Name : ____________________

Number : ______________

HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12 2008

MATHEMATICS EXTENSION 1

TRIAL HIGHER SCHOOL CERTIFICATE

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General Instructions

- Reading time : 5 minutes
- **Working time : 2 hours**
- Attempt **all** questions
- **Start a new sheet of paper for each question**
- All necessary working should be shown
- This paper contains 8 questions worth 10 marks each. Total Marks: **80 marks**
- Marks may not be awarded for careless or badly arranged work
- Board approved calculators and mathematical templates may be used
- This examination paper must **not** be removed from the examination room

Marks

QUESTION 1. *Start a new sheet of paper.*

a) Let A $(-2,3)$ and B $(-5,-3)$ be points on the number plane. Find the coordinates of the point P which divides the interval AB externally in the ratio 3:1. **2**

b) Find the acute angle between the lines $2x-3y+1=0$ and $y=2x-1$ to the nearest degree. **2**

c) Solve the inequality $\dfrac{3}{x+2}\le 1$ **3**

d) Solve $|x+1|>|x-2|$ **3**

QUESTION 2. *Start a new sheet of paper.*

a) Use mathematical induction to prove, for all positive integers n, that **3**

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\dots\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

b) Find the Cartesian equation of the curve defined by the parametric equations $x=\sin\theta$ and $y=\cos^2\theta-3$. **2**

c) The points $P(2ap,\,ap^2)$ and $Q(2aq,\,aq^2)$ lie on the parabola $x^2=4ay$.
The equation of the normal to the parabola at P is $x+py=2ap+ap^3$ and the equation of the normal at Q is similarly given by $x+qy=2aq+aq^3$.

(i) Show that the normals at P and Q intersect at the point R whose coordinates are $\left(-apq[p+q],\,a\left[p^2+pq+q^2+2\right]\right)$. **2**

(ii) The equation of the chord PQ is $y=\frac{1}{2}(p+q)x-apq$. (Do NOT show this.) **1**
If the chord PQ passes through $(0,\,a)$, show that $pq=-1$.

(iii) Find the equation of the locus of R if the chord PQ passes through $(0,\,a)$. **2**

Marks

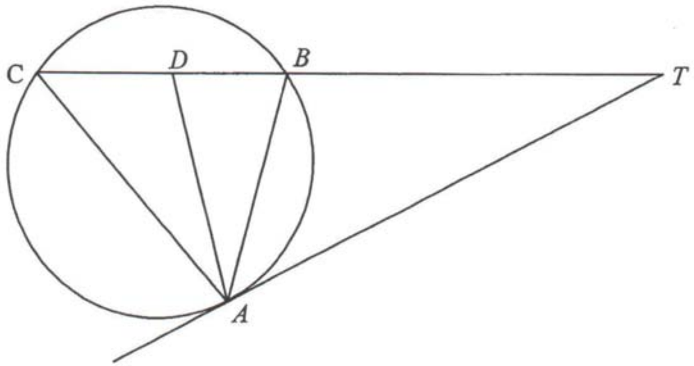
QUESTION 3. *Start a new sheet of paper.*

a) If $y = e^{kx}$ is a solution of the equation **2**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$$

determine the value(s) of k.

b) **3**



TA is a tangent to the circle above. AD bisects $\angle BAC$.
Prove that $TA = TD$.

c) On a warm summer afternoon, two cans of drink, initially at 30°C, are put into a refrigerator whose inside temperature is maintained at 4°C. The cooling rate of the soft drink is proportional to the difference between the temperature of the refrigerator and the temperature, T, of the can of drink. This means that T satisfies the equation

$$\frac{dT}{dt} = -k(T-4)$$

where t is the number of minutes after the can of drink is placed in the refrigerator

(i) Show that $T = 4 + Ae^{-kt}$ satisfies the above equation. **1**

(ii) After 20 minutes, one of the cans is removed. It is found that its temperature is now 15°C. Find the values of A and k. **2**

(iii) How long will it take for the other can to cool down to 8°C? {Time is measured from when it was put in the refrigerator} **2**

Marks

QUESTION 4. *Start a new sheet of paper.*

a) State the domain of $y = \cos^{-1} 3x$. Sketch the graph of $y = \cos^{-1} 3x$ **2**

b) Evaluate $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **2**

c) By writing an expression for $\tan(\alpha - \beta)$, show that **3**

$$\tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

d) If $f(x) = \tan^{-1}\left(\frac{2}{x}\right)$ find the equation of the tangent to the curve at the point where $x = 2$. **3**

QUESTION 5. *Start a new booklet.*

a) Evaluate $\int_0^{\frac{\pi}{4}} \sin^2 x\,dx$ **2**

b) Use the substitution $u = 1-2x$ to evaluate $\int_0^{\frac{1}{2}} 2x\sqrt{1-2x}\,dx$ **3**

c) If $\int_0^t \frac{1}{1+x^2}\,dx = 0.9$, find t correct to two decimal places **2**

d) Show that $\int_0^{\frac{\pi}{4}} \tan x\,dx = \frac{1}{2}\ln 2$ **3**

Marks

QUESTION 6. *Start a new sheet of paper.*

a) i) Noting that $2\cos^2 x \equiv 1+\cos 2x$, prove that $8\cos^4 x \equiv 3+4\cos 2x+\cos 4x$ 2

ii) Hence, find the volume generated when the area bounded by the curve $y=\cos^2 x$, the x axis and $x=0$ and $x=\frac{\pi}{4}$ is rotated about the x axis 2

b) Prove the identity: $\frac{\sin A}{\cos A+\sin A}+\frac{\sin A}{\cos A-\sin A}\equiv \tan 2A$ 2

c) Solve the equation $2\sin^2\theta = \sin 2\theta$.
Give your answer in general form. 2

d) Solve $\sin x+\cos x=1$ for x where $0\le x\le 2\pi$ 2

QUESTION 7. *Start a new sheet of paper.*

The function $f(x)$ is defined by $f(x)=\frac{1}{2}\left(e^{x}-e^{-x}\right)$

a) State the domain of $f(x)$ 1

b) Prove that $f(x)$ is an increasing function and hence state its range 3

c) Explain why the function has an inverse function $f^{-1}(x)$. 1

d) By interchanging x and y, show that $f^{-1}(x)=\ln\left(x+\sqrt{x^2+1}\right)$ 3

e) Find $\frac{d}{dx}\left(f^{-1}(x)\right)$ and hence find $\int\frac{dx}{\sqrt{1+x^2}}$ 2

Marks

QUESTION 8. *Start a new sheet of paper.*

a) Let each different arrangement of all the letters of D E L E T E D be called a word.

i) How many words are possible? 1

ii) In how many of these words will the D's be separated? 1

b) Use polynomial division to find the remainder when x^4-3x^2+4x-8 is divided by x^2+1 2

c) i) Show that the function $f(x)=x^3-x^2-x-1$ has a zero between 1 and 2 1

ii) Taking x = 2 as a first approximation to this zero, use Newton's method to calculate a second approximation 1

iii) Give a geometrical interpretation of the process used in ii). Why is $x=1$ unsuitable as a first approximation to this zero? 2

d) In each of the following, use the given information to obtain the real polynomial $P(x)$ in the form $P(x)=p_0x^n+p_1x^{n-1}+p_2x^{n-2}+\dots\dots+p_n$ where $n, p_0, p_1, p_2, \dots\dots, p_n$ are to be given numerical values.

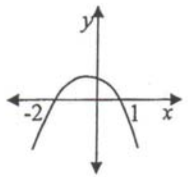
i) $P(x)$ is quadratic, $P(0)=32$, and $P(2^t)=0$ has roots $t=1$ and $t=3$ 1

ii) $P(x)$ has degree 4, has factors $(x-2)^2$ and $(x+2)^2$, and has a remainder of 50 when divided by $(x-3)$. 1

Question No. 1 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 solves problems involving inequalities
H5 applies appropriate techniques from the study of calculus & trigonometry

Outcome	Solutions	Marking Guidelines
PE3	(a) External point of division $\therefore$ ratio of $3:-1$ Point of division is $\left(\frac{lx_1+kx_2}{k+l},\frac{ly_1+ky_2}{k+l}\right)=\left(\frac{-2\times-1+-5\times3}{3-1},\frac{3\times-1-3\times3}{3-1}\right)$ $=\left(\frac{-13}{2},-6\right)$	2 marks : correct answer 1 mark : significant progress towards answer
H5	(b) For $2x-3y+1=0,\ y=\frac{2x}{3}+\frac{1}{3},\ m=\frac{2}{3}$ For $y=2x-1,\ m=2$ Using $\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert,\ \tan\theta=\left\lvert\frac{2-\frac{2}{3}}{1+2\times\frac{2}{3}}\right\rvert$ $\therefore \tan\theta=\frac{4}{7},\ \theta=30°$ (to the nearest degree)	2 marks : correct answer 1 mark : significant progress towards answer
PE3	(c) $\frac{3}{x+2}\le 1 \quad x\ne -2$ $\frac{3(x+2)^2}{x+2}\le (x+2)^2$ $3(x+2)-(x+2)^2\le 0$ $(x+2)(1-x)\le 0$  $\therefore x\le -2$ or $x\ge 1$ But $x\ne -2$, $\therefore x<-2$ or $x\ge 1$	3 marks : multiply both sides by denominator squared and correctly factorise and correctly solve inequality 2 marks : 2 of above 1 mark : one of above **or** 3 marks : correct answer with justification 2 marks : correct answer 1 mark : partially correct answer
PE3	(d) $\lvert x+1\rvert>\lvert x-2\rvert$ $\sqrt{(x+1)^2}>\sqrt{(x-2)^2}$ $(x+1)^2>(x-2)^2$ (squaring both sides as both sides positive) $x^2+2x+1>x^2-4x+4$ $6x>3$ $\therefore x>\frac{1}{2}$	3 marks : square both sides and correctly expand and correctly solve inequality 2 marks : 2 of above 1 mark : one of above **or** 3 marks : correct answer with justification 2 marks : correct answer with limited justification 1 mark : solves inequality, incorrect method

Question No. 2 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE2 uses inductive reasoning in the construction of proofs
PE3 solves problems involving (permutations and combinations, inequalities, polynomials, circle geometry and) **parametric representations**

Outcome	Solutions	Marking Guidelines
HE2	(a) Show true for $n=1$ $LHS=\frac{1}{1(1+1)}=\frac{1}{2}\quad RHS=\frac{1}{1+1}=\frac{1}{2}$ $\therefore$ true for $n=1$ Assume for $n=k$ ie $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots+\frac{1}{k(k+1)}=\frac{k}{k+1}$ Prove true for $n=k+1$ $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$ $LHS=\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$ $=\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)}$ $=\frac{k(k+2)+1}{(k+1)(k+2)}$ $=\frac{k^2+2k+1}{(k+1)(k+2)}$ $=\frac{(k+1)^2}{(k+1)(k+2)}$ $=\frac{k+1}{k+2}$ $=RHS$ $\therefore$ true by mathematical induction	**3 marks** : correct solution **2 marks** : substantially correct solution **1 mark** : partially correct solution
PE3	(b) $x=\sin\theta$ and $y=\cos^2\theta-3$ $y=(1-\sin^2\theta)-3$ $y=(1-x^2)-3$ $y=-2-x^2$	**2 marks** : correct solution **1 mark** : correct approach with no more than one algebraic error.

PE3

(c) (i) $x + py = 2ap + ap^3 \qquad \ldots(1)$

$x + qy = 2aq + aq^3 \qquad \ldots(2)$

$(1)-(2)$:

$$py - qy = 2ap + ap^3 - 2aq - aq^3$$

$$(p-q)y = 2a(p-q) + a(p-q)(p^2+pq+q^2)$$

$$y = 2a + a(p^2+pq+q^2)$$

$$= a(2+p^2+pq+q^2)$$

Sub into (1):

$$x + py = 2ap + ap^3$$

$$x + pa(2+p^2+pq+q^2) = 2ap + ap^3$$

$$x = -ap^2q - apq^2$$

$$= -apq(p+q)$$

$$\therefore R \text{ is } \left(-apq[p+q],\ a[p^2+pq+q^2+2]\right)$$

2 marks : Correct solution

1 mark : Applies correct method but obtains incorre result. (substantially correc

(ii) Substitute $(0,a)$ in $y = \frac{1}{2}(p+q)x - apq$

$$a = \frac{1}{2}(p+q).0 - apq$$

$$a = -apq$$

$$pq = -1 \qquad (\text{since } a \neq 0)$$

1 mark : correct solution

(iii) $x = -apq(p+q), \qquad$ from (i)

$= a(p+q), \qquad$ using $pq = -1$

so $p + q = \frac{x}{a} \qquad \ldots(1)$

$y = a(p^2+pq+q^2+2), \quad$ from (i)

$= a(p^2+2pq+q^2-pq+2)$

$= a([p+q]^2+3), \quad$ using $pq=-1$

$= a\left(\left[\frac{x}{a}\right]^2+3\right), \quad$ using (1)

$= \frac{1}{a}(x^2+3a^2)$

$x^2 + 3a^2 = ay$

$x^2 = a(y-3a)$

2 marks : Correct soluti

1 mark : Applies correc method but obtains inco result. (substantially co

note: need some sort of in x,y before mark awar

Question No. 3 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

HE3 uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay

H3 manipulates algebraic expressions involving logarithmic and exponential functions

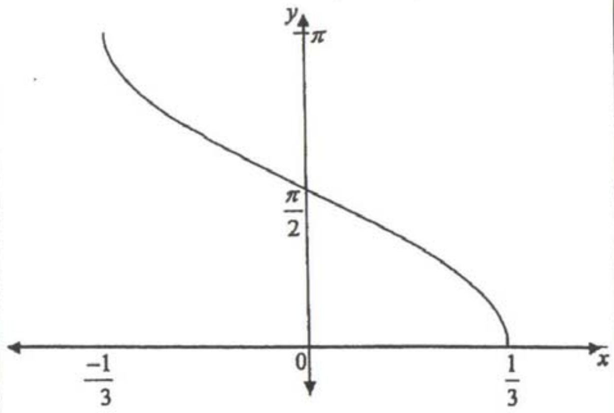
Part	Solutions	Marking Guidelines
(a) H3	$y = e^{kx},\ \frac{dy}{dx} = ke^{kx},\ \frac{d^2y}{dx^2} = k^2e^{kx}$ $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0 \Rightarrow k^2e^{kx} - 4ke^{kx} + 3e^{kx} = 0$ $\therefore e^{kx}(k^2-4k+3) = 0$ $\therefore k^2-4k+3=0 \quad (\because e^{kx} \neq 0)$ $\therefore (k-1)(k-3)=0$ $\therefore k = 1$ or 3	**Award 2** Correct solution. **Award 1** Finds the first and second derivatives only.
(b) PE3	$\angle TAB = \angle ACB$ (Angle in Alternate Segment) $= y^0$ Let $\angle CAD = x^0 = \angle BAD$ (AD bisects $\angle BAC$) $\therefore \angle TDA = (x+y)^0$ (Exterior Angle of ΔDCA) But $\angle TAD = \angle TAB + \angle BAD = (x+y)^0$ $\therefore TA = TD$ (Sides opposite equal angles)	**Award 3** Correct solution. **Award 2** Correct approach but with insufficient reasoning. **Award 1** Correct approach but with incorrect (or no) reasoning.
(c) (i) H3	LHS $= \frac{dT}{dt} = -k.Ae^{-kt}$ RHS $= -k(T-4) = -k(4+Ae^{-kt}-4)$ $= -k.Ae^{-kt}$ $=$ LHS $\therefore$ Solution to given differential equation.	**Award 1** Correct solution.
(ii) HE3	$t=0,\ T=30$ $\therefore 30 = 4 + Ae^{-k.0}$ $\therefore A = 26$ $t=20,\ T=15$ $15 = 4 + 26e^{-20k}$ $e^{-20k} = \frac{11}{26}$ $k = -\frac{1}{20}\ln\left(\frac{11}{26}\right) \approx -0.04301006326$	**Award 2** Correct solution. **Award 1** Finds only one of A or k or Attempts to use the correct method

ii) HE3	$T = $ ⁻ $8 = 4 + 26e^{\frac{1}{20}\ln\left(\frac{11}{26}\right)t}$ $e^{\frac{1}{20}\ln\left(\frac{11}{26}\right)t} = \frac{4}{26}$ $\frac{1}{20}\ln\left(\frac{11}{26}\right)t = \ln\left(\frac{4}{26}\right)$ $t = \frac{\ln\left(\frac{4}{26}\right)}{\frac{1}{20}\ln\left(\frac{11}{26}\right)} \approx 43.5200982$ minutes	**Award 2** Correct solution. **Award 1** Substantial progress tow solution.

Question No. 4 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE4 Uses the relationship between trig functions, their inverse functions and their derivatives

Outcome	Solutions	Marking Guidelines
	(a) Domain of $y = \cos^{-1} x$ is $-1 \le x \le 1$ $\therefore$ for $y = \cos^{-1} 3x$, need $-1 \le 3x \le 1$ $\therefore$ domain of $y = \cos^{-1} 3x$ is $\frac{-1}{3} \le x \le \frac{1}{3}$ 	2 marks : correct domain and correct graph for given domain 1 mark : one of above
	(b) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\pi}{3}$ (note $\frac{-\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$)	2 marks : correct answer 1 mark : significant progress towards correct answer
	(c) Let $\alpha = \tan^{-1}\frac{1}{2}$ and $\beta = \tan^{-1}\frac{1}{4}$. $\therefore \tan\alpha = \frac{1}{2}, \quad \tan\beta = \frac{1}{4}$ As $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$, $\tan(\alpha - \beta) = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2} \times \frac{1}{4}}$ $\therefore \tan(\alpha - \beta) = \frac{2}{9}$ $\therefore \alpha - \beta = \tan^{-1}\left(\frac{2}{9}\right)$ $\therefore \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}\left(\frac{2}{9}\right)$	3 marks : writes expansion for $\tan(\alpha - \beta)$ and proves result 2 marks : writes expansion for $\tan(\alpha - \beta)$ and makes significant progress towards proving result 1 mark : writes expansion for $\tan(\alpha - \beta)$

(d) $f(x)=\tan^{-1}\left(\dfrac{2}{x}\right)$

$$f'(x)=\frac{\frac{d}{dx}\left(2x^{-1}\right)}{1+\left(\frac{2}{x}\right)^2} \quad \text{since } \frac{d}{dx}\left(\tan^{-1}f(x)\right)=\frac{f'(x)}{1+\left(f(x)\right)^2}$$

$$=\frac{\frac{-2}{x^2}}{1+\frac{4}{x^2}}$$

$$\therefore f'(x)=\frac{-2}{x^2+4} \text{ and } \therefore f'(2)=\frac{-2}{2^2+4}=\frac{-1}{4}$$

When $x=2$, $=\tan^{-1}\left(\dfrac{2}{2}\right)=\tan^{-1}1=\dfrac{\pi}{4}$

Equation of tangent with $m=\dfrac{-1}{4}$, passing through

$\left(2,\dfrac{\pi}{4}\right)$ is $y-\dfrac{\pi}{4}=\dfrac{-1}{4}(x-2)$

$4y-\pi=-x+2$

Tangent is $x+4y-\pi-2=0$

3 marks : cor
derivative; c
finds gradien
and correctly
equation
2 marks : tw
1 mark : one

Question No. 5 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE6: Determines integrals by reduction to a standard form through a given substitution

	Solutions	Marking Guidelines
5(a)	$\cos 2x=1-2\sin^2 x$ $\therefore \sin^2 x=\frac{1}{2}(1-\cos 2x)$ $\int_0^{\frac{\pi}{4}}\sin^2 x\,dx=\frac{1}{2}\int_0^{\frac{\pi}{4}}(1-\cos 2x)\,dx$ $=\frac{1}{2}\left[x-\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{2}\left\{\left(\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{2}\right)-(0)\right\}$ $=\frac{1}{2}\left(\frac{\pi}{4}-\frac{1}{2}\right)$	**2 marks:** Fully correct solution **1 mark:** Correct integration, but incorrect answer.
(b)	$\int_0^{\frac{1}{2}}2x\sqrt{1-2x}\,dx$ $=\int_1^0(1-u)\sqrt{u}\,.\frac{du}{-2}$ $=-\frac{1}{2}\int_1^0(1-u)\sqrt{u}\,du$ $=\frac{1}{2}\int_0^1 u^{\frac{1}{2}}-u^{\frac{3}{2}}\,du$ $=\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right]_0^1$ $=\frac{1}{2}\left(\frac{2}{3}-\frac{2}{5}\right)$ $=\frac{2}{15}$ $u=1-2x$; $2x=1-u$ $du=-2dx$ $dx=\frac{du}{-2}$ when $x=\frac{1}{2}$, $u=0$ when $x=0$, $u=1$	**3 marks:** Fully correct solution **2 marks:** Substantial progress towards correct result **1 mark:** Some progress towards correct result.
(c)	$\int_0^t\frac{1}{1+x^2}\,dx=\left[\tan^{-1}x\right]_0^t$ $=\tan^{-1}t-\tan^{-1}0$ $=\tan^{-1}t$ *i.e.* $\tan^{-1}t=0.9$ $\therefore t=1.26$	**2 marks:** Fully correct **1 mark:** Correct integration, but incorrect answer.
(d)	$\int_0^{\frac{\pi}{4}}\tan x\,dx=\int_0^{\frac{\pi}{4}}\frac{\sin x}{\cos x}\,dx$ $=\left[-\ln(\cos x)\right]_0^{\frac{\pi}{4}}$ $=\left(-\ln\left(\cos\frac{\pi}{4}\right)\right)-\left(-\ln(\cos 0)\right)$ $=-\ln\frac{1}{\sqrt{2}}+\ln 1$ $=-\ln 2^{-\frac{1}{2}}$ $=\frac{1}{2}\ln 2$	**3 marks:** Fully correct solution **2 marks:** Substantial progress towards correct result **1 mark:** Some progress towards correct result.

Question No.6 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form
H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
HE7	a (i) $8\cos^4 x = 2\left(2\cos^2 x\right)^2$ $= 2\left(1+\cos 2x\right)^2$ $= 2\left(1+2\cos 2x+\cos^2 2x\right)$ but $\cos^2 2x = \frac{1}{2}(1+\cos 4x)$ $= 2\left(1+2\cos 2x+\frac{1}{2}(1+\cos 4x)\right)$ $= 2+4\cos 2x+1+\cos 4x$ $= 3+4\cos 2x+\cos 4x$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
H8	(ii) $V = \pi\int_0^{\frac{\pi}{4}} \cos^4 x\, dx$ $V = \pi\int_0^{\frac{\pi}{4}} \frac{1}{8}(3+4\cos 2x+\cos 4x)dx$ from part (i) $V = \frac{\pi}{8}\left[3x+2\sin 2x+\frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{4}}$ $V = \frac{\pi}{8}\left[3\left(\frac{\pi}{4}\right)+2\sin\left(\frac{\pi}{2}\right)+\frac{1}{4}\sin\pi\right]-0$ $V = \frac{\pi}{8}\left(\frac{3\pi^2}{4}+2\right)u^3$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
HE7	b $LHS = \frac{\sin A}{\cos A+\sin A}+\frac{\sin A}{\cos A-\sin A}$ $= \frac{\sin A(\cos A-\sin A)}{(\cos A+\sin A)} \times \frac{\sin A(\cos A+\sin A)}{(\cos A-\sin A)}$ $= \frac{\sin A\cos A-\sin^2 A+\sin A\cos A+\sin^2 A}{\cos^2 A-\sin^2 A}$ $= \frac{2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $= \frac{\sin 2A}{\cos 2A}$ $= \tan 2A$ $= RHS$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
HE7	c $2\sin^2\theta-\sin 2\theta = 0$ $2\sin^2\theta-2\sin\theta\cos\theta = 0$ $2\sin\theta(\sin\theta-\cos\theta) = 0$ $\sin\theta = 0, \sin\theta = \cos\theta$ $\tan\theta = 1$ $\theta = n\pi, \theta = n\pi+\frac{\pi}{4}$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
HE7	d $\sin\theta+\cos\theta = 1$ $\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right) = 1$ $\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) = 1$ $\sin\left(\theta+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $\theta+\frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$ $\theta = 0, \frac{\pi}{2}, 2\pi$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution

Question No.8 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations
PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

Outcome	Solutions	Marking Guidelines
PE3/6	a (i) $\frac{7!}{3!2!} = 420$	1 mark correct answer
PE3 **PE6**	(ii) D's together $\frac{6!}{3!2!}$ D's separated $\frac{7!}{3!2!}-\frac{6!}{3!2!} = 420-120 = 300$	1 mark correct answer
PE3 **PE6**	b $x^2 \quad -4$ $x^2+1\overline{)x^4+0x^3-3x^2+4x-8}$ $x^4 \quad +x^2$ $-4x^2+4x-8$ $-4x^2 \quad -4$ $4x-4$ $R(x) = 4x-4$	2 marks correct method leading to correct answer 1 mark substantially correct solution
PE3 **PE6**	c $f(x) = x^3-x^2-x-1$ $f(1) = 1^3-1^2-1-1<0$ $f(2) = 2^3-2^2-2-2>0$ $\therefore$ root between 1 and 2. $x_{n+1} = x_n-\frac{f(x_n)}{f'(x_n)}$ $f(2) = 1$ $f'(x) = 3x^2-2x-1$ $f'(2) = 3(2)^2-2(2)-1 = 7$ $x_2 = 2-\frac{1}{7}$ $x_2 = 1\frac{6}{7}$	1 mark correct answer 1 mark correct answer
HE7	iii Newtons method finds where the tangent to the curve cuts the x axis its value should be closer to the root than the previous approximation. $f'(x) = 3x^2-2x-1$ $f'(1) = 3(1)^2-2(1)-1 = 0$ x=1 is a turning point therefore the tangent is parallel to the x axis and does not cut the axis.	2 marks correct reasoning and solution 1 mark substantially progress towards correct solution
PE3	d (i) $P(x)$ has $P(2^1) = 0 \therefore x = 2$ is a root $P(x)$ has $P(2^3) = 0 \therefore x = 3$ is a root $\therefore P(x) = a(x-2)(x-8)$ $P(0) = 32$ $a(0-2)(0-8) = 32$ $a = 2$ $\therefore P(x) = 2(x-2)(x-8)$ $P(x) = 2x^2-20x+32$ (ii) $\therefore P(x) = a(x-2)^2(x+2)^2$ $P(3) = 50$ $a(3-2)^2(3+2)^2 = 50$ $a = 2$ $\therefore P(x) = 2(x-2)^2(x+2)^2$ $P(x) = 2x^4-16x^2+32$	1 mark correct answer 1 mark correct answer