HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate
Trial Examination Term 3 2017

STUDENT NUMBER: ______________________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- A reference sheet is provided separately
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination room.

Total marks – 70

Section I Pages 3 – 6

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 7 – 13

60 marks

Attempt Questions 11 – 14

Start each question in a new writing booklet

Write your student number on every writing booklet

Question	*1-10*	*11*	*12*	*13*	*14*	*Total*
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/70*

This assessment task constitutes 45% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

---

1 What is the domain of $y = \sin^{-1}\dfrac{x}{2}$?

(A) $-\pi \le x \le \pi$

(B) $-2 \le x \le 2$

(C) $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$

(D) $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$

2 A particle is moving in one dimension. Given that $v^2 = 4x^2 - 8$, where *x* is displacement in metres and *v* is velocity in metres per second, the acceleration of the particle at 2 metres right of the origin is:

(A) $0\ \text{ms}^{-2}$

(B) $2\ \text{ms}^{-2}$

(C) $4\ \text{ms}^{-2}$

(D) $8\ \text{ms}^{-2}$

3 The number of followers of an Instagram account (*N*) grows according to the formula $N = 80 + 20e^{kt}$, where *t* is time in hours and $k = 0.2$.

How long will it take for the Instagram account to have 1000 more followers than it started with?

(A) 19 hours 8 minutes

(B) 19 hours 34 minutes

(C) 19 hours 40 minutes

(D) 20 hours 6 minutes

4 Which group of three numbers could be the roots of the polynomial equation $x^3 + 9x^2 + 8x + d = 0$?

(A) $-6, -5, 2$

(B) $-20, 6, 5$

(C) $-3, -3, -3$

(D) $-1, -2, -6$

5 The acute angle between the lines $3x + 2y - 7 = 0$ and $2x - y + 1 = 0$ is closest to:

(A) $14°$

(B) $41°$

(C) $60°$

(D) $76°$

6 What is the derivative of $2\cos^{-1} 3x$?

(A) $\dfrac{2}{\sqrt{1-9x^2}}$

(B) $\dfrac{-2}{\sqrt{1-9x^2}}$

(C) $\dfrac{-6}{\sqrt{1-9x^2}}$

(D) $\dfrac{6}{\sqrt{1-9x^2}}$

7 Let $|b| \le 1$. What is the general solution to $\cos\dfrac{x}{2} = b$?

(A) $k\pi \pm \cos^{-1} b$

(B) $2k\pi \pm \cos^{-1} b$

(C) $4k\pi \pm \cos^{-1} b$

(D) $4k\pi \pm 2\cos^{-1} b$

8 The point P divides the interval $A(-2,-2)$ to $B(6,4)$ externally in the ratio $1:3$.

The y-value of P is:

(A) -5

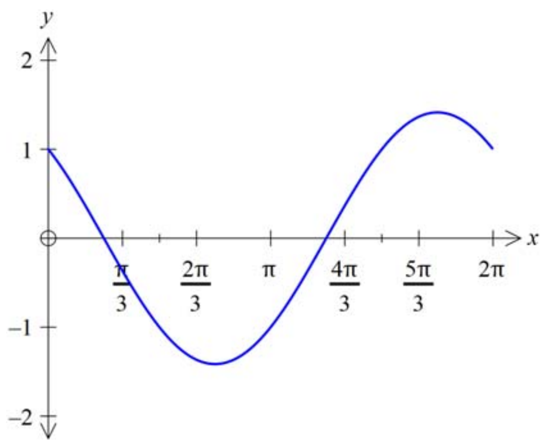
(B) -1

(C) 1

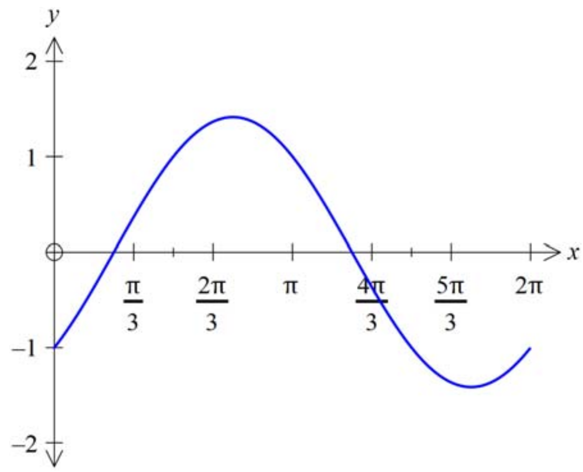
(D) 2

9 The graph of $y=\cos x-\sin x$ for $0\le x\le 2\pi$ is:

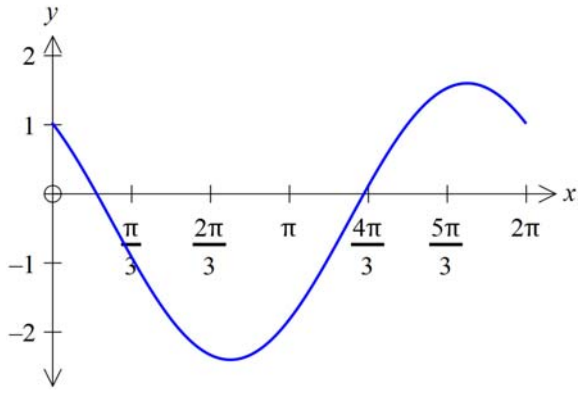
(A)



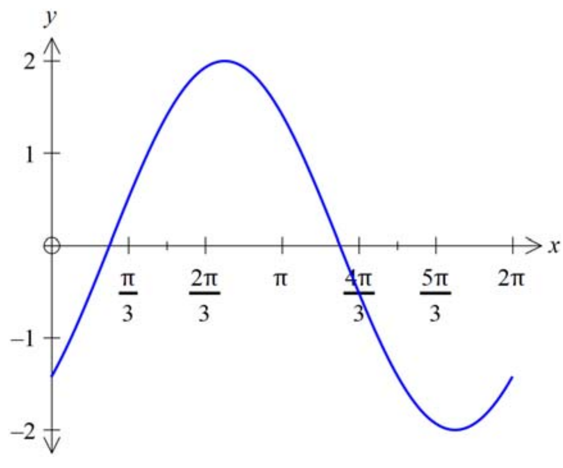
(B)



(C)



(D)



10 The primitive function of $\dfrac{1}{x^2+2x+3}$ is:

(A) $\log_e\left(x^2+2x+3\right)+C$

(B) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{x+1}{2}\right)+C$

(C) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x+1}{\sqrt{2}}\right)+C$

(D) $\dfrac{1}{4}\tan^{-1}\left(\dfrac{x+1}{4}\right)+C$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

(a) Solve $\dfrac{2}{x-1} \geq x$. **3**

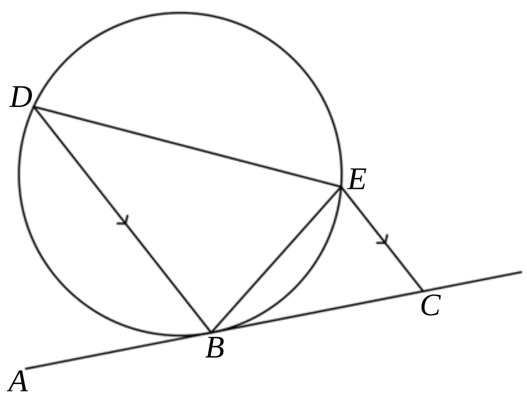
(b) Find the exact value of $\int_0^{\pi} \cos^2 3x \, dx$. **3**

(c) (i) Expand $\left(x+\dfrac{2}{x}\right)^5$, giving your answer in simplest form. **2**

(ii) Hence, simplify $\left(x+\dfrac{2}{x}\right)^5 + \left(x-\dfrac{2}{x}\right)^5$. **2**

(d) Evaluate $\displaystyle\int_0^1 \frac{e^x}{\sqrt{e^x+1}} dx$, using the substitution $u = e^x$, giving your answer in exact form. **3**

(e) In the diagram below, AC is a tangent to the circle at B and $BD \parallel CE$. **2**



NOT TO SCALE

Prove that $\angle BCE = \angle BED$

Question 12 (15 marks) Start a new writing booklet

(a) The depth of water in a harbour varies during the day and is given by the equation $d = 16 + 7\sin\left(\frac{\pi}{12}t\right)$, where d is measured in metres and t is the number of hours after midnight.

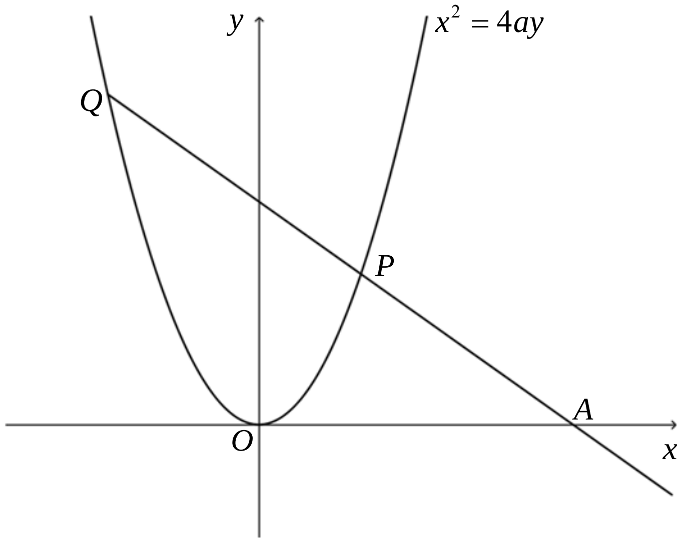
(i) Find the depth of the water at low and high tide. **1**

(ii) At what time(s) does high tide occur during the day? **1**

(iii) It is safe for a cruise ship to be in the harbour when the depth is more than 20 metres. Find between what times during the day it is safe for the cruise ship to be in the harbour. **2**

Give your answer correct to the nearest minute.

(b) The chord QP joining the points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ on $x^2 = 4ay$ always passes through the point A when produced.



NOT TO SCALE

(i) Show that the equation of the chord PQ is $(p+q)x - 2y = 2apq$. **2**

(ii) A has coordinates $(2a, 0)$. Show that $p + q = pq$. **1**

Let $M(x, y)$ be the midpoint of QP.

(iii) Show that the Cartesian Equation of the locus of M is a parabola with equation $(x-a)^2 = 2ay + a^2$. **2**

(iv) Hence, find the focus and directrix of the locus of M. **2**

Question 12 continues on page 9

Question 12 (continued)

(c) Let $f\left(x\right)=\log_e\left(x-1\right)+\log_e 3$, for $x>1$.

(i) Show that $f^{-1}\left(x\right)=\dfrac{e^x}{3}+1$. **1**

(ii) Show that $y=f\left(x\right)$ and $y=f^{-1}\left(x\right)$ intersect when $3x-3=e^x$. **1**

(iii) Hence, show graphically, or otherwise, that $y=f\left(x\right)$ and $y=f^{-1}\left(x\right)$ do not intersect. **2**

End of Question 12

Question 13 (15 marks) Start a new writing booklet

(a) 12 people need to be split up into teams for a quiz.

(i) Explain why the number of ways of splitting them into two groups of the same size is $\frac{1}{2}\,{}^{12}C_6$. **1**

(ii) How many ways are there of splitting them into two groups of any size (but there must be at least one person in each group)? **2**

(iii) At the end of the quiz, the 12 people sit around a circular table to have dinner. In how many ways can they be seated? **1**

(b) Consider the polynomial $P(x) = x^3 + (p-2)x^2 + (5-2p)x - 10$.

(i) Show that $(x-2)$ is a factor of $P(x)$. **1**

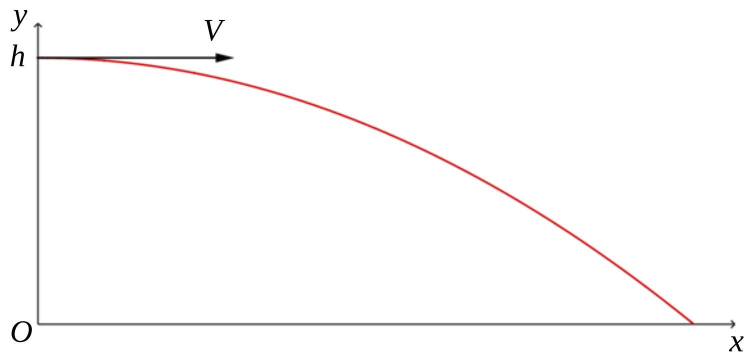
(ii) By performing long division, show that $P(x) = (x-2)(x^2 + px + 5)$. **1**

(iii) Find the exact values of p for which $P(x) = 0$ has only two distinct real roots. **2**

(iv) For the larger of the two values of p found in (iii), sketch $y = P(x)$ showing the x- and y-intercept(s). **2**

Question 13 continues on page 11

(c) A particle is projected horizontally with velocity V ms^{-1}, from a point h metres above the ground. Take $g = 10$ ms^{-2} as the acceleration due to gravity.



NOT TO SCALE

Taking the origin as the point on the ground vertically below the point of projection, horizontal displacement is given by $x = Vt$. **Do not prove this result.**

(i) Derive the expression for vertical displacement y. **1**

(ii) Show that the equation of the path is given by $y = h - \dfrac{5x^2}{V^2}$. **1**

(iii) Show that the range of the particle is $\dfrac{V\sqrt{h}}{\sqrt{5}}$ metres. **1**

(iv) If the projectile hits the ground at an angle of $\dfrac{\pi}{4}$ with the negative direction of the x-axis, find V in terms of h. **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet

(a) The expansion of $(a+x)^n$ has the form $a^n + \ldots + \alpha x^r + \beta x^{r+1} + \gamma x^{r+2} + \ldots + x^n$, where n is a positive integer, a is a non-zero constant and α, β and γ are coefficients.

(i) Show that $\dfrac{\alpha}{\beta} = \dfrac{a(r+1)}{n-r}$. **2**

(ii) Show that if $a = 1$ and the expansion contains two consecutive terms with equal coefficients, then n will be odd. **1**

(iii) Using the result from (i), deduce an expression for $\dfrac{\beta}{\gamma}$. **1**

(iv) Prove that there are no values for a such that the expansion of $(a+x)^n$ has three consecutive terms with the same coefficient. **2**

Question 14 continues on page 13

Question 14 (continued)

(b) The function f is defined by $f(x) = e^{px}(x+1)$ where p is a non-zero real number.

(i) Show that $f'(x) = e^{px}\left[p(x+1)+1\right]$. **1**

(ii) Let $f^{(n)}(x)$ denote the result of differentiating $f(x)$ with respect to x, n times. **2**

Use mathematical induction to prove that for any positive integer n that

$f^{(n)}(x) = p^{n-1}e^{px}\left[p(x+1)+n\right]$.

(iii) When $p = \sqrt{3}$, the graph of the function has exactly one point of inflexion. **1**
Without proving a change in concavity, find the exact value of the x-coordinate of the point of inflexion.

Consider the function when $p = \dfrac{1}{2}$, that is $f(x) = e^{\frac{x}{2}}(x+1)$.

The graph of $y = e^{\frac{x}{2}}(x+1)$ has a minimum turning point at $\left(-3, \dfrac{-2}{e\sqrt{e}}\right)$, and a point of inflexion at $\left(-5, \dfrac{-4}{e^2\sqrt{e}}\right)$.

(iv) Sketch $y = e^{\frac{x}{2}}(x+1)$, showing the stationary point, the point of inflexion, the intercepts with the coordinate axes and any asymptotes. **2**

It is known that $\dfrac{d}{dx}\left[\dfrac{e^{px}}{p}(x+1) - \dfrac{e^{px}}{p^2}\right] = e^{px}(x+1)$. **Do not prove this result.**

(v) Let R be the region enclosed by the curve $y = e^{\frac{x}{2}}(x+1)$, the x-axis and the lines $x = -2$ and $x = 2$. **3**

Show that the area of R is $\dfrac{2\left(e^2 + 4\sqrt{e} - 3\right)}{e}$ square units.

End of Paper

HSC Mathematics Extension 1 Trial Examination 2017 HGHS Solutions

Multiple Choice

1.

$y = \sin^{-1}\frac{x}{2}$

$-1 \le \frac{x}{2} \le 1$

$-2 \le x \le 2$

(B)

2.

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(2x^2 - 4\right)$

$= 4x$

When $x = 2$, $\ddot{x} = 8$

(D)

3.

$N = 80 + 20e^{kt}$

Let $t = 0$

$N = 80 + 20$

100

Let $N = 1100$

$1100 = 80 + 20e^{kt}$

$e^{kt} = \frac{1020}{20}$

$t = \frac{1}{k}\ln\frac{1020}{20}$

$= 19.65...$

$= 19$ hours 40 minutes

(C)

4.

$-6 + -5 + 2 = -9$

$-6 \times -5 + 2 \times -5 + 2 \times -6 = 8$

(A)

5.

$3x+2y-7=0$

$2x-y+1=0$

$m_1=\frac{-3}{2}$

$m_2=2$

$$\tan\theta=\left|\frac{2+\frac{3}{2}}{1+2\times\frac{-3}{2}}\right|$$

$$=\frac{7}{4}$$

$\theta=60°$ (nearest degree)

(C)

6.

$y=2\cos^{-1}u, u=3x$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

$$=\frac{-2}{\sqrt{1-u^2}}\times 3$$

$$=\frac{-6}{\sqrt{1-(3x)^2}}$$

$$=\frac{-6}{\sqrt{1-9x^2}}$$

(C)

7.

$$\cos\frac{x}{2}=b$$

$$\frac{x}{2}=2k\pi\pm\cos^{-1}b$$

$$x=4k\pi\pm2\cos^{-1}b$$

(D)

8.

$y_1=-2,\ y_2=4$

$m=-1, n=3$

$$y=\frac{-1\times4+3\times-2}{-2+3}$$

$$=\frac{-10}{2}$$

$$=-5$$

(A)

9.

$$y = \cos x - \sin x$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)$$

$$= \sqrt{2}\left(\cos\left(x + \frac{\pi}{4}\right)\right)$$

By observation, (A) or (B) match amplitude

Check $x = 0$

$$y = \sqrt{2}\left(\cos\frac{\pi}{4}\right)$$

$$= 1$$

(A)

10.

$$\int \frac{1}{x^2 + 2x + 3}\,dx = \int \frac{1}{2 + (x+1)^2}dx$$

$$= \frac{1}{\sqrt{2}}\tan\left(\frac{x+1}{\sqrt{2}}\right) + C$$

(C)

Question 11

(a)

$$\frac{2}{x-1} \geq x \quad x \neq 1$$

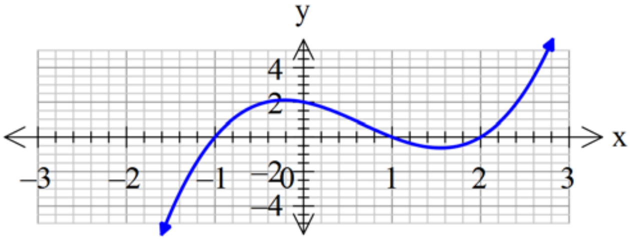
$$2(x-1) \geq x(x-1)^2$$

$$0 \geq x(x-1)^2 - 2(x-1)$$

$$(x-1)[x(x-1)-2] \leq 0$$

$$(x-1)(x^2-x-2) \leq 0$$

$$(x-1)(x-2)(x+1) \leq 0$$



From graph $x \leq -1 \, or \; 1 < x \leq 2$

(b)

$$\cos 2x = 2\cos^2 x - 1$$

$$\frac{1}{2}(\cos 2x + 1) = \cos^2 x$$

$$\cos^2 3x = \frac{1}{2}(\cos 6x + 1)$$

$$\begin{aligned}
\int_0^{\pi} \cos^2 3x dx &= \frac{1}{2}\int_0^{\pi} (\cos 6x + 1)\, dx \\
&= \frac{1}{2}\left[\frac{1}{6}\sin 6x + x\right]_0^{\pi} \\
&= \frac{1}{2}\left[\left(\frac{1}{6}\sin 6\pi + \pi\right) - \left(\frac{1}{6}\sin 0 + 0\right)\right] \\
&= \frac{1}{2} \times \pi \\
&= \frac{\pi}{2}
\end{aligned}$$

(c)

(i)

$$\begin{aligned}
\left(x + \frac{2}{x}\right)^5 &= x^5 + 5x^4 \times \frac{2}{x} + 10x^3 \times \frac{4}{x^2} + 10x^2 \times \frac{8}{x^3} + 5x \times \frac{16}{x^4} + \frac{32}{x^5} \\
&= x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}
\end{aligned}$$

(ii)

$$\left(x - \frac{2}{x}\right)^5 = x^5 - 10x^3 + 40x - \frac{80}{x} + \frac{80}{x^3} - \frac{32}{x^5}$$

$\therefore\left(x+\frac{2}{x}\right)^5+\left(x-\frac{2}{x}\right)^5=2\left(x^5+40x+\frac{80}{x^3}\right)$ $=2x^5+80x+\frac{160}{x^3}$	
(d) Let $u=e^x$ $\frac{du}{dx}=e^x$ $du=e^x dx$ When $x=0$, $u=1$ When $x=1, u=e$ $\int_0^1\frac{e^x}{\sqrt{e^x+1}}dx=\int_1^e(u+1)^{\frac{-1}{2}}du$ $=2\left[\sqrt{u+1}\right]_1^e$ $=2\left(\sqrt{e+1}-\sqrt{1+1}\right)$ $=2\sqrt{e+1}-2\sqrt{2}$	
(e) Let $\angle ABD=\alpha$ $\angle BCE=\angle ABD$ (corresponding $\angle$'s, $BD \parallel CE$) $=\alpha$ $\angle DEB=\angle ABD$ ($\angle$ in the alternate segment) $=\alpha$ $\therefore\angle BCE=\angle BED$	

Question 12	
(a) $d = 16 + 7\sin\left(\frac{\pi}{12}t\right)$ (i) Max depth is 16+7 = 23 m, minimum depth 9 m	
(ii) Let $d = 23$ $23 = 16 + 7\sin\left(\frac{\pi}{12}t\right)$ $7 = 7\sin\left(\frac{\pi}{12}t\right)$ $\sin\left(\frac{\pi}{12}t\right) = 1$ $\frac{\pi}{12}t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$ $t = 6, 30, \ldots$ 6:00am is the only time of the day.	
(iii) Let $d = 20$ $20 = 16 + 7\sin\left(\frac{\pi}{12}t\right)$ $\frac{4}{7} = \sin\left(\frac{\pi}{12}t\right)$ $\frac{\pi}{12}t = \sin^{-1}\frac{4}{7}, \pi - \sin^{-1}\frac{4}{7}$ $t = \frac{12}{\pi}\sin^{-1}\frac{4}{7}, \frac{12}{\pi}\left(\pi - \sin^{-1}\frac{4}{7}\right)$ $= 2 \text{ hours } 19 \text{ minutes}, 9 \text{ hours } 41 \text{ minutes}$ Therefore, between 2:19 am and 9:41 am.	
(b) (i) $P\left(2ap, ap^2\right) \quad Q\left(2aq, aq^2\right)$ $m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$ $= \frac{a(p-q)(p+q)}{2a(p-q)}$ $= \frac{p+q}{2}$ Equation of *PQ*:	

$y-ap^2=\frac{p+q}{2}(x-2ap)$ $2y-2ap^2=(p+q)(x-2ap)$ $2y-2ap^2=x(p+q)-2ap(p+q)$ $2y-2ap^2=x(p+q)-2ap^2-2apq$ $x(p+q)-2y=2apq$	
(ii) Sub $x=2a$, $y=0$ into equation of chord $2a(p+q)=2apq$ $p+q=pq$	
(iii) Consider the midpoint $M(x,y)$ $x=a(p+q)$ $\therefore \frac{x}{a}=p+q$ $y=\frac{ap^2+aq^2}{2}$ $y=\frac{a(p^2+q^2)}{2}$ $=\frac{a}{2}\left[(p+q)^2-2pq\right]$ $=\frac{a}{2}\left[\frac{x^2}{a^2}-2(p+q)\right]$ $=\frac{a}{2}\left[\frac{x^2}{a^2}-\frac{2x}{a}\right]$ $\frac{2y}{a}=\frac{x^2-2xa}{a^2}$ $2ay=x^2-2xa$ $2ay+a^2=x^2-2xa+a^2$ $(x-a)^2=2a\left(y+\frac{a}{2}\right)$	
(iv) $Vertex\left(a,\frac{-a}{2}\right), Focal\ length\ \frac{a}{2},$ Focus $(a,0)$, $Directrix\ \ y=-a$	

(c)

$f(x) = \ln(x-1) + \ln 3$

(i)

Finding inverse:

$x = \ln(y-1) + \ln 3$

$x = \ln\left[3(y-1)\right]$

$e^x = 3(y-1)$

$\frac{e^x}{3} = y-1$

$y = 1 + \frac{e^x}{3}$

$f^{-1}(x) = \frac{e^x}{3} + 1$

(ii)

$y = f^{-1}(x)$ and $y = f(x)$ intersect on $y = x$

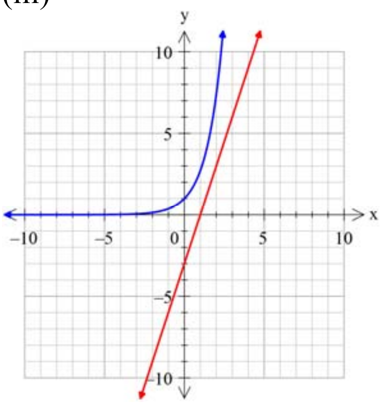
Let $f^{-1}(x) = x$

$\frac{e^x}{3} + 1 = x$

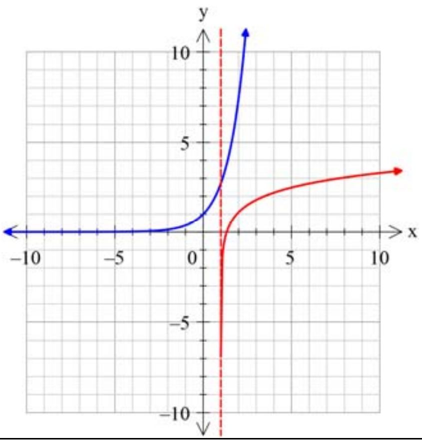
$e^x + 3 = 3x$

$3x - 3 = e^x$

(iii)



Therefore $y = f(x)$ and $y = f^{-1}(x)$ do not intersect



Question 13

(a)

(i)

We want to select 6 from 12, ${}^{12}C_6$. When we do this, it leaves out a group of 6 (a team).
Therefore for each combination we are counting twice the number of teams.

$\therefore \frac{1}{2} \times {}^{12}C_6$

(ii)

Number of groups:

$\frac{1}{2} \times \sum_{r=1}^{11} {}^{12}C_r = 2047$

Or

$\frac{1}{2}\,{}^{12}C_6 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_4 + {}^{12}C_5$

(iii)

$(12-1)! = 11!$

$= 39916800$

(b)

$P(x) = x^3 + (p-2)x^2 + (5-2p)x - 10$

(i)

$P(2) = 2^3 + (p-2)\times 2^2 + (5-2p)\times 2 - 10$

$= 8 + 4(p-2) + 2(5-2p) - 10$

$= 8 + 4p - 8 + 10 - 4p - 10$

$= 0$

$\therefore (x-2)$ is a factor of $P(x)$

(ii)

$x^2 + px + 5$

$x - 2 \;)\; x^3 + (p-2)x^2 + (5-2p)x - 10$

$x^3 - 2x^2$

$px^2 + (5-2p)x$

$px^2 - 2px$

$5x - 10$

$5x - 10$

0

$\therefore (x-2)(x^2 + px + 5) = P(x)$

(iii)

$x = 2$ is a root.

We need one root from $x^2 + px + 5 = 0$

$\Delta = p^2 - 4\times 5\times 1$

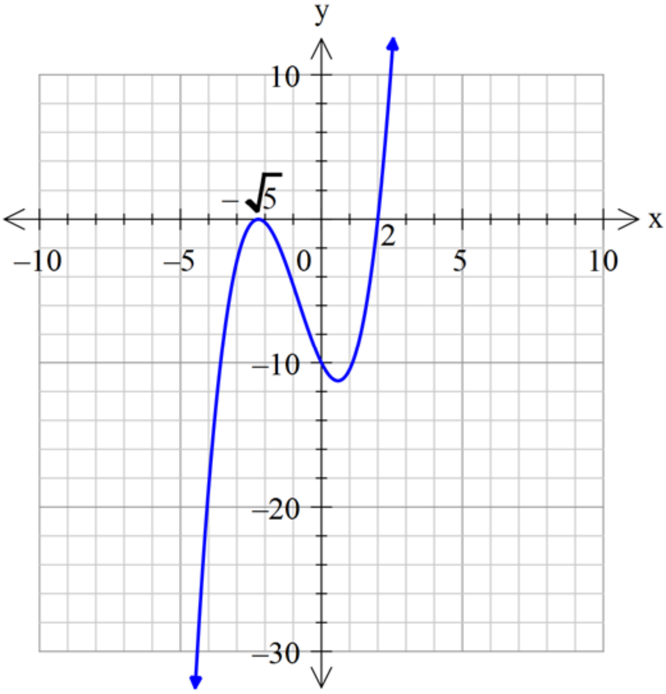
$= p^2 - 20$

Let $\Delta = 0$

$p^2 - 20 = 0$

$p = \pm\sqrt{20}$

$= \pm 2\sqrt{5}$

Let $p = 2\sqrt{5}$ 	
(c) (i) $\ddot{y} = -g$ $\dot{y} = -gt + c_1$ When $t = 0$, $\dot{y} = V\sin 0°$ $= 0$ $\therefore c_1 = 0$ $\dot{y} = -gt$ $y = -\frac{gt^2}{2} + c_2$ When $t = 0$, $y = h$ $h = 0 + c_2$ $c_2 = h$ $\therefore y = -\frac{gt^2}{2} + h$ $= -5t^2 + h$	
(ii) $x = Vt$ $t = \frac{x}{V}$ Substitute into y: $y = -10\left(\frac{x}{V}\right)^2 + h$ $= h - 5 \times \frac{x^2}{V^2}$ $= h - \frac{5x^2}{V^2}$	

(iii) Let $y=0$ $0=h-\frac{5x^2}{V^2}$ $\frac{5x^2}{V^2}=h$ $5x^2=hV^2$ $x^2=\frac{h}{5}V^2$ $x=V\sqrt{\frac{h}{5}}$ $=V\frac{\sqrt{h}}{\sqrt{5}}$	
(iv) $\frac{dy}{dx}=\frac{-10x}{V^2}$ When $x=\frac{V\sqrt{h}}{\sqrt{5}}$, $\frac{dy}{dx}=\frac{-10}{V^2}\times\frac{V\sqrt{h}}{\sqrt{5}}$ $=\frac{-10\sqrt{h}}{\sqrt{5}V}$ But $\frac{dy}{dx}=\tan 135°$ $=-1$ $\therefore \frac{-10\sqrt{h}}{\sqrt{5}V}=-1$ $-10\sqrt{h}=-\sqrt{5}V$ $\sqrt{5}V=\sqrt{5}\sqrt{2}\sqrt{10}\sqrt{h}$ $V=\sqrt{20h}$	

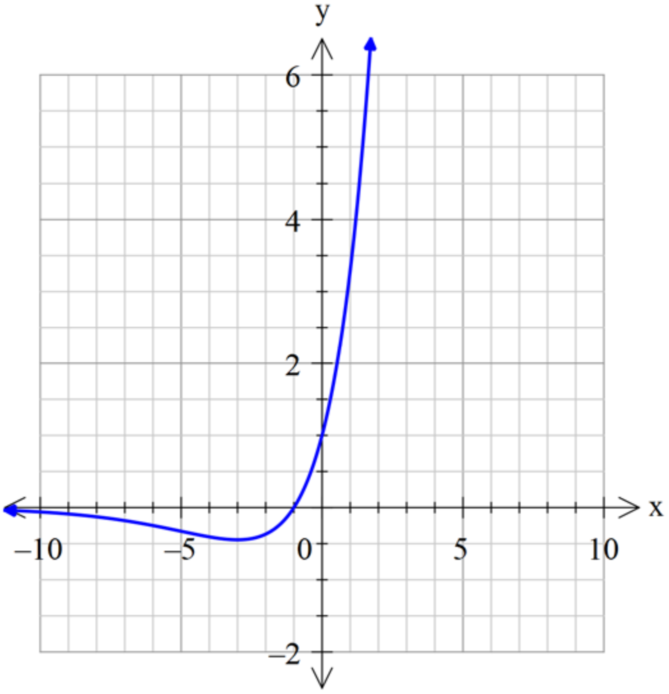
Question 14	
(a) $(a+x)^n = \sum_{k=0}^{n} {}^nC_k a^{n-k} x^k$ (i) $\frac{\alpha}{\beta} = \frac{{}^nC_r a^{n-r}}{{}^nC_{r+1} a^{n-(r+1)}}$ $= \frac{n!}{(n-r)!r!} \div \frac{n!}{(n-r-1)!(r+1)!} \times \frac{a^{n-r}}{a^{n-r-1}}$ $= a \times \frac{n!}{(n-r)!r!} \times \frac{(n-r-1)!(r+1)!}{n!}$ $= \frac{a(r+1)}{n-r}$	
(ii) Let $a=1$ and $\frac{\alpha}{\beta}=1$ $1 = \frac{r+1}{n-r}$ $n-r=r+1$ $n=2r+1$ Since *r* is a non-negative integer, $2r+1$ will be an odd number. Therefore there will be two consecutive coefficients with *n* is odd.	
(iii) $\frac{\beta}{\gamma} = \frac{a((r+1)+1)}{n-(r+1)}$ $= \frac{a(r+2)}{n-r-1}$	
(iv) For three consecutive equal coefficients: $\frac{\alpha}{\beta} = \frac{\beta}{\gamma} = 1$ $\frac{a(r+2)}{n-r-1} = \frac{a(r+1)}{n-r}$ $\frac{r+2}{n-r-1} = \frac{r+1}{n-r}$ $(r+2)(n-r) = (n-r-1)(r+1)$ $nr - r^2 + 2n - 2r = nr + n - r^2 - r - r - 1$ $2n-2r = n-2r-1$ $2n = n-1$ $n=-1$ But *n* is a positive integer. Therefore there will be no values of *a* such that three consecutive terms have the same coefficient.	
(b) $f(x) = e^{px}(x+1)$	

(i) $f'(x) = pe^{px}(x+1) + 1 \times e^{px}$ $\quad = e^{px}(p(x+1)+1)$	
(ii) Let $n=1$ Proven above Assume true for $n=k$ $f^{(k)}(x) = p^{k-1}e^{px}(p(x+1)+k)$ Let $n=k+1$ RTP: $f^{(k+1)}(x) = p^k e^{px}(p(x+1)+(k+1))$ $f^{(k+1)}(x) = \frac{d}{dx}\left(f^{(k)}(x)\right)$ $\quad = p^{k-1} \times pe^{px}(p(x+1)+k) + p^{k-1}e^{px} \times p$ $\quad = p^k e^{px}(p(x+1)+k) + p^k e^{px}$ $\quad = p^k e^{px}(p(x+1)+k+1)$ As required. $\therefore$ True for $n=k+1$ if true for $n=k$ By the principle of mathematical induction, the statement is true for integer $n \geq 1$.	
(iii) Let $p=\sqrt{3}$ $f(x) = e^{\sqrt{3}x}(x+1)$ $f''(x) = \sqrt{3}e^{\sqrt{3}x}\left(\sqrt{3}(x+1)+2\right)$ Let $f''(x)=0$ $\sqrt{3}e^{\sqrt{3}x}\left(\sqrt{3}(x+1)+2\right)=0$ $\sqrt{3}(x+1)+2=0$ $x+1 = \frac{-2}{\sqrt{3}}$ $x = -1-\frac{2}{\sqrt{3}}$ $\quad = -\left(\frac{3+2\sqrt{3}}{3}\right)$	
(iv) $f(x) = e^{\frac{x}{2}}(x+1)$ Let $f(x)=0$	

$$e^{\frac{x}{2}}\left(x+1\right)=0$$

$$x+1=0 \qquad \left(e^{\frac{x}{2}}>0\right)$$

$$x=-1$$



(v)

$$A=-\int_{-2}^{-1} e^{\frac{x}{2}}\left(x+1\right)dx+\int_{-1}^{2} e^{\frac{x}{2}}\left(x+1\right)dx$$

$$=\left[2e^{\frac{x}{2}}\left(x+1\right)-4e^{\frac{x}{2}}\right]_{-1}^{-2}+\left[2e^{\frac{x}{2}}\left(x+1\right)-4e^{\frac{x}{2}}\right]_{-1}^{2}$$

$$=2\left(\left[e^{\frac{x}{2}}\left(x+1-2\right)\right]_{-1}^{-2}+\left[e^{\frac{x}{2}}\left(x+1-2\right)\right]_{-1}^{2}\right)$$

$$=2\left(\left[e^{\frac{x}{2}}\left(x-1\right)\right]_{-1}^{-2}+\left[e^{\frac{x}{2}}\left(x-1\right)\right]_{-1}^{2}\right)$$

$$=2\left[\left(e^{-1}\times -3\right)-\left(e^{\frac{-1}{2}}\times -2\right)+\left(e\times 1\right)-\left(e^{\frac{-1}{2}}\times -2\right)\right]$$

$$=2\left(\frac{-3}{e}+\frac{2}{\sqrt{e}}+e+\frac{2}{\sqrt{e}}\right)$$

$$=2\left(e+\frac{4}{\sqrt{e}}-\frac{3}{e}\right)$$

$$=2\left(\frac{e^{2}+4\sqrt{e}-3}{e}\right)$$