2018

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Hunters Hill High School

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen only
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 14 show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I Questions 1 – 10 **10 marks**
Allow about 15 minutes for this section

Section II Questions 11 – 14 **60 marks**
Allow about 1 hour and 45 minutes for

Name:____

Section 1 **10 marks**

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Which of the following expressions is equivalent to $\cos x + \sqrt{3}\sin x$?

(A) $2\cos\left(x + \frac{\pi}{3}\right)$

(B) $2\cos\left(x - \frac{\pi}{3}\right)$

(C) $2\cos\left(x + \frac{\pi}{6}\right)$

(D) $2\cos\left(x - \frac{\pi}{6}\right)$

2 If $(x-2)$ and $(x+1)$ are factors of $x^3 + x^2 + bx + c$ what is the value of $(b+2c)$?

(A) -4

(B) 12

(C) -8

(D) -12

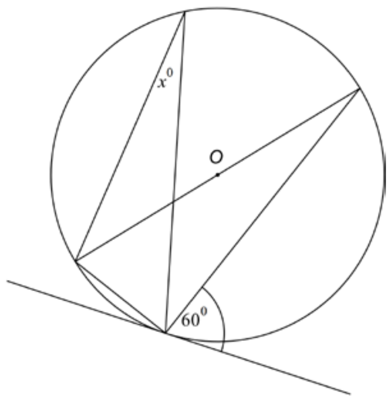
3 The diagram below shows a circle with tangent. If *O* is the centre, find the value of x°. Diagram is not to scale.

(A) 60°

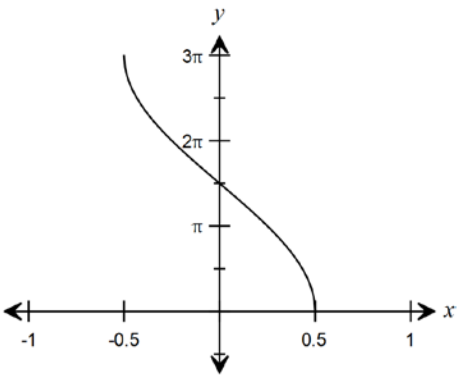
(B) 30°

(C) 15°

(D) 20°



4 Which of the following equations represents the graph below?



(A) $y = 2\cos^{-1}(3x)$

(B) $y = 2\left|\sin^{-1}(3x)\right|$

(C) $y = 3\cos^{-1}(2x)$

(D) $y = 3\sin^{-1}(2x)$

5 Evaluate $\int_0^1 \frac{e^x}{1 + e^x}\,dx$.

(A) $\frac{e}{1 + e}$

(B) $\frac{e^2}{1 + e^2}$

(C) $\ln(1 + e)$

(D) $\ln\left(\frac{1 + e}{2}\right)$

6 A parabola has the parametric equations $x = 6t$, $y = 3t^2$. Hence $\frac{dy}{dx}$, in Cartesian form, is equal to which of the following?

(A) $6t$

(B) $\frac{x^2}{12}$

(C) $\frac{x}{6}$

(D) $\frac{2x}{9}$

7 What is the general solution of $\cos 2\alpha = \dfrac{1}{\sqrt{2}}$

(A) $\alpha = \dfrac{\pi}{8} + n\pi$ or $\alpha = -\dfrac{\pi}{8} + n\pi$, for $n \in \square$.

(B) $\alpha = \dfrac{\pi}{8} + 2n\pi$ or $\alpha = \dfrac{7\pi}{8} + 2n\pi$, for $n \in \square$.

(C) $\alpha = \dfrac{\pi}{4} + n\pi$ or $\alpha = \dfrac{3\pi}{4} + n\pi$, for $n \in \square$.

(D) $\alpha = \dfrac{\pi}{4} + 2n\pi$ or $\alpha = \dfrac{3\pi}{4} + 2n\pi$, for $n \in \square$.

8 A bag contains 5 identical blue marbles, 6 identical black marbles and 3 identical red marbles. Three marbles are drawn at random. Which expression below gives the correct probability that exactly two blue marbles are drawn?

(A) $\dfrac{{}^{5}C_{2}}{{}^{14}C_{3}}$

(B) $\dfrac{{}^{5}C_{2} \times {}^{9}C_{1}}{{}^{14}C_{3}}$

(C) $\dfrac{2}{5} \times \dfrac{1}{9}$

(D) $\dfrac{2}{14} \times \dfrac{1}{9}$

9 If $f(x)=\dfrac{3+e^{2x}}{5}$, which of the following is $f^{-1}(x)$?

(A) $\ln(5x-3)$

(B) $\dfrac{1}{2}\ln(5x-3)$

(C) $\ln(5x)-\ln(3)$

(D) $\dfrac{1}{2}\left(\ln(5x)-\ln(3)\right)$

10 The population, P, of animals in an environment in which there are scarce resources is increasing such that $\dfrac{dP}{dt}=P(100-P)$, where t is time. The initial population is 20 animals. Which of the following is true?

(A) $P=100-80e^{100t}$

(B) The population is increasing most rapidly when $P=50$.

(C) The population is increasing most rapidly when $t=50$.

(D) The maximum population is $P=50$.

End of Section 1

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE page. Extra writing booklets are available.

In questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Determine the acute angle between the lines $x+y-5=0$ and $x-2y-5=0$, correct to the nearest minute. **2**

(b) Evaluate $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \cos^2 2x \, dx$ leaving your answer in exact form. **3**

(c) (i) Sketch $y=\dfrac{x-1}{x+1}$, showing all asymptotes and intercepts. **3**

(ii) Hence, or otherwise, solve $\dfrac{x-1}{x+1}>1$. **2**

(d) *a*, *b* and *c* are the roots of the polynomial

$3x^3+4x^2-5x-8=0$

Find the value of $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$. **2**

(e) Find the exact value of $\sin 2\left(\tan^{-1}\dfrac{1}{2}\right)$ **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the expansion of $\left(2x^2 - \frac{1}{x}\right)^9$.

(i) Find the coefficient of x^6. **2**

(ii) Determine the size of the greatest coefficient. **2**

(b) Use the substitution $u = 9 - x$ to evaluate $\int_0^5 x\sqrt{9-x}\,dx$ **3**

(c) The point $P(1,2)$ divides the interval *AB* in the ratio $k:1$. If A is the point $(-3,6)$and B is the point $(7,-4)$, determine the value of *k*? **2**

(d) Two metals, *A* and *B*, are heated in separate ovens. Metal *A* is heated to a temperature of 175° and metal *B* is heated to a temperature of 275°. The metals are taken out of the ovens at the same time and left in a room to cool. The metals cool at different rates. The temperature of metal *A* is given by $T_A = 25 + 150e^{-\left(\frac{1}{20}\ln\frac{3}{2}\right)t}$ where t is the time in minutes after the metals have been removed from the ovens. The temperature of metal *B* is given by $T_B = 25 + 250e^{-kt}$

(i) Twenty minutes after being removed from the oven the temperature of metal *B* is 175°. Show that **1**

$$k = \frac{1}{20}\ln\frac{5}{3}.$$

(ii) How many minutes after being removed from the ovens will the metals have the same temperature? Write your answer to the nearest minute. **2**

(e) Prove by mathematical induction that

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots\cdots + \frac{n}{(n+1)!} = \frac{(n+1)! - 1}{(n+1)!}$$

n is any positive integer, where $n \geq 1$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch $f(x)=\sin^{-1}\left(\frac{5x}{3}\right)$. **2**

(ii) Find $f'(x)$ for the function. **2**

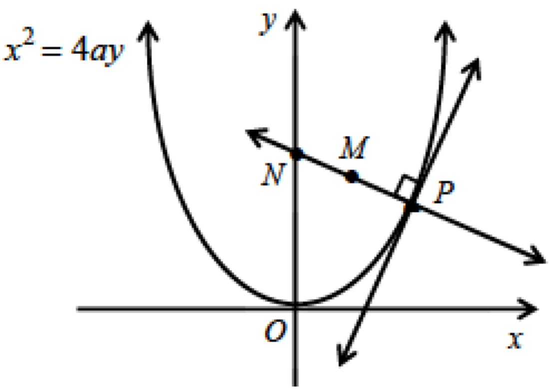
(b) Evaluate $\int_0^{\frac{1}{\sqrt{3}}} \frac{2}{9+4x^2}\,dx$. **3**

(c) An office network consisting of 20 computers has been attacked by a computer virus. The probability that any particular computer has been affected by the virus is $\frac{1}{5}$. A computer technician has to check each computer individually. **3**

(i) Write down an expression for the probability that exactly six of the computers have been affected.. **1**

(ii) Find the probability that exactly six of the computers have been affected and they are the first six computers that the technician checks. **2**

(d) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$. The normal to the parabola at P cuts the y-axis at N. M is the midpoint of PN.



(i) Show that N has coordinates $(0, ap^2+2a)$. **1**

(ii) Determine the coordinates of M. **1**

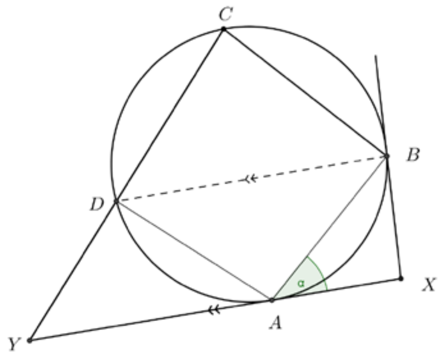
(iii) Hence, find the locus of M as P moves on the parabola. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) If $3\sin x + \sqrt{3}\cos x \equiv R\sin(x+\alpha)$, determine the values of R and α. **2**

(b)



The diagram above shows the cyclic quadrilateral *ABCD.* The tangents drawn from point *X* touch the circle at *A* and *B*. *XA* produced meets *CD* produced at *Y*. The chord *DB* is parallel to *YX*.

(i) Given $\angle BAX = \alpha$ show that $\angle BCD = 2\alpha$. **3**

(ii) Show that *BXYC* is a cyclic quadrilateral. **2**

(c) A four letter employee password is formed from the letters *A, B, C, D, E* and *F*.

(i) If repetition of letters is not allowed, how many different passwords beginning with *A* can be formed? **1**

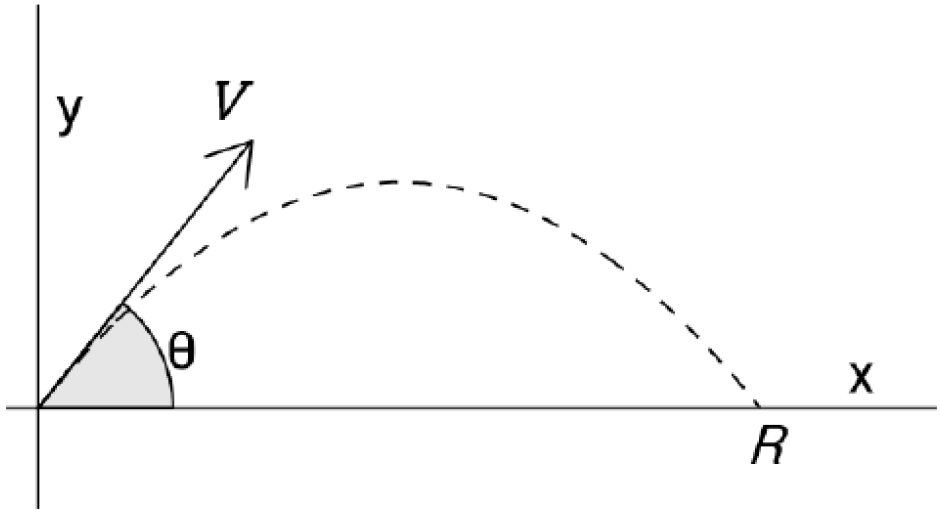
(ii) If repetition is allowed, how many different passwords can be formed if exactly one of the letters must appear twice ? **2**

Question 14 continues on page 16

Question 14 (continued)

(d) The projectile is fired on the x-y plane with initial velocity V and an angle of projection θ. You are given that the Cartesian equation of the projectile is

$$y = tan\theta - \frac{gx^2}{2V^2} sec^2\theta \quad \text{(Do not prove this.)}$$



(i) Show that the range is given by $R = \frac{V^2 sin2\theta}{g}$. **2**

(ii) A projectile falls 100 metres short of a target when the angle of projection is 15°and lands 558.8 metres past the target when the angle of projection is 30°. Find the angle of projection required to hit the target giving your answer correct to the nearest minute. **3**

End of Paper.

DRAFT SOLUTIONS

Name:____________________________________

2018

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Multiple-Choice Answer Sheet

Select the alternative A, B, C, or D that best answers the question by placing an **X** in the box.

	A	B	C	D
1		X		
2				X
3		X		
4			X	
5				X
6			X	
7	X			
8		X		
9		X		
10		X		

Section 1 10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

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(B) $2\cos\left(x-\frac{\pi}{3}\right)$

(C) $2\cos\left(x+\frac{\pi}{6}\right)$

(D) $2\cos\left(x-\frac{\pi}{6}\right)$

$\equiv R\cos(x-\alpha)$

$R=2$

$\tan\alpha=\sqrt{3}$

$\alpha=\frac{\pi}{3}$

B

2 If $(x-2)$ and $(x+1)$ are factors of x^3+x^2+bx+c what is the value of $(b+2c)$?

(A) –4

(B) 12

(C) –8

(D) –12

$8+4+2b+c=0$

$2b+c=-12$ — (1)

$-1+1-b+c=0$

$-b+c=0$ — (2)

$b+2c=-12$

D

3 The diagram below shows a circle with tangent. If O is the centre, find the value of $x°$. Diagram is not to scale.

(A) 60°

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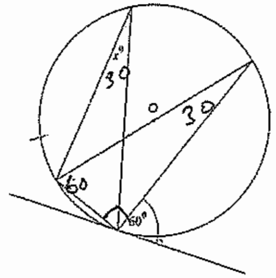

B

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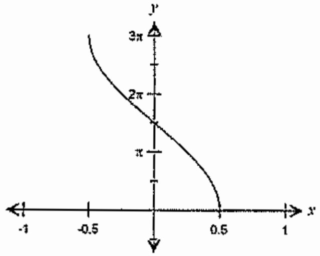

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(C) $\ln(5x)-\ln(3)$

(D) $\frac{1}{2}(\ln(5x)-\ln(3))$

$y=\frac{3+e^{2x}}{5}$

$\therefore$ inverse $x=\frac{3+e^{2y}}{5}$

$5x-3=e^{2y}$

$2y=\ln|5x-3|$ $\therefore y=\frac{1}{2}\ln(5x-3)$

B

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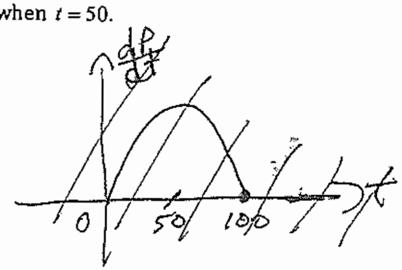


End of Section 1

B

Q.11

(a) $x+y-5=0$, $x-2y-5=0$

$m_1=-1$, $m_2=\frac{1}{2}$

$\therefore$ using $\tan\alpha=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$=\left|\frac{-1-\frac{1}{2}}{1-\frac{1}{2}}\right|$

$=\left|\frac{-\frac{3}{2}}{\frac{1}{2}}\right|$

$=|-3|$

$\alpha=71^\circ 34'$

2

(b) $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}}\cos^2 2x\,dx$

$=\frac{1}{2}\int_{\frac{\pi}{16}}^{\frac{\pi}{8}}[1+\cos 4x]\,dx$

$=\frac{1}{2}\left[x+\frac{1}{4}\sin 4x\right]_{\frac{\pi}{16}}^{\frac{\pi}{8}}$

$=\frac{1}{2}\left[\left(\frac{\pi}{8}+\frac{1}{4}\sin\frac{\pi}{2}\right)-\left(\frac{\pi}{16}+\frac{1}{4}\sin\frac{\pi}{4}\right)\right]$

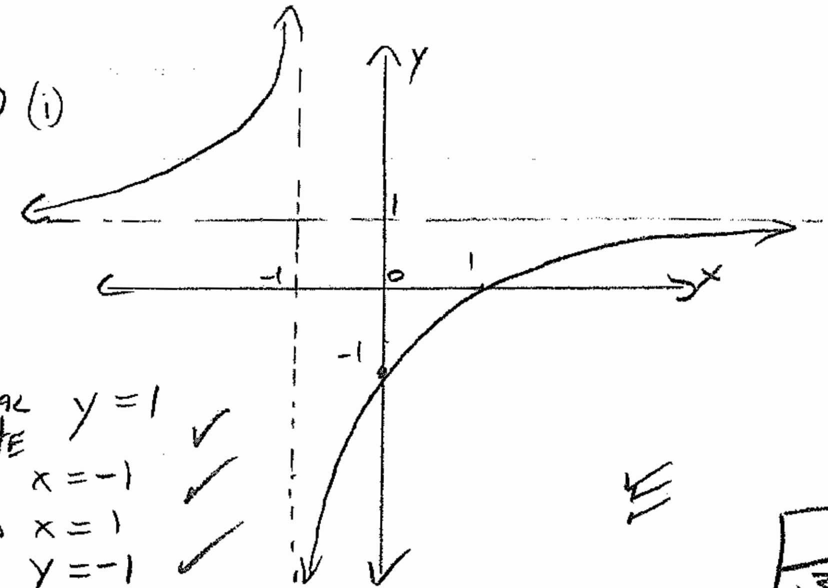
$=\frac{1}{2}\left[\frac{\pi}{8}+\frac{1}{4}-\frac{\pi}{16}-\frac{1}{4}\times\frac{1}{\sqrt{2}}\right]$

$=\frac{1}{2}\left[\frac{\pi}{16}+\frac{1}{4}-\frac{1}{4\sqrt{2}}\right]$

$=\frac{1}{8}\left[\frac{\pi}{4}+1-\frac{1}{\sqrt{2}}\right]$

3

(c) (i)



Horizontal Asymptote $y = 1$ ✓

Vertical $x = -1$ ✓

Intercepts $x = 1$ ✓

$y = -1$ ✓

[3]

(ii) $\frac{x-1}{x+1} > 1$

Solution $x < -1$

[2]

(d) $3x^3 + 6x^2 - 5x - 8 = 0$

$\therefore$ Sum $a+b+c = -\frac{6}{3}$

Sum×2: $ab + ac + bc = -\frac{5}{3}$

Prod $abc = \frac{8}{3}$

$\therefore \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{bc + ac + ab}{abc}$ ✓

$= \frac{-\frac{5}{3}}{\frac{8}{3}}$

$= -\frac{5}{8}$ ✓

[2]

(e) $\sin 2\theta$ where $\theta = \tan^{-1}\frac{1}{2}$

$= 2\sin\theta\cos\theta$ ✓ $\tan\theta = \frac{1}{2}$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ (triangle: $\sqrt{5}$, 1, 2) ✓

$= \frac{4}{5}$ ✓

[2]

Q12

(a) $\left(2x^2 - \frac{1}{x}\right)^9$

(i) $T_{r+1} = {}^9C_r\,(2x^2)^{9-r} \cdot \left(-\frac{1}{x}\right)^r$

$= {}^9C_r\, 2^{9-r} \cdot x^{18-2r} (-1)^r \cdot x^{-r}$ ✓

$= (-1)^r\,{}^9C_r\, 2^{9-r}\, x^{18-3r}$

[2]

let $18 - 3r = 6$

$3r = 12$

$r = 4$

$\therefore$ Coeff $= {}^9C_4 (2^5)$ ✓

$= 4032$

(ii) Consider $\frac{T_{r+1}}{T_r} \geq 1$ where $\left(2x^2 + \frac{1}{x}\right)^9$ Ignore (−)

$\therefore \frac{{}^9C_r\,(2x^2)^{9-r} \cdot \left(+\frac{1}{x}\right)^r}{{}^9C_{r-1}\,(2x^2)^{10-r}\left(+\frac{1}{x}\right)^{r-1}}$

$\therefore \frac{{}^9C_r}{{}^9C_{r-1}} \cdot \left(\frac{1}{2}\right) \cdot \left(+\frac{1}{x}\right)^{1-1}$

$= \frac{\frac{9!}{r!(9-r)!}}{\frac{9!}{(r-1)!(10-r)!}} \times \frac{1}{2} = \left[\frac{10-r}{r} \times \frac{1}{2}\right]$

Use $\left[\frac{n-r+1}{r} \cdot \frac{b}{a} \geq 1\right]$

$\frac{10-r}{r}\,\frac{(+1)}{2} \geq 1$

$\frac{10-r}{2r} \geq 1$

$10 - r \geq 2r$

$3r \leq 10$

$r \leq \frac{10}{3}$

$r = 3$

$\therefore$ Greatest Coeff is $\frac{{}^9C_3}{{}^9C_2} \times \frac{1}{2}$

12 (b) $\int_0^5 x\sqrt{9-x}\,dx$ $\quad u = 9-x$, $\frac{du}{dx} = -1$

$= \int_9^4 (9-u)\sqrt{u}\cdot -1\,du$

$= \int_4^9 \left[9u^{\frac{1}{2}} - u^{\frac{3}{2}}\right] du$

$= \left[\frac{9u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2}\right]_4^9$

$= \left[6u^{3/2} - \frac{2}{5}u^{5/2}\right]_4^9$

$= 6(9)^{3/2} - \frac{2}{5}(9)^{5/2} - 6(4)^{3/2} + \frac{2}{5}(4)^{5/2}$

$= \frac{148}{5}$

(c) Ratio $k:1$ $\quad A(-3,6)$, $B(7,-4)$

$\therefore \left[\frac{7k-3}{k+1}, \frac{-4k+6}{k+1}\right] = [1, 2]$

$\therefore \frac{7k-3}{k+1} = 1 \qquad \frac{-4k+6}{k+1} = 2$

$7k-3 = k+1 \qquad -4k+6 = 2k+2$

$6k = 4 \qquad 6k = 4$

$k = \frac{2}{3} \qquad k = \frac{2}{3}$

[illegible]

12(d) $T_A = 25 + 150e^{-\left[\frac{1}{10}\ln\frac{3}{2}\right]t}$

$T_B = 25 + 250e^{-kt}$

(i) For B $\quad t = 20 \quad T = 175$

$\therefore 175 = 25 + 250e^{-kt}$

$250e^{-kt} = 150$

$e^{-kt} = \frac{15}{25}$

$\therefore e^{-20k} = \frac{3}{5}$

$-20k = \ln\left(\frac{3}{5}\right)$

$k = \frac{1}{20}\ln\frac{5}{3}$

(ii) Let $25 + 150e^{-\left(\frac{1}{10}\ln\frac{3}{2}\right)t} = 25 + 250e^{-\left(\frac{1}{20}\ln\frac{5}{3}\right)t}$

$150e^{-\left(\frac{1}{10}\ln\frac{3}{2}\right)t} = 250e^{-\left(\frac{1}{20}\ln\frac{5}{3}\right)t}$

$\therefore e^{-\left(\frac{1}{10}\ln\frac{3}{2}\right)t + \left(\frac{1}{20}\ln\frac{5}{3}\right)t} = \frac{25}{15}$

$\therefore t\left[\frac{1}{20}\ln\frac{5}{3} - \frac{1}{10}\ln\frac{3}{2}\right] = \ln\frac{5}{3}$

$t \times \ln\left(\frac{5/3}{3/2}\right) = \frac{100}{7}\,20\times\ln\frac{5}{3}$

$t = \frac{\frac{100}{7}\,20\ln\frac{5}{3}}{\ln\left(\frac{10}{9}\right)} = 97$ min

12(e) $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = \frac{(n+1)!-1}{(n+1)!}$, $n \geq 1$

(i) Test for $n=1$

$LHS = \frac{1}{2!} = \frac{1}{2}$

$RHS = \frac{2!-1}{2!} = \frac{2-1}{2!} = \frac{1}{2}$

$\therefore$ True for $n=1$

(ii) Assume true for $n=k$

$\therefore \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{k}{(k+1)!} = \frac{(k+1)!-1}{(k+1)!}$ *

(iii) Test for $n=k+1$

$\therefore LHS = \underbrace{\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{k}{(k+1)!}}_{*} + \frac{(k+1)}{(k+2)!} = \frac{(k+2)!-1}{(k+2)!}$

Now $LHS = \frac{(k+1)!-1}{(k+1)!} + \frac{k+1}{(k+2)!}$ from assumption *

$= \frac{((k+1)!-1)(k+2) + k+1}{(k+2)!}$

$= \frac{(k+2)! - k - 2 + k + 1}{(k+2)!}$ as $(k+1)!(k+2) = (k+2)!$

$= \frac{(k+2)!-1}{(k+2)!} = RHS$

$\therefore$ if true for $n=k$ hence true for $n=k+1$

But as it is true for $n=1$ it must be true for $n=2,3,\ldots$ and hence true for all n, by Mathematical Induction

MARKING
1 - Step (i) + Conclusion
2 - Steps (ii) + (iii)

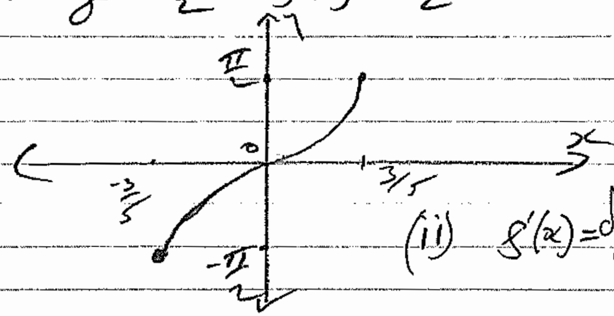
Q13

(a) (i) $f(x) = \sin^{-1}\left(\frac{5x}{3}\right)$

Domain: $-1 \leq \frac{5x}{3} \leq 1$

$-\frac{3}{5} \leq x \leq \frac{3}{5}$

Range $-\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$



[2]

(ii) $f'(x) = \frac{d\left[\sin^{-1}\frac{x}{3/5}\right]}{dx}$

$= \frac{1}{\sqrt{\frac{9}{25} - x^2}}$

$= \frac{5}{\sqrt{9-(5x)^2}}$

[2]

(b) $\int_0^{\frac{1}{\sqrt{3}}} \frac{2}{9+4x^2}\,dx$

$= \int_0^{\frac{1}{\sqrt{3}}} \frac{2}{9+(2x)^2}\,dx$

$= \frac{1}{3}\left[\tan^{-1}\frac{2x}{3}\right]_0^{\frac{1}{\sqrt{3}}}$

$= \frac{1}{3}\left[\tan^{-1}\frac{2}{3\sqrt{3}} - \tan^{-1}0\right]$

$= \frac{1}{3}\left\{\tan^{-1}\frac{2\sqrt{3}}{9}\right]$

[3]

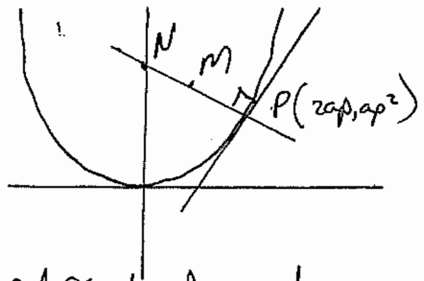
13(c)(i) $P(\text{exactly six}) = {}^{20}C_6 \left(\frac{1}{5}\right)^6 \left(\frac{4}{5}\right)^{14}$

$= 0.000002816 74$ ✓ [1]

(ii) $\left(\frac{1}{5}\right)^6 \times \left(\frac{4}{5}\right)^{14}$ ✓✓ [2]

check

(14)



(i) From Ref Sheet Normal:

$x + py = ap^3 + 2ap$

let $x = 0$ $\therefore y = ap^2 + 2a$

$N(0, ap^2 + 2a)$ ✓ [1]

(ii) $M\left[\frac{0 + 2ap}{2}, \frac{ap^2 + 2a + ap^2}{2}\right]$

$= (ap, ap^2 + a)$ ✓ [1]

(iii) Locus $x = ap$ $\quad y = ap^2 + a$ ✓

$\therefore p = \frac{x}{a}$ $\quad \therefore y = a\left(\frac{x}{a}\right)^2 + a$ [3]

$= \frac{x^2}{a} + a$ ✓

$ay = x^2 + a^2$ ✓

$x^2 = a(y - a)$

Q14(a) $3\sin x + \sqrt{3}\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$

$\therefore$ $R\cos\alpha = 3$ --- (i)

$R\sin\alpha = \sqrt{3}$ --- (ii)

$\therefore (ii)^2 + (i)^2 \Rightarrow R^2(\sin^2\alpha + \cos^2\alpha) = 12$

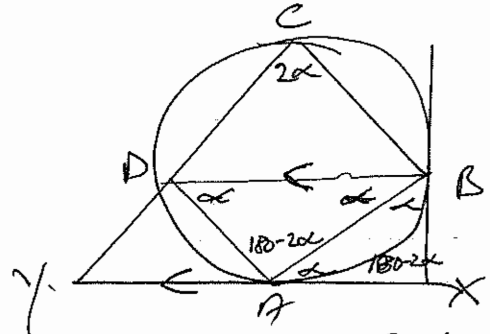
$R = \sqrt{12}$ ✓

$= 2\sqrt{3}$

(ii) ÷ (i) $\quad \tan\alpha = \frac{\frac{\sqrt{3}}{3}}{}$

$= \frac{1}{\sqrt{3}}$ $\quad \therefore \alpha = \frac{\pi}{6}$ ✓ [2]

(b)



(i) $\angle BDA = \alpha$ [Alt ∠ Theorem] ✓

$\angle DBA = \alpha$ [Alt ∠, DB // YX]

$\therefore \angle DAB = 180 - 2\alpha$ [∠ sum Δ] ✓ [2]

$\therefore \angle BCD = 2\alpha$ [ABCD is Cyclic Quad]

(ii) In $\triangle AXB$, $AX = BX$ [~~Lengths, Drawn~~ TANGENT LENGTHS EQUAL]

$\therefore \angle ABX = \alpha$

$\therefore \angle AXB = 180 - 2\alpha$

$\therefore$ BXYC is a cyclic QUAD ✓ [2]

AS $\angle BCA = 2\alpha$

14 (C) (i) [A] B C D E F

No. of 4 letter with A $= 1 \times 5 \times 4 \times 3$

$= \underline{60}$ ✓ [1]

(ii) $\dfrac{6 \times 5 \times 4}{2}$ ✓

or 6 ways to select letter

$\therefore 6 \times$ [A][A] $\times 5 \times 4 \times 6 = \underline{120} \times 6$ $= 720$ [2]

OR $6 \times {}^5C_2 \times \dfrac{4!}{2!} = \underline{\underline{720}}$

(d)(i) $y = \tan\theta x - \dfrac{gx^2}{2v^2}\sec^2\theta$

let $y = 0$ $\tan\theta - \dfrac{gx^2}{2v^2}\sec^2\theta = 0$

$\dfrac{g\sec^2\theta}{2v^2} \cdot x^2 = \tan\theta$

$x^2 = \dfrac{2v^2\tan\theta}{g\sec^2\theta}$

$= \dfrac{2v^2}{g} \cdot \dfrac{\sin\theta}{\cos\theta} \times \cos^2\theta$

$x^2 = v^2 \dfrac{2\sin\theta \cdot \cos\theta}{g}$

$\therefore R = \dfrac{v^2 \sin 2\theta}{g}$

if $R = x^2$

(ii) $R_1 = x - 100$ when $\theta = 150°$ $R_2 = 558.8 + x$ $\theta = 30°$

$\therefore \dfrac{v^2 \sin 30°}{g} = x - 100$ $\dfrac{v^2 \sin 60°}{g} = 558.8 + x$ --(*)

$\therefore \dfrac{x - 100}{\sin 30°} = \dfrac{558.8 + x}{\sin 60°}$

$\dfrac{\sqrt{3}}{2}(x - 100) = \dfrac{1}{2}(558.8 + x)$

$\sqrt{3}x - x = 658.8$

$x = \dfrac{658.8}{\sqrt{3} - 1}$

$= \underline{899.96}$

from * $\dfrac{v^2}{g} = \dfrac{558 + x}{\sqrt{3}/2}$

$= \dfrac{558 + 899.96}{\sqrt{3}/2}$

Now $\dfrac{v^2}{g} = 1683.68$

$\therefore \sin 2\theta = \dfrac{899.96}{1683.68}$ $2\theta = \sin^{-1}(0.5\ldots$ $\theta = \underline{16°}$