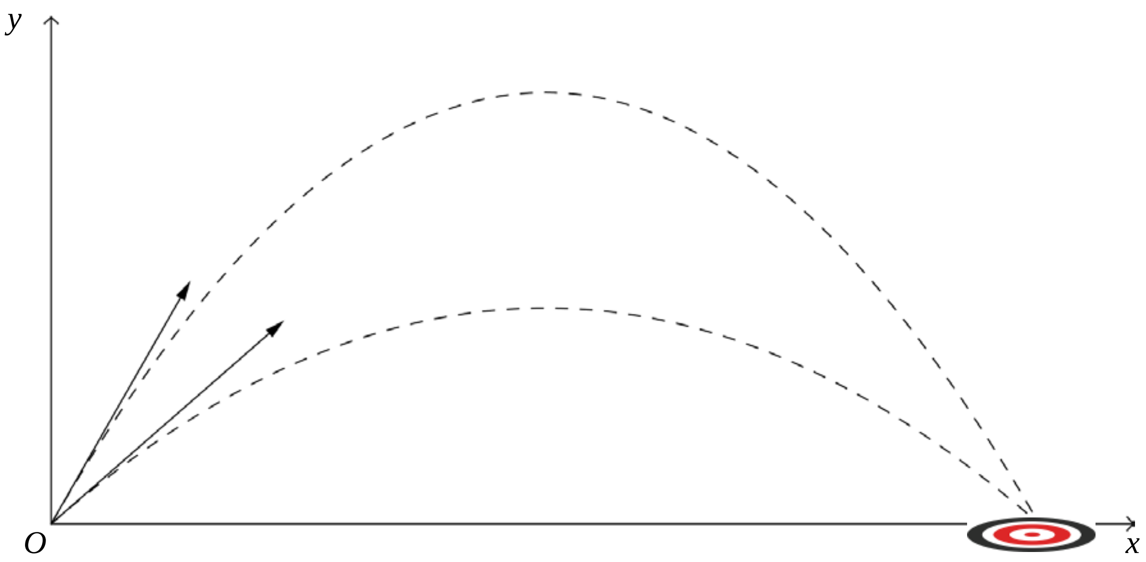
(c) Consider the series $\log_e \frac{a^3}{\sqrt{b}} + \log_e \frac{a^3}{b} + \log_e \frac{a^3}{b\sqrt{b}} + \log_e \frac{a^3}{b^2} + \ldots$.

(i) Prove that the series is an arithmetic series and state the first term and common difference. **2**

(ii) Find an expression for the sum of the first 23 terms of the series giving your answer in the form $\log_e \frac{a^m}{b^n}$ where m and n are integers. **2**

Question 14 continues on page 11

(d) Two archers at the same location each shoot one arrow at the same time at a target which lies on the same horizontal plane as the point of projection of the arrows.
The archers shoot with the same initial velocity $25\ \text{ms}^{-1}$ but with different angles of projection, with one archer's angle of projection $\frac{\pi}{3}$. Their arrows hit the target at different times.



NOT TO SCALE

The equations of motion when a projectile is projected from the origin with angle θ to the horizontal with initial velocity V are $x = Vt\cos\theta$ and $y = Vt\sin\theta - 5t^2$ where x and y are the horizontal and vertical positions in metres t seconds after projection.
Do NOT prove this result.

(i) Show that the distance of the target from the archers is $\frac{125\sqrt{3}}{4}$ metres. **2**

(ii) Find the angle of elevation of the other archer's arrow. **2**

(iii) Find the exact time which elapses between when the two arrows hit the target. **2**

End of Examination

Year 12 Extension 1 2016 Trial Examination Solutions
Multiple Choice
Question 1

$\alpha-\alpha+\beta=\frac{-1}{2}$

$\beta=\frac{-1}{2}$

(A)

Question 2

$x_1=-5$

$m=3$

$x_2=-2$

$n=-2$

$x=\frac{-2\times-5+3\times-2}{2-1}$

$=10-6$

$=4$

(C)

Question 3

$\cos 2x=\cos^2-\sin^2 x$

$=\cos^2 x-\left(1-\cos^2 x\right)$

$=2\cos^2 x-1$

$\cos 2x+1=2\cos^2 x$

$\cos^2 x=\frac{1}{2}\left(\cos 2x+1\right)$

$\cos^2 3x=\frac{1}{2}\left(\cos 6x+1\right)$

$I=\frac{1}{2}\int_0^{\pi}\left(\cos 6x+1\right)dx$

$=\frac{1}{2}\left[\frac{1}{6}\sin 6x+x\right]_0^{\pi}$

$=\frac{1}{2}\left[\frac{1}{6}\sin 6x+\pi-\frac{1}{6}\sin-0\right]$

$=\frac{\pi}{2}$

(D)

Question 4

$10^2 = x(x+6)$

$100 = x^2 + 6x$

$x^2 + 6x + 9 = 109$

$(x+3)^2 = 109$

$x = \sqrt{109} - 3 \ (x > 0)$

$x = 7.44030...$

(C)

Question 5

$-1 \le \frac{2x}{3} \le 1$

$-3 \le 2x \le 3$

$\frac{-3}{2} \le x \le \frac{3}{2}$

(D)

Question 6

$$\lim_{x\to\pi} \frac{\sin(x-\pi)\cos(2\pi - x)}{(x-\pi)} = \lim_{x\to\pi} \frac{\sin(x-\pi)}{(x-\pi)} \times \lim_{x\to\pi} \cos(2\pi - x)$$

$$= 1 \times \cos\pi$$

$$= -1$$

Question 7

$\ddot{x} = -4x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$

$\frac{1}{2}v^2 = -2x^2 + C$

When $x = 2, v = 0$

$0 = -2 \times 4 + C$

$C = 8$

$\frac{1}{2}v^2 = -2x^2 + 8$

$v^2 = -4x^2 + 16$

$= 16 - 4x^2$

(A)

Question 9

$\angle DEF = 39°$ (angle in the alternate segment theorem)

$\angle DAF = 2 \times 39°$ (angle at the centre is twice the angle at the circumference subtended by the same arc)

$= 78°$

(B)

Question 10:

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}\,dx$$

$$= \frac{1}{2}\sin^{-2}\frac{3x}{2} + C$$

(D)

Question 11

(a)

$$5! = 120$$

(b)

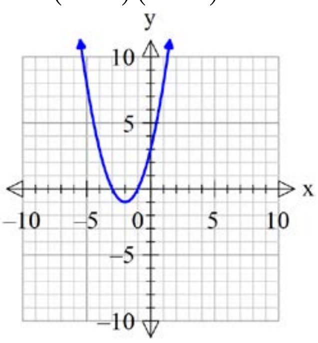
$$\frac{x-1}{x+1} \geq 2, \quad x \neq -1$$

$$(x+1)(x-1) \geq 2(x+1)^2$$

$$0 \geq 2(x+1)^2 - (x+1)(x-1)$$

$$0 \geq (x+1)[2x+2-x+1]$$

$$0 \geq (x+1)(x+3)$$



$$-3 \leq x < 1$$

(c)

$$P(2) = 8$$

$$P(-3) = -27$$

$$8 = (2a+b)^3$$

$$2a + b = 2 \quad \ldots (1)$$

$$-27 = (-3a+b)^3$$

$$-3a + b = -3 \quad \ldots (2)$$

(1)-(2):

$$5a = 5$$

$$a = 1$$

$$-3 + b = -3$$

$$\therefore b = 0, a = 1$$

(d)
(i)

$$\cos\left(x+\frac{\pi}{3}\right)=\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}$$
$$=\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x$$

(ii)

$$\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$$
$$\sqrt{3}\sin x=0$$
$$\sin x=0$$
$$\therefore x=-2\pi,\pi,0,\pi,2\pi$$

(e)

$$3x-4y=3$$
$$4y=3x-3 \qquad 2y=x-11$$
$$y=\frac{3}{4}x-\frac{3}{4} \qquad y=\frac{1}{2}x-\frac{11}{2}$$
$$m_1=\frac{3}{4} \qquad m_2=\frac{1}{2}$$

$$\tan\theta=\left|\frac{\frac{3}{4}-\frac{1}{2}}{1+\frac{3}{4}\times\frac{1}{2}}\right|$$
$$\theta=\tan^{-1}\left(\frac{2}{11}\right)$$
$$=10°18' \text{ (nearest minute)}$$

(f)

$$u = \frac{1}{2}x - 1$$

$$du = \frac{1}{2}dx \qquad \textit{When } x = 4, u = 1$$

$$dx = 2du \qquad \textit{When } x = 6, u = 2$$

$$\int_2^4 \frac{x}{\sqrt{\frac{1}{2}x - 1}}dx = 4\int_1^2 \frac{u+1}{\sqrt{u}}du$$

$$= 4\int_1^2 \left(u^{\frac{1}{2}} + u^{\frac{-1}{2}}\right)du$$

$$= 4\left[\frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}}\right]_1^2$$

$$= 4\left(\frac{2}{3}\sqrt{8} + 2\sqrt{2} - \frac{2}{3} - 2\right)$$

$$= 4\left(\frac{4}{3}\sqrt{2} + \frac{6\sqrt{2}}{3} - \frac{8}{3}\right)$$

$$= \frac{4}{3}\left(10\sqrt{2} - 8\right)$$

$$= \frac{8}{3}\left(5\sqrt{2} - 4\right)$$

Question 12

(a)

Let $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ and $\beta = \cos^{-1}\left(\frac{3}{5}\right)$ α in 1st quad, β in first quad

$$\therefore \tan\beta = \frac{4}{3}$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{\frac{3}{4} + \frac{4}{3}}{1 - \frac{3}{4}\times\frac{4}{3}}$$

$$= \frac{\frac{3}{4} + \frac{4}{3}}{0}$$

$$\therefore \alpha + \beta = \frac{\pi}{2} \text{ since } \alpha, \beta \textit{ acute}$$

(b)

$f(x) = x^2 - 6x + 2$

(i)

$f(x) = x^2 - 6x + 9 - 7$

$= (x-3)^2 - 7$

Range is $f(x) \geq -7$

(ii)

Largest domain is $x \geq 3$ if it contains $x = 5$

Finding inverse:

$x = (y-3)^2 - 7$

$x + 7 = (y-3)^2$

$y = 3 \pm \sqrt{x+7}$

$but\ y \geq 3$

$\therefore f^{-1}(x) = 3 + \sqrt{x+7}$

(iii)

$Solve\,(x-3)^2 - 7 = x$

$x^2 - 7x + 2 = 0$

$$x = \frac{7 \pm \sqrt{49 - 4\times 1\times 2}}{2\times 1}$$

$$= \frac{7 \pm \sqrt{41}}{2}$$

$$= \frac{7 + \sqrt{41}}{2} \text{ since } x \geq 3$$

$$\therefore \left(\frac{7+\sqrt{41}}{2}, \frac{7+\sqrt{41}}{2}\right) \text{is the point of intersection}$$

(c)

(i)

$$\frac{dr}{dt} = \frac{k}{\sqrt{r}}$$

$$2 = \frac{k}{\sqrt{4}}$$

$k = 4$

(ii)

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$115 = 2\pi r \times \frac{4}{\sqrt{r}}$$

$$\sqrt{r} = \frac{115}{8\pi r}$$

$$r = \frac{115^2}{64\pi^2}$$

$$= \frac{13225}{64\pi^2}$$

$$= 65.8\,cm^2$$

(d)

(i)

$$m_{PQ} = 1$$

$$\frac{aq^2 - ap^2}{2aq - 2ap} = 1$$

$$\frac{a(q-p)(q+p)}{2a(q-p)} = 1$$

$$\frac{q+p}{2} = 1$$

$$q + p = 2$$

(ii)

$$M = \left(\frac{2aq + 2ap}{2}, \frac{aq^2 + ap^2}{2} \right)$$

$$x_M = a(p+q)$$

$$= 2a$$

Therefore the equation of the locus of M is the vertical line $x = 2a$

Question 13

(a)

(i)

$$T = 20 + Ae^{kt}$$

$$\frac{dT}{dt} = kAe^{kt}$$

$$= k\left(Ae^{kt} + 20 - 20\right)$$

$$= k\left(T - 20\right)$$

(ii)

$$When\ t = 0, T = 4$$

$$4 = 20 + Ae^{0}$$

$$\therefore A = -16$$

$$T = 20 - 16Ae^{kt}$$

Let $t = 15, T = 7$

$$7 = 20 - 16e^{15k}$$

$$-13 = -16e^{15k}$$

$$\frac{13}{16} = e^{15k}$$

$$15k = \ln\left(\frac{13}{16}\right)$$

$$k = \frac{1}{15}\ln\left(\frac{13}{16}\right)$$

(iii)

Let $T = 15$

$$15 = 20 - 16e^{kt}$$

$$-5 = -16e^{kt}$$

$$\frac{5}{16} = e^{kt}$$

$$kt = \ln\frac{5}{16}$$

$$t = \frac{1}{k}\ln\frac{5}{16} \quad where\ k = \frac{1}{5}\ln\left(\frac{13}{16}\right)$$

$t = 84.0267555..$

Not safe to eat after

$84.0267555 + 60$

$= 144$ minutes (nearest minute)

(b)
(i)
$x = b + a\cos nt$
Period=12 hours

$$\frac{2\pi}{n} = 12$$

$$n = \frac{2\pi}{12}$$

$$= \frac{\pi}{6}$$

At $t = 0$, $x = 8$
$8 = b + a ...(1)$
At $t = 6, x = 2$

$$2 = b + a\cos\frac{\pi}{6}\times 6$$

$2 = b - a ..(2)$
(1)+(2):
$10 = 2b$
$b = 5$
Put $b = 5$ into (1)
$8 = 5 + a$
$a = 3$

(ii)
Let $x = 7$

$$7 = 5 + 3\cos\frac{\pi}{6}t$$

$$\frac{2}{3} = \cos\frac{\pi}{6}t$$

$$\frac{\pi}{6}t = \cos^{-1}\left(\frac{2}{3}\right)$$

$$t = \frac{6}{\pi}\cos^{-1}\left(\frac{2}{3}\right)$$

=1 hour 36 minutes

Let $x = 3$

$$3 = 5 + 3\cos\frac{\pi}{6}t$$

$$\frac{-2}{3} = \cos\frac{\pi}{6}t$$

$$\frac{\pi}{6}t = \cos^{-1}\left(\frac{-2}{3}\right)$$

$$t = \frac{6}{\pi}\cos^{-1}\left(\frac{-2}{3}\right)$$

$= 4\ hours\ 24\ \min utes$

Therefore the ship can leave safely between 5:36 am and 8:24 am

(c)
(i)
In ΔFCD and ΔFEB
$\angle FCD = \angle FEB$ (exterior angle equals the opposite interior angle in cyclic quadrilateral)
Similarly, $\angle FDC = \angle FBE$
$\angle CFD = \angle EFB$ (common)
$\therefore \Delta FCD \,|||\, \Delta FEB$ (equiangular)

(ii)
$BE : CD = 2:1$

$$A_{FCD} = \frac{1}{2} \times FC \times FD \times \sin \angle CFD$$

$$\therefore \frac{1}{2} \times FC \times FD = \frac{6}{\sin \angle CFD}$$

$$A_{FBE} = \frac{1}{2} \times FE \times FB \times \sin \angle CFD$$

$$= \frac{1}{2} \times FC \times FD \times 4 \sin \angle CFD \ (\textit{matching sides are in the same ratio}, \Delta FCD \,|||\, \Delta FEB)$$

$$= \frac{6}{\sin \angle CFD} \times 4 \sin \angle CFD$$

$$= 24\, units^2$$

$$A_{CDEB} = 24 - 6$$

$$= 18\, units^2$$

Question 14
(a)
Step 1: Prove true for $n = 1$
$1^3 + 5 \times 1 = 6$
$= 3 \times 2$ which is divisible by 3

Step 2: Assume true for $n = k$
$k^3 + 5k = 3M$ where M is an integer

Step 3: Let $n = k + 1$

$$(k+1)^3 + 5(k+1) = k^3 + 3k^2 + 3k + 1 + 5k + 5$$

$$= 3M - 5k + 3k^2 + 3k + 6 + 5k$$

$$= 3M + 3k^2 + 3k + 6$$

$$= 3(M + k^2 + k + 2) \ \textit{where } M + k^2 + k + 2 \text{ is an integer}$$

$$\therefore \textit{Divisble by } 3$$

Therefore true for $n = k + 1$ is true for $n = k$

Therefore by the principle of mathematical induction it is true for all positive integers n

(b)
${}^{11}P_5 \times 3! \times 12 = 3991680$

(c)
(i)

$$\ln\frac{a^3}{\sqrt{b}}+\ln\frac{a^3}{b}+\ln\frac{a^3}{b\sqrt{b}}+\ldots$$

$$T_2-T_1=\ln\frac{a^3}{b}-\ln\frac{a^3}{\sqrt{b}}$$

$$=\ln\left(\frac{a^3}{b}\times\frac{\sqrt{b}}{a^3}\right)$$

$$=\ln\frac{1}{\sqrt{b}}$$

$$T_3-T_2=\ln\frac{a^3}{b\sqrt{b}}-\ln\frac{a^3}{b}$$

$$=\ln\left(\frac{a^3}{b\sqrt{b}}\times\frac{b}{a^3}\right)$$

$$=\ln\frac{1}{\sqrt{b}}$$

$$\therefore T_3-T_2=T_2-T_1$$

$$\therefore \textit{Arithmetic Series with } a=\ln\frac{a^3}{\sqrt{b}},\ d=\ln\frac{1}{\sqrt{b}}$$

(ii)
Sum of the first 23 term

$$=\frac{23}{2}\left(2\times\ln\frac{a^3}{\sqrt{b}}+22\ln\frac{1}{\sqrt{b}}\right)$$

$$=23\left(\ln\frac{a^3}{\sqrt{b}}+11\ln\frac{1}{\sqrt{b}}\right)$$

$$=23\left(\ln\frac{a^3}{\sqrt{b}}+\ln b^{-\frac{11}{2}}\right)$$

$$=23\left(\ln\frac{a^3b^{-\frac{11}{2}}}{b^{\frac{1}{2}}}\right)$$

$$=23\ln\left(a^3b^{-6}\right)$$

$$=\ln\left(\left(\frac{a^3}{b^6}\right)^{23}\right)$$

$$=\ln\left(\frac{a^{69}}{b^{138}}\right)$$

(d)
(i)
Put $y=0$ in
$$y = Vt\sin\theta - 5t^2$$
$$Vt\sin\theta - 5t^2 = 0$$
$$t = 0 \; or \; t = \frac{V\sin\theta}{5}$$

When $V = 25, \sin\theta = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$
Therefore,
$$t = \frac{25}{5}\times\frac{\sqrt{3}}{2}$$
$$= \frac{5\sqrt{3}}{2}$$
Sub into x
$$x = Vt\cos\theta$$
$$= 25\times\frac{5\sqrt{3}}{2}\times\cos\frac{\pi}{3}$$
$$= 25\times\frac{5\sqrt{3}}{2}\times\frac{1}{2}$$
$$= \frac{125\sqrt{3}}{4}$$
The distance of the target from the archers is $\frac{125\sqrt{3}}{4}$ *metres*

(ii) Let the other archers angle of project be α
Therefore time of flight is $\frac{25\sin\alpha}{5} = 5\sin\alpha$
Sub $t = 5\sin\alpha, x = \frac{125\sqrt{3}}{4}$
$$\frac{125\sqrt{3}}{4} = 25\times 5\sin\alpha\times\cos\alpha$$
$$\frac{\sqrt{3}}{4} = \sin\alpha\cos\alpha$$
$$\frac{\sqrt{3}}{4} = \frac{1}{2}\sin 2\alpha$$
$$\frac{\sqrt{3}}{2} = \sin 2\alpha$$
$$2\alpha = \frac{\pi}{3}$$
$$\alpha = \frac{\pi}{6}$$

(iii)

$$t_1 = \frac{5\sqrt{3}}{2}$$

$$t_2 = 5\sin\frac{\pi}{6}$$

$$= \frac{5}{2}$$

Therefore the time that elapses between the two arrows hitting their target is $\frac{5}{2}\left(\sqrt{3}-1\right)\text{seconds}$.