HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate
Trial Examination Term 3 2019

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- A reference sheet is provided separately
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination room.

Total marks – 70

Section I Pages 3 – 6

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 7 – 14

60 marks

Attempt Questions 11 – 14

Start each question in a new writing booklet

Write your student number on every writing booklet

Question	***1-10***	***11***	***12***	***13***	***14***	***Total***
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/70*

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 The point P that divides the interval AB internally in the ratio 3:1 where $A = (-2,3)$ and $B = (10,11)$ has coordinates:

(A) $(4,7)$

(B) $(1,5)$

(C) $(7,9)$

(D) $(5,7)$

2 Eight athletes are running a race. In how many different ways can the first three places be filled?

(A) 3
(B) 8
(C) 336
(D) 40320

3 A circular disc is cut into twelve sectors whose angles are in an arithmetic sequence.
The angle of the largest sector is twice the angle of the smallest sector.

The angle of the smallest sector is:

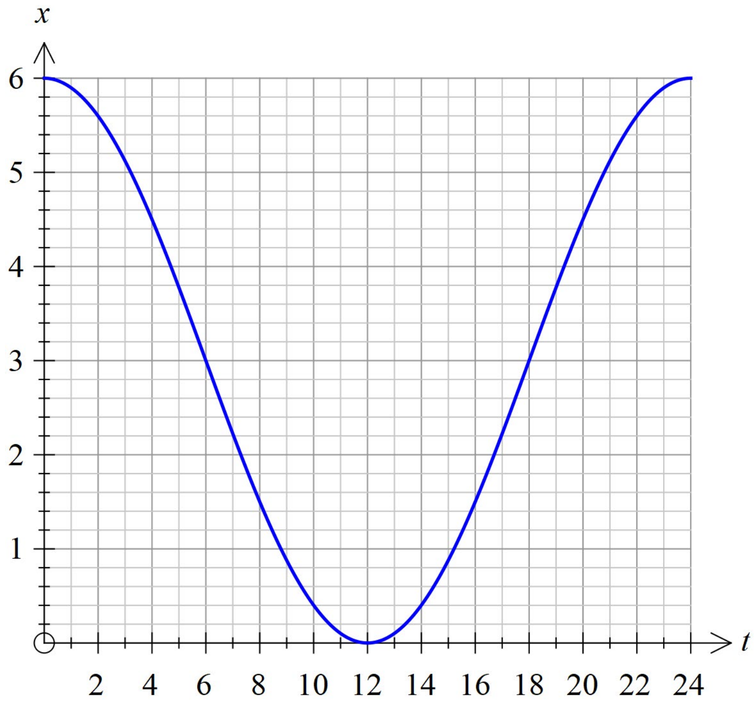
(A) 20°

(B) 30°

(C) 45°

(D) 60°

4 The graph below shows the depth of water below a walkway as a function of time. The equation of the graph is of the form $x = a\cos(bt) + m$.



The values of a, b and m are:

(A) $a = 3,\ b = \dfrac{\pi}{12}, m = 3$

(B) $a = 6,\ b = \dfrac{\pi}{12}, m = 3$

(C) $a = 3,\ b = \dfrac{\pi}{24}, m = 3$

(D) $a = 6,\ b = \dfrac{\pi}{24}, m = 3$

5 The primitive of $\sin^2 4x$ is:

(A) $8\sin 4x \cos 4x$

(B) $\dfrac{x}{2} - \dfrac{1}{4}\sin 2x + C$

(C) $\dfrac{x}{2} - \dfrac{1}{8}\sin 4x + C$

(D) $\dfrac{x}{2} - \dfrac{1}{16}\sin 8x + C$

6 The general solutions to equation $\cos x\left(2\sin x-1\right)=0$ are:

(A) $x=\dfrac{k\pi}{2}$ *or* $x=k\pi+\left(-1\right)^{k}\dfrac{\pi}{6}$, where k is an integer.

(B) $x=\left(2k+1\right)\dfrac{\pi}{2}$ *or* $x=k\pi+\left(-1\right)^{k}\dfrac{\pi}{6}$, where k is an integer.

(C) $x=\dfrac{k\pi}{2}$ *or* $x=k\pi+\left(-1\right)^{k}\dfrac{\pi}{3}$, where k is an integer.

(D) $x=\left(2k+1\right)\dfrac{\pi}{2}$ *or* $x=k\pi+\left(-1\right)^{k}\dfrac{\pi}{3}$, where k is an integer.

7 The roots of the equation are $x^3-3x+1=0$ are α , β and γ .

The value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$ is :

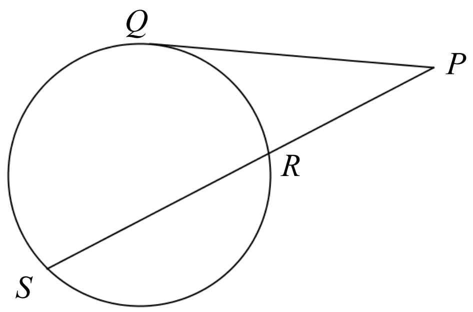
(A) 1

(B) -1

(C) 3

(D) -3

8 *PQ* is a tangent to a circle *QRS*, while *PRS* is a secant intersecting the circle at *R* and *S*, as in the diagram.



NOT TO SCALE

Given that $PQ=6$ cm, $RS=5$ cm, the length of *PR* is:

(A) 1 cm

(B) 4 cm

(C) 5 cm

(D) 9 cm

9 A cappuccino at temperature $T\ °\mathrm{C}$ loses heat when placed in a cooler environment, such that $T = 26 + Ae^{kt}$ after t minutes. Initially, the cappuccino was $70°\mathrm{C}$ and in 20 minutes the temperature is $42°\mathrm{C}$.

The value of k, correct to 2 significant figures is:

(A) -0.018

(B) -0.019

(C) -0.050

(D) -0.051

10 Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 3x \tan 3x}{5x^2}$

(A) 1

(B) $\dfrac{3}{5}$

(C) $\dfrac{9}{5}$

(D) 3

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

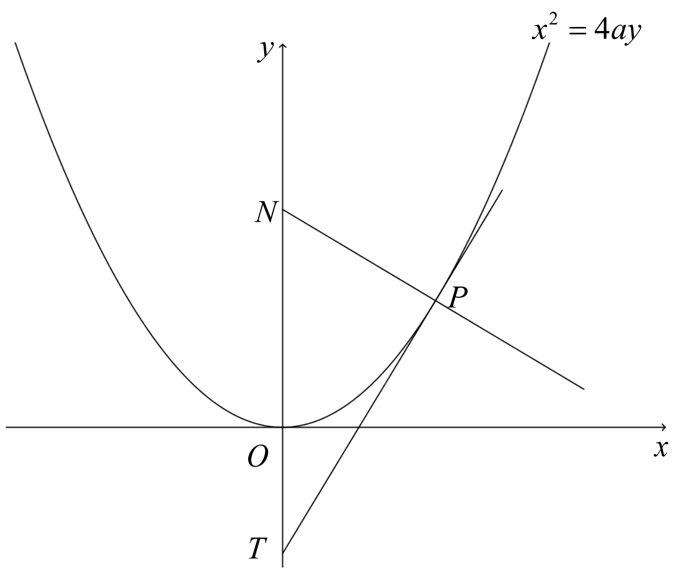
(a) Solve the inequality $\dfrac{x}{1+x} > 2$. **2**

(b) Differentiate $x^2 \sin^{-1} x$ with respect to x. **2**

(c) Using the substitution $u = 4 - x$, evaluate $\displaystyle\int_2^3 \left(\frac{x}{4-x}\right)^2 dx$. **3**

(d) The angle between the lines $y = mx + 1$ and $2y + x - 1 = 0$ is $60°$. Find the possible value(s) of m. **3**

(e) The point $P\left(2ap, ap^2\right)$ lies on the parabola $x^2 = 4ay$, as shown below.
The y-intercept of the tangent at P is T and the y-intercept of the normal at P is N.



NOT TO SCALE

(i) Show that the equation of the tangent to the parabola at P is $y = px - ap^2$ **2**

(ii) Find the coordinates of T and N. **2**

(iii) Find the length of NT. **1**

Question 12 (15 marks) Start a new writing booklet

(a) Prove by the method of mathematical induction that for positive integer n, that **3**

$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

(b) A particle in moving in a straight line with displacement x metres at time t seconds given by $x = 3\cos(2t+5)+1$.

(i) Show that the motion is Simple Harmonic and state the centre of motion. **2**

(ii) Find the maximum velocity of the particle **1**

(iii) Find the first time, in seconds, when the particle has displacement -2 metres, giving your answer to 3 significant figures. **2**

(c) A class of 30 pupils are lining up in three rows of ten for a class photograph. How many different arrangements are possible? **2**

(d) Consider the points of intersection of the graphs of $y = x^2 - 2x - 1$ and $y = \frac{6}{x}$.

(i) Show that the x-value(s) of the points of intersection satisfy $x^3 - 2x^2 - x - 6 = 0$. **1**

Let $P(x) = x^3 - 2x^2 - x - 6$.

(ii) Show that $(x-3)$ is a factor of $P(x)$. **1**

(iii) Show that $P(x)$ factorises into one linear factor and one quadratic factor which is positive-definite. **2**

(iv) Hence, or otherwise, find the coordinates of the point(s) of intersection of the graphs. **1**

Question 13 (15 marks) Start a new writing booklet

(a) Given that $\tan\left(x+\dfrac{\pi}{3}\right)=\dfrac{1}{2}$, show that $\tan x = 8-5\sqrt{3}$ **3**

(b) Consider the function $f(x)=\dfrac{3x-1}{2x-3}, x \neq \dfrac{3}{2}$.

(i) Show, using $f'(x)$, that $f(x)$ has an inverse function for all values of x in its domain. **2**

(ii) By finding the inverse function $f^{-1}(x)$, show that $f^{-1}(x)=\dfrac{3x-1}{2x-3}=f(x)$ **1**

(iii) Show algebraically that $f\left[f^{-1}(x)\right]=x$ **2**

(c) A particle P is moving along the x-axis. At time t seconds, P is x metres from O, has velocity v ms^{-1} and acceleration $\ddot{x}=-\dfrac{1}{2}e^{-x}$. Initially P is at the origin and is moving with velocity 1 ms^{-1} in the positive direction.

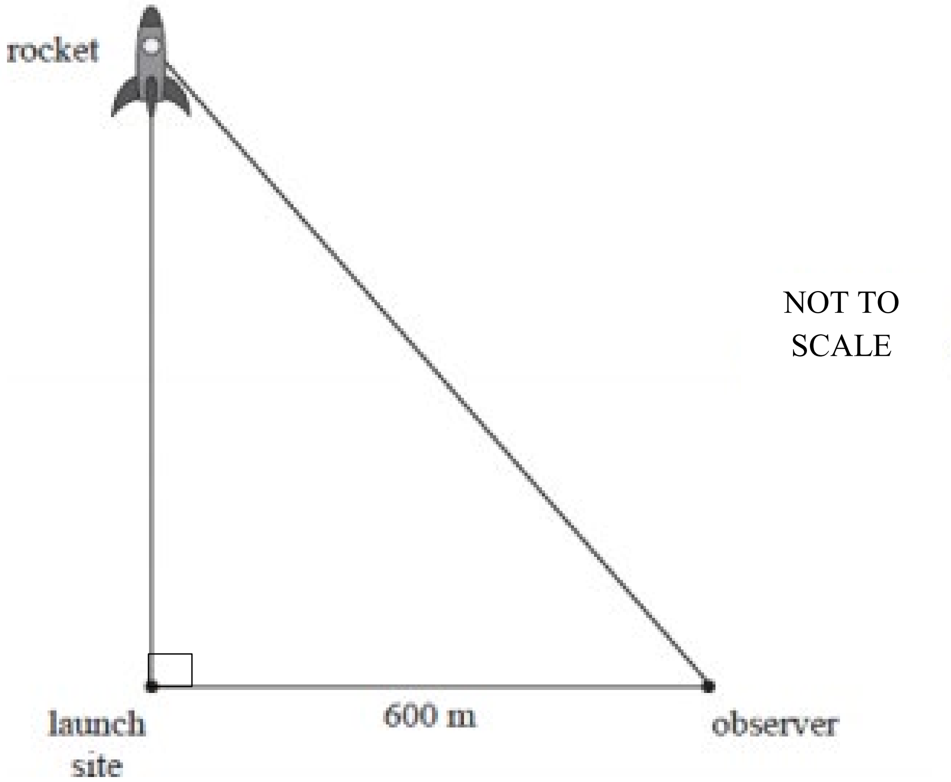
(i) Show that $v=e^{-\frac{x}{2}}$. **2**

(ii) Hence find displacement in terms of time. **2**

Question 13 continues on page 10

(d) An observer is 600 m from the launch site of a rocket and on the same horizontal level as the launch site, as shown in the diagram below.

Let x be the distance from the observer to the rocket and h be the vertical distance between the rocket and the launch site.



(i) Show that $\dfrac{dh}{dx} = \dfrac{x}{\sqrt{x^2 - 600^2}}$. **1**

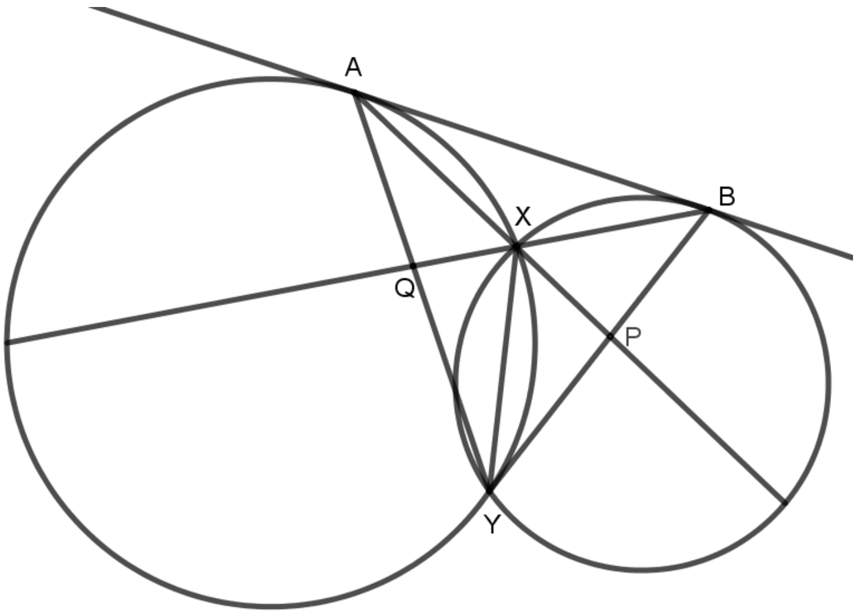
The rocket is rising vertically at a speed of 300 ms^{-1} when it is 800 metres directly above the launch site.

(ii) Calculate the rate of change of the distance between the rocket and the observer when it is 800 metres directly above the launch site. **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet

(a) In the diagram below, AB is a common tangent and XY is a common chord. Extend BX to meet AY at Q and extend AX to meet BY at P.



NOT TO SCALE

Copy or trace the diagram into your writing booklet.

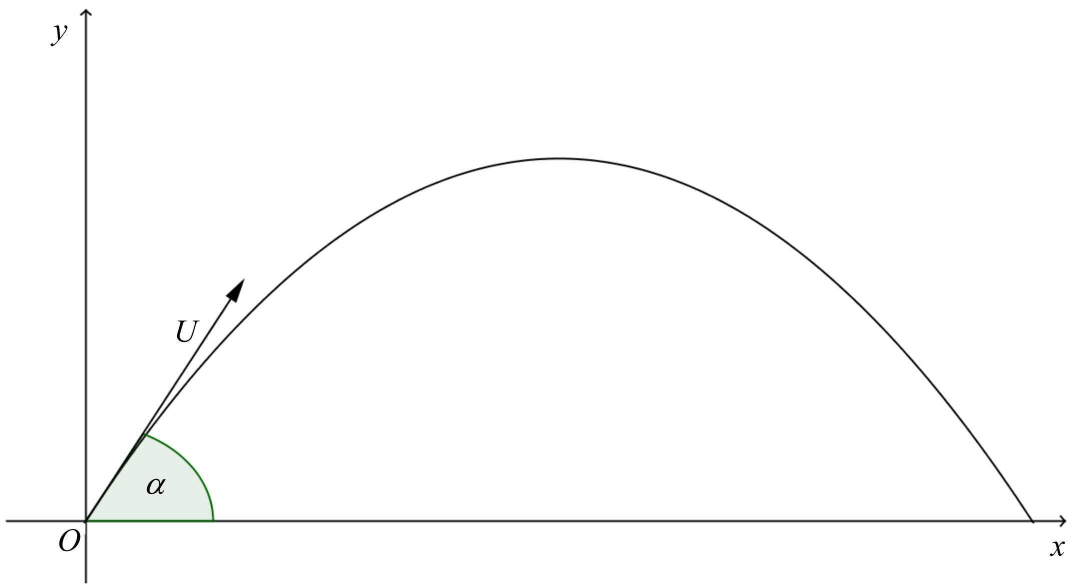
(i) Explain why $\angle BAX = \angle AYX$. **1**

(ii) Prove that $PXQY$ is a cyclic quadrilateral. **2**

(iii) Show that PQ is parallel to AB. **2**

Question 14 continues on page 12

(b) A particle is projected from a point with speed U metres per second at an angle of elevation α. When the particle has moved a horizontal distance x metres, its height above the point of projection is y metres.



NOT TO SCALE

The equations of motion are $x = Ut\cos\alpha$ and $y = Ut\sin\alpha - \frac{1}{2}gt^2$, where t is time in seconds.

Do NOT prove these results

(i) Show that the Cartesian equation of the path of the particle is **2**

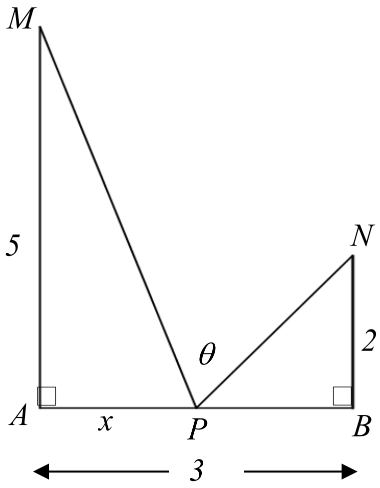
$$y = x\tan\alpha - \frac{gx^2}{2U^2}\left(1 + \tan^2\alpha\right).$$

A particle is projected from a point A on a horizontal plane, with speed 28ms^{-1} at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 32 m from A and at a height of 8 metres above the plane. (Use $g = 9.8\text{ ms}^{-2}$)

(ii) Find the two possible values of α, giving your answers in exact form. **3**

Question 14 continues on page 13

(c) In the diagram below, AB has length three metres. AM and BN are perpendicular to AB and have lengths 5 metres and 2 metres, respectively. The point P is lies on AB such that $AP = x$ and $\angle MPN = \theta$.



NOT TO SCALE

Copy or trace the diagram into your writing booklet

(i) Show that $\theta = \pi - \tan^{-1}\left(\dfrac{5}{x}\right) - \tan^{-1}\left(\dfrac{2}{3-x}\right)$. **1**

(ii) Show that $\dfrac{d\theta}{dx} = \dfrac{5}{x^2+25} - \dfrac{2}{13+x^2-6x}$. **2**

The position of P is chosen such that θ is a maximum.

(iii) Find the length of AP and justify that θ is a maximum. **2**

End of Paper

Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Trial Examination 2019
Solutions

Multiple Choice

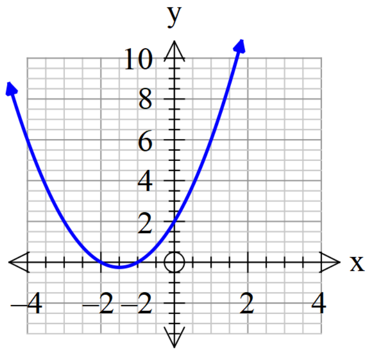
1. $x_1 = -2 \qquad x_2 = 10$ $y_3 = 3 \qquad y_2 = 11$ $m = 3 \qquad n = 1$ $x = \dfrac{-2\times1+3\times10}{3+1} = \dfrac{28}{4} = 7$ $y = \dfrac{3\times1+3\times11}{3+1} = \dfrac{36}{4} = 9$ $\therefore P(7,9)$ Answer: C	
2. 1st place = 8 possible athletes 2nd place = 7 possible athletes 3rd place = 6 possible athletes $\text{number of ways} = 8\times7\times6 = 336$ Answer: C	
3. $T_1 = \theta$ $T_{12} = 2\theta$ $S_{12} = 360$ $360 = \dfrac{12}{2}(\theta+2\theta)$ $360 = 6(3\theta)$ $\theta = 20$ Smallest angle is $20°$ Answer: A	

4. $amplitude = \frac{0+6}{2}$ $= 3$ $period = 24$ $\frac{2\pi}{b} = 24$ $b = \frac{2\pi}{24}$ $b = \frac{\pi}{12}$ Answer: A	
5. $\cos 2x = \cos^2 - \sin^2 x$ $\cos 2x = 1 - \sin^2 x - \sin^2 x$ $\cos 2x = 1 - 2\sin^2 x$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\sin^2 4x = \frac{1}{2}(1 - \cos 8x)$ $\int \sin^2 2x = \frac{1}{2}\int (1 - \cos 8x)\,dx$ $= \frac{1}{2}\left(x - \frac{1}{8}\sin 8x\right) + C$ $= \frac{x}{2} - \frac{1}{16}\sin 8x + C$ Answer: D	
6. $\cos x(2\sin x - 1) = 0$ $\cos x = 0$ $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\left(odd\ multiples\ of\ \frac{\pi}{2}\right)$ $x = (2k+1) \times \frac{\pi}{2}$ $\sin x = \frac{1}{2}$ $x = 2k\pi + (-1)^k \times \frac{\pi}{6}$ Answer: B	

7. $x^3+0x^2-3x+1=0$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$ $=\frac{-3}{-1}$ $=3$ Answer: C	
8. Let $PR=x$ $36=x(x+5)$ $x^2+5x=36$ $x^2+5x-36=0$ $(x+9)(x-4)=0$ $x=4\ \ (x>0)$ Answer: B	
9. $T=26+Ae^{kt}$ When $t=0$, $T=70$ $A=70-26$ $=44$ $T=26+44e^{kt}$ Let $T=42$ $42=26+44e^{20k}$ $\frac{16}{44}=e^{20k}$ $k=\frac{1}{20}\ln\left(\frac{16}{44}\right)$ $=-0.051$ Answer: D	

10. $\lim_{x\to 0}\frac{\sin 3x\tan 3x}{5x^2}$ $=\lim_{x\to 0}\frac{\sin 3x}{5x}\times\frac{\sin 3x}{x}\times\frac{1}{\cos 3x}$ $=\frac{1}{5}\lim_{x\to 0}\frac{3\sin 3x}{3x}\times\frac{3\sin 3x}{3x}\times\frac{1}{\cos 3x}$ $=\frac{1}{5}\times 3\times 3\times 1$ $=\frac{9}{5}$ Answer: C	

Question 11

(a) $\frac{x}{1+x} > 2$ $x(1+x) > 2(1+x)^2$ $0 > 2(1+x)^2 - x(1+x)$ $0 > (1+x)[2+2x-x]$ $0 > (1+x)(2+x)$  From graph: $-2 < x < -1$	Generally well done. Some students made arithmetic errors with the inside bracket after factoring. Be careful when copying down question that you copy it down correctly from the question paper.
(b) $y = x^2 \sin^{-1} x$ $\frac{dy}{dx} = 2x \times \sin^{-1} x + x^2 \times \frac{1}{\sqrt{1-x^2}}$ $= 2x \sin^{-1} x + \frac{x^2}{\sqrt{1-x^2}}$	Well done. Correct use of product rule.
(c) $u = 4 - x$ $\frac{du}{dx} = -1$ $-du = dx$ When $x = 2, u = 2$ When $x = 3, u = 1$	This was not done well. Students left off the square from the bracket, or forgot to square the denominator of $\left(\frac{4-u}{u}\right)^2$, or incorrectly expanded the perfect square. Many students incorrectly integrated $\frac{8}{u}$, not noting that $\frac{8}{u} = 8 \times \frac{1}{u}$

$I = -\int_2^1 \left(\frac{4-u}{u}\right)^2 du$ $= \int_1^2 \left(\frac{4-u}{u}\right)^2 du$ $= \int_1^2 \frac{16-8u+u^2}{u^2} du$ $= \int_1^2 \left(16u^{-2} - \frac{8}{u} + 1\right) du$ $= \left[-\frac{16}{u} - 8\ln u + u\right]_1^2$ $= (-8 - 8\ln 2 + 2) - (-16 - 1)$ $= -6 + 15 - 8\ln 2$ $= 9 - 8\ln 2$	
(d) $y = mx + 1$ $2y + x - 1 = 0$ $2y = -x + 2$ $y = \frac{-1}{2}x + 1$ $m_2 = -\frac{1}{2}$ $\tan 60° = \left\|\frac{m + \frac{1}{2}}{1 - \frac{m}{2}}\right\|$ $\sqrt{3} = \left\|\frac{2m+1}{2-m}\right\|$ $\frac{2m+1}{2-m} = \sqrt{3}$ $2m + 1 = 2\sqrt{3} - \sqrt{3}m$ $2m + \sqrt{3m} = 2\sqrt{3} - 1$ $m\left(2 + \sqrt{3}\right) = 2\sqrt{3} - 1$ $m = \frac{2\sqrt{3} - 1}{2 + \sqrt{3}}$ $\frac{2m+1}{2-m} = -\sqrt{3}$ $2m + 1 = -2\sqrt{3} + \sqrt{3}m$ $2m - \sqrt{3m} = -2\sqrt{3} - 1$ $m\left(2 - \sqrt{3}\right) = -2\sqrt{3} - 1$ $m = \frac{-2\sqrt{3} - 1}{2 - \sqrt{3}}$ $= \frac{2\sqrt{3} + 1}{\sqrt{3} - 2}$ $\therefore m = \frac{2\sqrt{3} - 1}{2 + \sqrt{3}} \text{ or } m = \frac{2\sqrt{3} + 1}{\sqrt{3} - 2}$ $m = 5\sqrt{3} - 8$ or $m = -5\sqrt{3} - 8$	Most students successfully found the gradient of the given straight line and substituted correctly into the formula. Students who left off the absolute values signs did not find both solutions. Generally, students who squared both sides instead of looking at cases were more successful. It was pleasing to see a good grasps of algebraic manipulation of equations involving surds. Students who attempted to cover both cases using a $\pm$ through their working made mistakes when needing to flip the sign – this approach is not advised.

(e)	
(i) $x^2 = 4ay$ $y = \frac{x^2}{4a}$ $\frac{dy}{dx} = \frac{2x}{4a}$ $= \frac{x}{2a}$ At $P\left(2ap, ap^2\right)$, $\frac{dy}{dx} = \frac{2ap}{2a}$ $= p$ Equation of tangent: $y - y_1 = m\left(x - x_1\right)$ $y - ap^2 = p\left(x - 2ap\right)$ $y - ap^2 = px - 2ap^2$ $y = px - ap^2$	As this is a show question, you are expected to show your steps. If you used the chain rule, it should be included in your working $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ Generally, very well done.
(ii) From reference sheet, equation of normal is $x + py = ap^3 + 2ap$ Let $x = 0$ $py = ap^3 + 2ap$ $y = ap^2 + 2a$ $N\left(0, ap^2 + 2a\right)$ Let $x = 0$ in tangent $y = -ap^2$ $\therefore T\left(0, -ap^2\right)$	Almost all students derived the normal rather than being familiar with the reference sheet – please be familiar with the sheet to save time. The one student who did use the formula struggled to change the *t's* for the *p's*.
(iii) $NT = OT + ON$ $= ap^2 + 2a + ap^2$ $= 2ap^2 + 2a \; units$	This was well done if candidates realised it was vertical distance and that distance formula was not required. The main errors here were poor algebra if using distance formula OR not realising that $OT = \left\lvert -ap^2 \right\rvert$ $= ap^2$

Question 12

(a) Step 1: Prove true for $n=1$ $LHS = 1^2$ $= 1$ $RHS = \frac{1}{6}\times 1\times(1+1)\times(2\times 1+1)$ $= \frac{1}{6}\times 2\times 3$ $= 1$ $= LHS$ Step 2: Assume true for $n=k$ $1^2+2^2+3^2+...+k^2 = \frac{1}{6}k(k+1)(2k+1)$ Step 3: Prove true for $n=k+1$ RTP: $1^2+2^2+3^2+...+k^2+(k+1)^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$ $1^2+2^2+3^2+...+k^2+(k+1)^2 = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ $= \frac{1}{6}(k+1)[k(2k+1)+6(k+1)]$ $= \frac{1}{6}(k+1)(2k^2+k+6k+6)$ $= \frac{1}{6}(k+1)(2k^2+7k+6)$ $= \frac{1}{6}(k+1)(2k^2+3k+4k+6)$ $= \frac{1}{6}(k+1)(k(2k+3)+2(2k+3))$ $= \frac{1}{6}(k+1)(k+2)(2k+3)$ as required Therefore true for $n=k+1$ if true for $n=k$. By principle of mathematical induction, the statement is true for all positive integers *n*.	Generally well done. Some poor setting out.

(b)	
(i) $x = 3\cos(2t+5)+1$ $\dot{x} = -6\sin(2t+5)$ $\ddot{x} = -12\cos(2t+5)$ $\quad = -4\left[3\cos(2t+5)+1-1\right]$ $\quad = -2^2(x-1)$ The particle is moving in simple harmonic motion as it satisfies $\ddot{x} = -n^2 X$, with centre of motion $x=1$.	Some students were not answering the question ie state the centre of motion.
(ii) $-1 \le \sin(2t+5) \le 1$ $-6 \le 6\sin(2t+5) \le 6$ $-6 \le \dot{x} \le 6$ Therefore, maximum velocity is 6 metres per second.	A lot solve correctly, gave final answer as -6
(iii) Let $x=-2$ $-2 = 3\cos(2t+5)+1$ $-3 = 3\cos(2t+5)$ $-1 = \cos(2t+5)$ $2t+5 = \pi, 3\pi, \ldots$ $t = \dfrac{\pi-5}{2}, \dfrac{3\pi-5}{2}, \ldots$ $\therefore$ *first time* $t = 2.21$ seconds $(3sf)$	Many students only found one value $t = \dfrac{\pi-5}{2}$, but this was not actual answer.
(c) Choosing first row: ${}^{30}C_{10}$ Choosing second row: ${}^{20}C_{10}$ Choosing first row: ${}^{10}C_{10} = 1$ Arranging each row amongst themselves: 10! Number of arrangements $= {}^{30}C_{10} \times {}^{20}C_{10} \times 10! \times 10! \times 10!$ Alternative Solution: Label each chair 1 – 30. The question becomes arrangement of 30 people into 30 seats – arrangements in a line. $30! = {}^{30}P_{30}$	

(d)	
(i) $y = x^2 - 2x - 1 \ldots (1)$ $y = \dfrac{6}{x} \ldots (2)$ *Equating* (1) *and* (2) $\dfrac{6}{x} = x^2 - 2x - 1$ $6 = x^3 - 2x^2 - x$ $0 = x^3 - 2x^2 - x - 6$ Therefore the x values of the points of intersection satisfy the given equation.	This was a show question – all three steps required to get the mark.
(ii) $P(3) = 3^3 - 2\times 3^2 - 3 - 6$ $= 27 - 2\times 9 - 9$ $= 27 - 27$ $= 0$ $\therefore (x-3)$ is a factor of $P(x)$	
(iii) By short division $x^3 - 2x^2 - x - 6 = (x-3)(x^2 + x + 2)$ For the quadratic factor $\Delta = 1 - 4\times 1\times 2 < 0$ and $a > 0$ Therefore polynomial factors to one linear and one quadratic positive-definite factor	Needed to indicate the coefficient of x^2 - many students did not!
(iv) The only point of intersection is $x = 3$ When $x = 3$, $y = \dfrac{6}{3}$ $= 2$ Therefore, only point of intersection is $(3, 2)$	

Question 13

(a) $\tan\left(x+\frac{\pi}{3}\right)=\frac{1}{2}$ $\frac{\tan x+\tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}=\frac{1}{2}$ $\frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x}=\frac{1}{2}$ $2\tan x+2\sqrt{3}=1-\sqrt{3}\tan x$ $2\tan x+\sqrt{3}\tan x=1-2\sqrt{3}$ $\left(2+\sqrt{3}\right)\tan x=1-2\sqrt{3}$ $\tan x=\frac{1-2\sqrt{3}}{2+\sqrt{3}}$ $=\frac{1-2\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$ $=2-\sqrt{3}-4\sqrt{3}+6$ $=8-5\sqrt{3}$	
(b) $f\left(x\right)=\frac{3x-1}{2x-3}$	
(i) $f'\left(x\right)=\frac{\left(2x-3\right)\times 3-2\left(3x-1\right)}{\left(2x-3\right)^2}$ $=\frac{6x-9-6x+2}{\left(2x-3\right)^2}$ $=-\frac{7}{\left(2x-3\right)^2}<0$ *for all x in domain* Therefore, the function is monotonically decreasing and therefore each *y-value has only one x value.*	Many students need to learn how to explain more thoroughly about the existence of an inverse function.
(ii) $y=\frac{3x-1}{2x-3}$ Finding inverse:	

$x=\frac{3y-1}{2y-3}$ $2xy-3x=3y-1$ $2xy-3y=3x-1$ $y(2x-3)=3x-1$ $y=\frac{3x-1}{2x-3}$ $\therefore f^{-1}(x)=\frac{3x-1}{2x-3}=f(x)$	
(iii) $f\left[f^{-1}(x)\right]=\frac{3f^{-1}(x)-1}{2f^{-1}(x)-3}$ $=\frac{3\left(\frac{3x-1}{2x-3}\right)-1}{2\left(\frac{3x-1}{2x-3}\right)-3}$ $=\frac{3(3x-1)-(2x-3)}{2x-3}\div\frac{2(3x-1)-3(2x-3)}{2x-3}$ $=\frac{9x-3-2x+3}{6x-2-6x+9}$ $=\frac{7x}{7}$ $=x$	Some students need to strengthen their algebraic tehcniques
(c) $\ddot{x}=-\frac{1}{2}e^{-x}$	
(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=\ddot{x}$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=-\frac{1}{2}e^{-x}$ $\frac{1}{2}v^2=\frac{1}{2}e^{-x}+C$ $v^2=e^{-x}+2C$ When $x=0, v=1$ $1=e^0+C$ $C=1-1$ $=0$ $v^2=e^{-x}$ $v=\pm\sqrt{e^{-x}}$ *but initially* $v>0$ $v=\sqrt{e^{-x}}$ $v=e^{\frac{-x}{2}}$	Students must justify why $v>0$

(ii) $\frac{dx}{dt} = e^{\frac{-x}{2}}$ $\frac{dt}{dx} = e^{\frac{x}{2}}$ $t = 2e^{\frac{x}{2}} + D$ $When\ t = 0, x = 0$ $0 = 2e^{0} + D$ $D = -2$ $t = 2e^{\frac{x}{2}} - 2$ $t + 2 = 2e^{\frac{x}{2}}$ $\frac{t+2}{2} = e^{\frac{x}{2}}$ $\frac{x}{2} = \ln\left(\frac{t+2}{2}\right)$ $x = 2\ln\left(\frac{t+2}{2}\right)$	

(d)	
(i) $x^2 = h^2 + 600^2$ $h^2 = x^2 - 600^2$ x $h = \left(x^2 - 600^2\right)^{\frac{1}{2}}$ $\frac{dh}{dx} = \frac{1}{2}\left(x^2 - 600^2\right)^{\frac{-1}{2}} \times 2x$ $= \frac{x}{\sqrt{x^2 - 600^2}}$	
(ii) When $h = 800, \frac{dh}{dt} = 300$ $\frac{dx}{dt} = \frac{dx}{dh} \times \frac{dh}{dt}$ $x^2 = 600^2 + 800^2$ $x^2 = 1000^2$ $x = 1000$ When $x = 1000$, $\frac{dh}{dx} = \frac{1000}{\sqrt{1000^2 - 600^2}}$ $= \frac{1000}{800}$ $= \frac{5}{4}$ $\frac{dx}{dt} = \frac{4}{5} \times 300$ $= 240\ ms^{-1}$	

Question 14

(a)	
(i) $\angle BAX = \angle AYX$ (angle between a chord and tangent at point of contact is equal to the angle in the alternate segment)	Students should ensure that they do not use abbreviations and give full reasons not a shortened form
(ii) Let $\angle BAX = \alpha$. Similarly to (i), $\angle ABX = \angle XYP = \beta$ $\angle AXQ = \angle BAX + \angle ABX$ $= \alpha + \beta$ (exterior angle of ΔAXB) $\angle QYP = \angle QYX + \angle XYP$ $= \alpha + \beta$ Therefore, *XYPQ* is a cyclic quadrilateral (exterior angle is equal to the opposite interior angle) OR $\angle AXB = 180° - (\angle XAB + \angle XBA)$ $= 180° - (\alpha + \beta)$ (angle sum of a triangle is $180°$) $\angle QXP = \angle AXB = 180° - (\alpha + \beta)$ (vertically opposite angles) $\therefore XYPQ$ is a cyclic quadrilateral (opposite angles are supplementary)	
(iii) $\angle QYX = \alpha = \angle QPX$ (angle in the same segment) $\angle BAP = \angle QPA = \alpha$ $\therefore AB \parallel QP$ (alternate angles are equal)	

(b)	
(i) $x = Ut\cos\alpha$ $t = \dfrac{x}{U\cos\alpha}$ Sub into y $y = \left(\dfrac{x}{U\cos\alpha}\right)U\sin\alpha - \dfrac{1}{2}\times g\times\left(\dfrac{x}{U\cos\alpha}\right)^2$ $= x\tan\alpha - \dfrac{g}{2}\times\dfrac{x^2}{U^2\cos^2\alpha}$ $= x\tan\alpha - \dfrac{gx^2}{2U^2}\sec^2\alpha$ $= x\tan\alpha - \dfrac{gx^2}{2U^2}\left(1+\tan^2\alpha\right)$	1 for making t the subject and substituting correctly 1 for showing thorough working for "show that" question
(ii) Let $g = 9.8, U = 28, x = 32, y = 8$ $8 = 32\times\tan\alpha - \dfrac{9.8\times 32^2}{2\times 28^2}\left(1+\tan^2\alpha\right)$ $8 = 32\tan\alpha - \dfrac{32}{5}\left(1+\tan^2\alpha\right)$ $1 = 4\tan\alpha - \dfrac{4}{5}\left(1+\tan^2\alpha\right)$ $5 = 20\tan\alpha - 4 - 4\tan^2\alpha$ $4\tan^2\alpha - 20\tan\alpha + 9 = 0$ $\tan\alpha = \dfrac{20\pm\sqrt{20^2-4\times4\times9}}{2\times4}$ $\tan\alpha = \dfrac{20\pm\sqrt{256}}{8}$ $\alpha = \tan^{-1}\left(\dfrac{20+16}{8}\right) or\ \alpha = \tan^{-1}\left(\dfrac{20-16}{8}\right)$ $\alpha = \tan^{-1}\left(\dfrac{9}{2}\right) or\ \alpha = \tan^{-1}\left(\dfrac{1}{2}\right)$ OR $4\tan^2\alpha - 20\tan\alpha + 9 = 0$ $4\tan^2\alpha - 2\tan\alpha - 18\tan\alpha + 9 = 0$ $2\tan\alpha\left(2\tan\alpha - 1\right) - 9\left(2\tan\alpha - 1\right) = 0$ $\left(2\tan\alpha - 1\right)\left(2\tan\alpha - 9\right) = 0$ $\tan\alpha = \dfrac{1}{2}\ or\ \tan\alpha = \dfrac{9}{2}$ $\alpha = \tan^{-1}\left(\dfrac{1}{2}\right) or\ \alpha = \tan^{-1}\left(\dfrac{9}{2}\right)$	1 for correct quadratic equation Lots of errors with operation symbols 8+6.4=14.4 NOT 8-6.4=1.6 1 for factorising or using quadratic formula 1 for solving

(c)	
(i) In ΔMAP $\tan\angle MPA=\dfrac{5}{x}$ $\angle MPA=\tan^{-1}\dfrac{5}{x}$ In ΔNBP $\tan\angle NPB=\dfrac{2}{3-x}$ $\angle NPB=\tan^{-1}\dfrac{2}{3-x}$ $\pi=\angle APM+\angle NPB+\theta$ (adjacent angles on a straight line) $\pi=\tan^{-1}\dfrac{5}{x}+\tan^{-1}\dfrac{2}{3-x}+\theta$ $\theta=\pi-\tan^{-1}\dfrac{5}{x}-\tan^{-1}\dfrac{2}{3-x}$	Done well 1 mark
(ii) $y=\tan^{-1}f(x)$ $\dfrac{dy}{dx}=\dfrac{f'(x)}{1+f(x)^2}$ $\dfrac{d\theta}{dx}=-\dfrac{\frac{d}{dx}\left(5x^{-1}\right)}{1+\left(\frac{5}{x}\right)^2}-\dfrac{\frac{d}{dx}\left(2(3-x)^{-1}\right)}{1+\left(\frac{2}{3-x}\right)^2}$ $(see *** below)$ $=\dfrac{5}{x^2\left(1+\frac{25}{x^2}\right)}-\dfrac{2}{(3-x)^2\left(1+\left(\frac{2}{3-x}\right)^2\right)}$ $=\dfrac{5}{x^2+25}-\dfrac{2}{(3-x)^2+4}$ $=\dfrac{5}{x^2+25}-\dfrac{2}{9-6x+x^2+4}$ $=\dfrac{5}{x^2+25}-\dfrac{2}{x^2-6x+13}$ *** $y=\tan^{-1}u$ where $u=\dfrac{5}{x}=5x^{-1}$ $\dfrac{dy}{du}=\dfrac{1}{1+u^2}$, $\dfrac{du}{dx}=-5x^{-2}$	Many students did not show sufficient working 1 for correct differentiation of each inverse tan function and simplifying

$$\frac{dy}{dx}=\frac{1}{1+u^2}\times-\frac{5}{x^2}$$

$$=\frac{1}{1+\left(\frac{5}{x}\right)^2}\times\frac{-5}{x^2}$$

$$=\frac{-5}{x^2+25}$$

$y=\tan^{-1}u$ where $u=\dfrac{2}{3-x}=2\left(3-x\right)^{-1}$

$$\frac{dy}{du}=\frac{1}{1+u^2}\ ,\qquad \frac{du}{dx}=-2\left(3-x\right)^{-2}\times-1$$

$$=\frac{2}{\left(3-x\right)^2}$$

$$\frac{dy}{dx}=\frac{1}{1+u^2}\times\frac{2}{\left(3-x\right)^2}$$

$$=\frac{1}{1+\left(\frac{2}{3-x}\right)^2}\times\frac{2}{\left(3-x\right)^2}$$

$$=\frac{2}{\left(3-x^2\right)+2^2}$$

(iii) Let $\frac{d\theta}{dx}=0$ $5\left(13+x^2-6x\right)-2\left(x^2+25\right)=0$ $65+5x^2-30x-2x^2-50=0$ $3x^2-30x+15=0$ $x^2-10x+5=0$ $x^2-10x+25=20$ $\left(x-5\right)^2=20$ $x=5\pm\sqrt{20}$ $x=5-\sqrt{20}\left(x<3\right)$ $x\approx 0.5278....$ When $x=0.5$, $\frac{d\theta}{dx}=\frac{5}{0.5^2+25}-\frac{2}{0.5^2-6\times 0.5+13}$ $=\frac{12}{4141}>0$ When $x=0.6$, $\frac{d\theta}{dx}=\frac{5}{0.6^2+25}-\frac{2}{0.6^2-6\times 0.6+13}$ $=\frac{-150}{19337}..<0$ Therefore $x=5-\sqrt{20}$ gives maximum value of θ $AP=5-\sqrt{20}\ m$	1 for solving for x. Many students did not consider $0<x<3$ 1 for justifying the maximum