HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS
EXTENSION 1
2005
YEAR 12

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

EXAMINERS ~ H. CAVANAGH, S. FAULDS, Z. PETHERS, G. RAWSON

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 2 hours.
- Attempt **all** questions.
- Questions are of equal value.
- **All** necessary working should be shown in every question.
- This paper contains seven (7) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and MathAids may be used.
- **Each question is to be started in a new answer booklet**.
- This examination paper must **NOT** be removed from the examination room.
- Write your student number on each booklet.

STUDENT NAME: ______________________________

TEACHER: ______________________________

QUESTION ONE 12 marks (Start a SEPARATE booklet) **Marks**

(a) Solve $\dfrac{2x+5}{x} \leq 1$ **3**

(b) Find, to the nearest degree, the acute angle formed by the lines $5x - y + 1 = 0$ and $x - 3y - 2 = 0$. **3**

(c) Differentiate $\tan^{-1}(x^2)$ **2**

(d) If $\tan\theta = 2$, and $0 < \theta < \dfrac{\pi}{2}$, find the exact value of $\sin\left(\theta + \dfrac{\pi}{4}\right)$ **2**

(e) For the function $y = 3\cos^{-1} 2x$, state the domain and the range. **2**

QUESTION TWO 12 marks (Start a SEPARATE booklet)

(a) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{5x}$. **1**

(b) The point P (5,7) divides the interval joining the points A (-1,1) and B (3,5) *externally* in the ratio m : 1. Find the value of m. **2**

(c) (i) Write $x^2 + 6x + 13$ in the form $(x+b)^2 + c$ **2**

(ii) Hence find $\int \frac{dx}{x^2 + 6x + 13}$. **1**

(d) (i) Express ${}^{12}P_r$ in factorial notation.

(ii) Hence, if ${}^{12}P_r = 120.{}^{12}C_r$, find r. **3**

(e) (i) Differentiate $y = \cos 2x$. **1**

(ii) For what values of x is the gradient of the curve $y = \cos 2x$ ($0 \le x \le 2\pi$) equal to $-\sqrt{3}$? **2**

QUESTION THREE 12 marks (Start a SEPARATE booklet) **Marks**

(a) Evaluate $\int_0^{\pi/2} \frac{\sin\theta \, d\theta}{3-2\cos\theta}$, using the substitution $u = 3-2\cos\theta$ **3**

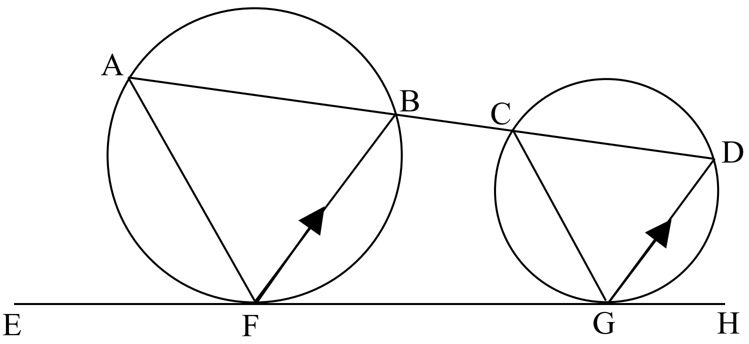
(b) Use the substitution $y=\sqrt{x}$ to find $\int \frac{dx}{\sqrt{x(1-x)}}$ **3**

(c) If $x^2 - x - 2$ is a factor of $x^4 + 3x^3 + ax^2 - 2x - b$, find a and b. **3**

(d) Given that a root of the equation $x + \ln x = 2$ lies close to $x = 1\cdot 5$, use Newton's Method to find a closer approximation to that root, correct to two decimal places. **3**

QUESTION FOUR 12 marks (Start a SEPARATE booklet) **Marks**

(a)

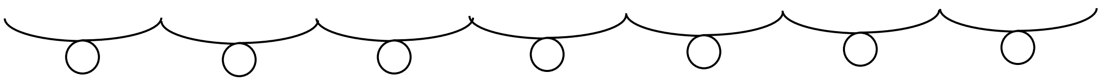


In the diagram above, FG is a common tangent and FBGD.

(i) Prove that FAGC. **3**

(ii) Prove that BCGF is a cyclic quadrilateral. **2**

(b)



String of coloured light globes

On a string of seven party lights there is one yellow, one green, one blue, one orange and three red globes. The globes are arranged at random.

(i) How many different arrangements of globes are possible on the string? **1**

(ii) Comment upon the following statement: **2**

"If globes are inserted at random, exactly 2 red globes will be next to each other in at least 50% of the arrangements."

(c) Prove that the expression **4**

$$3^n + 5$$

is divisible by 8 but only if n is an odd integer.

QUESTION FIVE 12 marks (Start a SEPARATE booklet) **Marks**

(a) (i) Express $2\cos\theta - \sqrt{5}\sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R > 0$. **2**

(ii) Hence, or otherwise, find all values of θ, where $0 \le \theta \le 360°$, for which $2\cos\theta - \sqrt{5}\sin\theta = 2$. **2**

(b) Use the substitution $t = \tan x$ to find all angles x, where $0 \le x \le 2\pi$, for which $\sin 2x + \cos 2x = 1$. **4**

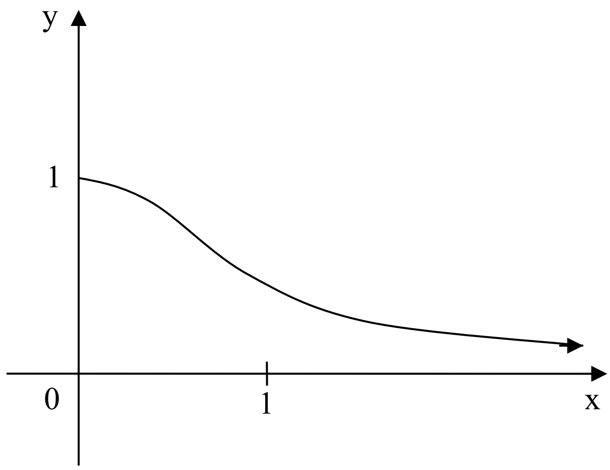
(c) (i) Write $\cos^2 x$ in terms of $\cos 2x$. **1**

(ii) Hence, or otherwise, find the volume of the solid generated when the curve $y = \cos x$ between $x = 0$ and $x = \dfrac{\pi}{2}$ is rotated around the x axis. **3**

QUESTION SIX 12 marks (Start a SEPARATE booklet) **Marks**

(a) The diagram below shows a sketch of the graph of $y = f(x)$, where $f(x) = \dfrac{1}{1+x^2}$,

for $x \geq 0$.



(i) Copy or trace this diagram. On the same set of axes, sketch the graph of the inverse function, $y = f^{-1}(x)$. **1**

(ii) State the domain of $f^{-1}(x)$. **1**

(iii) Find an expression for $y = f^{-1}(x)$ in terms of x. **2**

(b) (i) Sketch, on one number plane, graphs of $y = \sin^{-1} x$ and $y = \cos^{-1} x$, indicating their domains and ranges. **2**

(ii) Verify that the graphs in part (i) intersect where $x = \dfrac{1}{\sqrt{2}}$. **1**

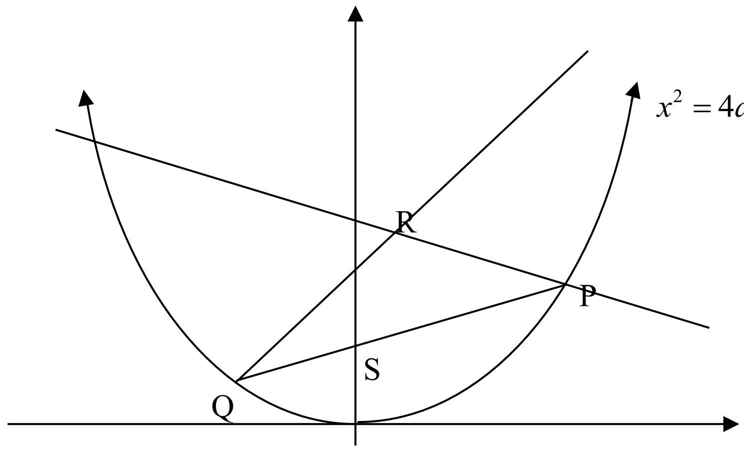
(c) $f(x) = (\cos^{-1} x)(\sin^{-1} x)$

(i) Find $f'(x)$. **2**

(ii) Prove that $f(x)$ has a maximum and find this value. **3**
[You may use any information from part (b)]

<u>QUESTION SEVEN 12 marks (Start a SEPARATE booklet)</u> **Marks**

(a) Two points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ with focus $S(0, a)$.

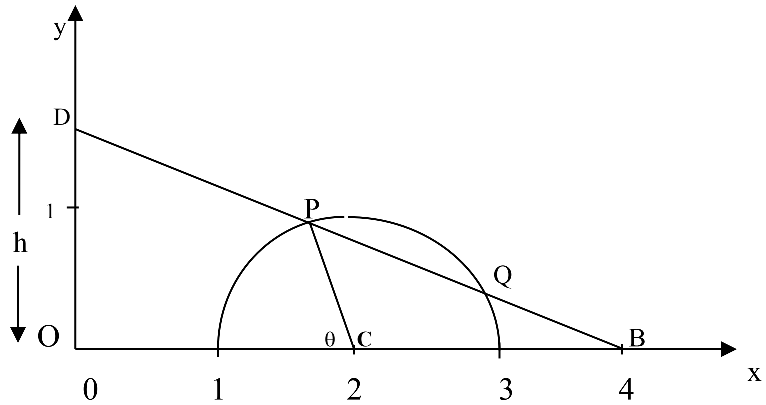


(i) Show that the equation of the normal at P is $x + py = ap^3 + 2ap$ **1**

(ii) Show that the normals at P and Q intersect at the point R where R has co-ordinates $[-apq(p+q),\ a(p^2 + pq + q^2 + 2)]$. **2**

(iii) Find the locus of R if PQ is a focal chord. **3**

(b) The diagram shows a semi-circle with centre C (2, 0) and a radius of 1 unit lying above the positive x-axis. A line from B (4, 0) cuts the semi-circle at Q and P and the y-axis at D. Let ∠ PCO = θ radians and the length OD = h units.



(i) By dropping a perpendicular from P to OB and considering similar triangles or otherwise, show that $h = \dfrac{4\sin\theta}{2 + \cos\theta}$. **2**

(ii) Find the value of θ for which $\frac{dh}{d\theta} = 0$. **2**

(iii) Show that h is a maximum for the value of θ found in part (ii). **2**