HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS

EXTENSION 1

2009

YEAR 12

TRIAL HSC EXAMINATION

EXAMINERS ~ H. CAVANAGH, D CRANCHER, J DILLON, G HUXLEY.

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 2 hours.
- Attempt **all** questions.
- Questions are of equal value.
- **All** necessary working should be shown in every question.
- This paper contains seven (7) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and MathAids may be used.
- **Each question is to be started in a new answer booklet**. Additional booklets are available if required.
- This examination must **NOT** be removed from the examination room

STUDENT NAME: ______________________

STUDENT NUMBER: ______________________

QUESTION 1 (12 marks) Start a new booklet **Marks**

(a) Find the acute angle, to the nearest degree, between the lines **2**

$$y = 5 - x \text{ and } y = \frac{3}{2}x + 5.$$

(b) (i) Show that $(x-2)$ is a factor of $x^3 - 3x^2 + 4$ **1**

(ii) Express $x^3 - 3x^2 + 4$ as a product of three linear factors **2**

(c) If $y = \log_{\frac{1}{a}}\left(\frac{1}{N}\right)$, where $a > 0$ and $N > 0$, show that $y = \log_a N$ **2**

(d) The point (0, 4) divides the interval from *(a, b)* to *(b, a)* internally in the ratio 3: 1. Find the values of *a* and *b*. **3**

(e) Find the Cartesian equation of the locus of a point *P(x, y)* **2**

where $x = 2\cos\theta$ and $y = \frac{1}{2}\sin\theta$

QUESTION 2 (12 marks) Start a new booklet **Marks**

(a) Find $\lim_{x \to 0} \frac{\sin x}{5x}$ **1**

(b) A particle moves in a straight line so that its velocity, v, in minutes per second at time t is given by $v = 4 - 2t$. Initially, the particle is at $x = 1$.

(i) Find the displacement x of the particle as a function of t. **1**

(ii) When is the particle at rest and what is its acceleration at that time? **2**

(c) Heat is applied to a metallic disc for t seconds.
Its area, A cm^2,increases at a rate given by:

$$\frac{dA}{dt} = t^2 - 2t + 1$$

(i) Find the rate at which the area of the metallic disc is increasing at the end of the third second. **1**

(ii) Before heat is applied the area of the metallic disc is $10\,\text{cm}^2$. **2**
Heat is then applied to the metallic disc for 3 seconds.
What is the area of this disc at the end of three seconds?

(d) Given that $0 < x < \frac{\pi}{4}$ prove that : $\tan\left(\frac{\pi}{4} + x\right) = \frac{\cos x + \sin x}{\cos x - \sin x}$ **3**

(e) Let $f(x) = \log_e\left[(x+2)(x-4)\right]$ **2**

What is the domain of $f(x)$?

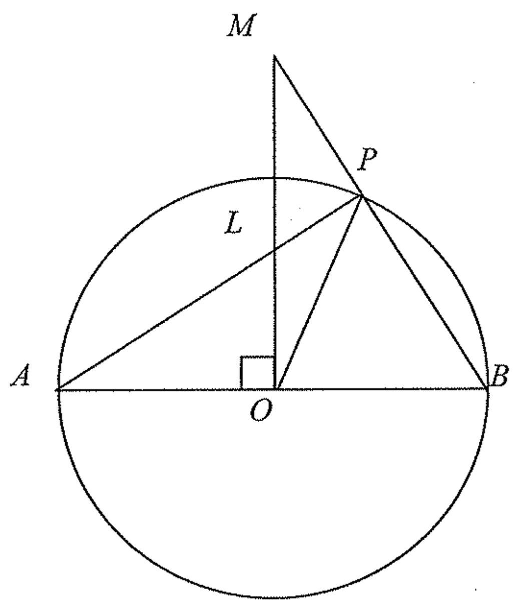
QUESTION 3 (12 marks) Start a new booklet **Marks**

(a) In how many ways can the letters of the word **SUCCESS** be arranged? All letters must be used. **2**

(b) The staff in an office consists of 4 males and 7 females. **2**

What is the probability that a randomly chosen committee of 5 staff can be formed which contains exactly 3 females?

(c)



O is the centre of the circle ABP. $MO \perp AB$. M, P and B are collinear. MO intersects AP at L.

(i) Prove that A, O, P and M are concyclic. **2**

(ii) Prove that $\angle OPA = \angle OMB$. **2**

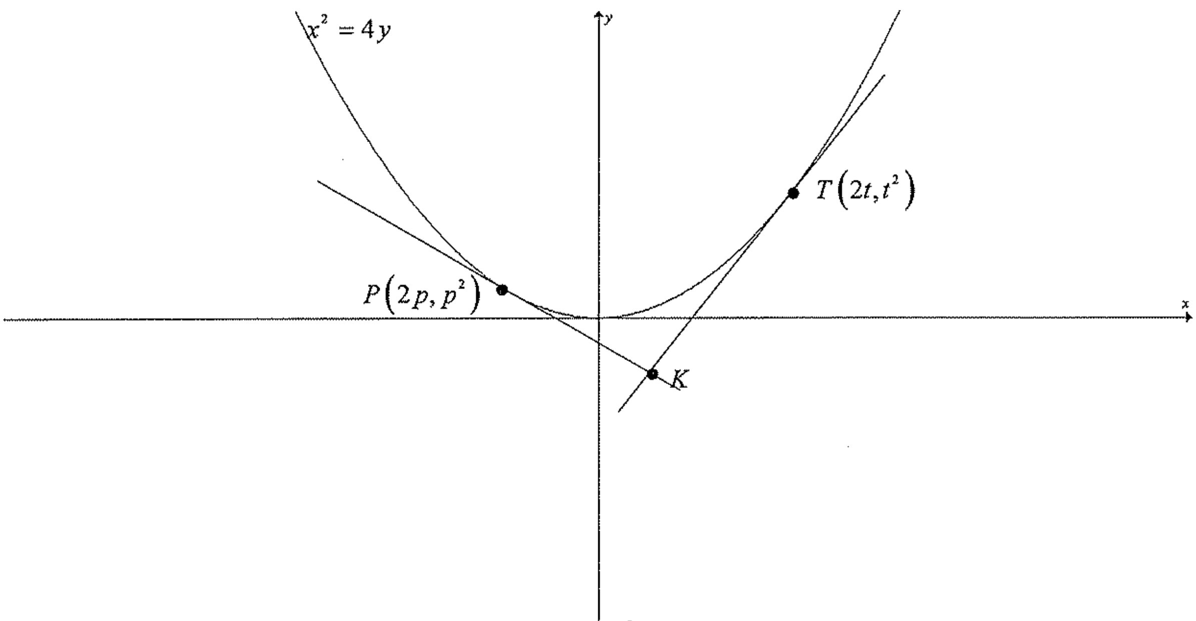
(d) (i) Show that $P(x) = x^3 - x^2 - x - 1$ has a zero between 1 and 2. **1**

(ii) Take $x = 2$ as a first approximation and use Newton's method to calculate a second approximation. **2**

(iii) Explain why $x = 1$ was not a suitable first approximation in this case. **1**

QUESTION 4 (12 marks) Start a new booklet **Marks**

(a)



The diagram shows the graph of the parabola $x^2 = 4y$ and also the tangents at $T(2t, t^2)$ and $P(2p, p^2)$. The tangents intersect at K.

(i) Prove that the equation of the tangent at T is $y = tx - t^2$ **2**

(ii) Show that the coordinates of the point K, the point where the tangents at T and P intersect, are $(p+t,\ pt)$. **2**

(iii) The angle TKP is a right angle. **1**
Show that the locus of K is a straight line.

(b) Solve the inequation: $\dfrac{x}{x-3} < 4$ **3**

(c) The polynomial $P(x) = 6x^3 - 7x^2 + ax + b$ has a zero at $x = -1$.
The remaining zeros are reciprocals.

(i) By examining the product of the three roots, determine the values of b, and hence of a. **2**

(ii) Find all zeros of $P(x)$. **2**

QUESTION 5 (12 marks) Start a new booklet **Marks**

(a) Use mathematical induction to prove that, for every positive integer n, $13 \times 6^n + 2$ is divisible by 5. **3**

(b) Differentiate with respect to x: $x e^{\sin x}$ **2**

(c) The curve $y = \sin 2x$ and the line $y = \frac{4x}{\pi}$ intersect at $x = 0$ and $x = \frac{\pi}{4}$. **2**

Find the area bounded by the curve $y = \sin 2x$ and the line $y = \frac{4x}{\pi}$ in the first quadrant. (Answer in terms of π).

(d) Find the exact volume of the solid formed when the area between the curve $y = \log_e x$ and the y-axis is rotated about the y-axis between $y = 0$ and $y = 3$. **2**

(e) Use the substitution $u = \sqrt{x}$ to evaluate $\int_0^3 \frac{dx}{\sqrt{x}(1+x)}$ **3**

QUESTION 6 (12 marks) Start a new booklet **Marks**

(a) Find the exact value of $\int_0^1 \frac{dx}{\sqrt{2-x^2}}$. **2**

(b) State the domain and range of the function $y = 2\sin^{-1}(3x)$. **3**
Sketch the graph.

(c) If $y = \cos^{-1}\left(\frac{1}{x}\right)$ for $0 \le y \le \frac{\pi}{2}$, show that: $\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$ **2**

(d) Consider the function $f(x) = e^{-x} - e^{x}$

(i) Show that $f(x)$ is decreasing for all values of x. **1**

(ii) Show that the inverse function is given by

$$f^{-1}(x) = \log_e\left(\frac{\sqrt{x^2+4}-x}{2}\right).$$ **3**

(iii) Hence, or otherwise, solve $e^{-x} - e^{x} = 6$ **1**

Give your answer correct to two decimal places.

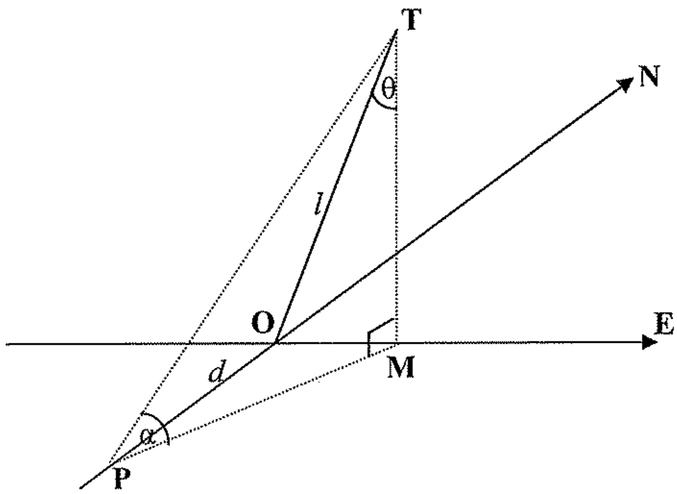
QUESTION 7 (12 marks) Start a new booklet **Marks**

(a) (i) Show that $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx = \frac{\pi+2}{8}$ **2**

(ii) The region under the curve $y = \cos x + \sec x$, above the x-axis, between $x = 0$ and $x = \frac{\pi}{4}$, makes a revolution about the x-axis. **3**

Show that the volume traced out is $\frac{5\pi(\pi+2)}{8}$ units^3.

(b)



A pole, OT, of length l metres, stands on horizontal ground.
The pole leans towards the east, making an angle of θ with the vertical.

From P, d metres south of O, the elevation of T is α

(i) Find expressions, in terms of l and θ, for OM and MT. **2**

(ii) Prove that $PM = l\cos\theta\cot\alpha$. **2**

(iii) Prove that $l^2 = \frac{d^2}{\cos^2\theta\,\cot^2\alpha - \sin^2\theta}$. **2**

(iv) Find the length of the pole, to the nearest metre, if $d = 25$, $\theta = 20^\circ$, $\alpha = 24^\circ$. **1**

END OF PAPER

Year 12 Mathematics Extension 1 Trial HSC 2009

Question No. 1 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3: solves problems involving polynomials and parametric representations
H3: manipulates algebraic expressions involving logarithmic and exponential functions
H5: applies appropriate techniques from the study of geometry and trigonometry to solve problems

Outcome	Solutions	Marking Guidelines
Q1(a) H5	$y = 5 - x$ and $y = \frac{3}{2}x + 5$ $m_1 = -1$ $\quad m_2 = \frac{3}{2}$ $\tan\theta = \left\lvert \frac{m_2 - m_1}{1 + m_1 m_2} \right\rvert = \left\lvert \frac{\frac{3}{2}+1}{1-\frac{3}{2}} \right\rvert = 5$ $\theta \doteqdot 79^\circ$	2 marks: Fully correct solution 1 mark: Attempts application of formula with correct gradients. OR arithmetic error in gradients, correct working for angle.
Q1(b)(i) PE3	$(x-2)$ is a factor of $P(x) = x^3 - 3x^2 + 4$ if $P(2) = 0$ $P(2) = 2^3 - 3(2)^2 + 4 = 0$ $\therefore (x-2)$ is a factor.	1 mark Correct use of factor theorem to prove the given result, or correct division.
Q1(b)(ii) PE3	Long division: $(x^3 - 3x^2 + 4) \div (x-2) = x^2 - x - 2$ $x^3 - 2x^2$ $-x^2 + 4$ $-x^2 + 2x$ $-2x + 4$ $-2x + 4$ .... $\therefore P(x) = (x-2)(x^2 - x - 2) = (x-2)(x+1)(x-2) = (x-2)^2(x+1)$	2 marks Correctly divides the given polynomial, and factorises the resulting quotient to show the 3 linear factors. OR Factorises correctly by inspection. 1 mark Substantial progress towards the correct solution
Q1(c) H3	$y = \log_{\frac{1}{a}}\left(\frac{1}{N}\right)$ $\left(\frac{1}{a}\right)^y = \frac{1}{N}$ Take reciprocals $a^y = N$ $\therefore y = \log_a N$	2 marks: Fully correct solution 1 mark: Sufficient progress towards correct result
Q1(d) PE3	At (0, 4), $\begin{cases} a + 3b = 0 \\ b + 3a = 16 \end{cases}$ Solving simultaneously, $a = 6, b = -2$	3 marks Correctly produces 2 equations, and correctly solves simultaneously. 2 marks Substantial progress towards the correct solution 1 mark indicates some knowledge of the process of division of an interval.
Q1(e) PE3	$\left.\begin{array}{l} x = 2\cos\theta \\ y = \frac{1}{2}\sin\theta \end{array}\right\} \rightarrow \left\{\begin{array}{l} \cos\theta = \frac{x}{2} \\ \sin\theta = 2y \end{array}\right.$ Since $\cos^2\theta + \sin^2\theta = 1$ $\frac{x^2}{4} + 4y^2 = 1$ $x^2 + 16y^2 = 4$	2 marks: Fully correct solution 1 mark: Some progress towards correct result, including use of the Pythagorean result.

Year 12 Extension 1 Mathematics Trial HSC 2009

Question No. 3 Solutions and Marking Guidelines

Outcomes Addressed in this Question

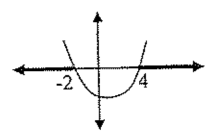
PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

	Solutions	Marking Guidelines
(a)	Number of ways $= \frac{7!}{3!2!} = 420$	Award 2 for correct solution. Award 1 for attempting to use an appropriate process.
(b)	Number of unrestricted committees of 5 $= \binom{11}{5}$ Number of committees of 3 females $= \binom{7}{3}\binom{4}{2}$ $\therefore$ Probability $= \frac{\binom{7}{3}\binom{4}{2}}{\binom{11}{5}} = \frac{5}{11}$	Award 2 for correct solution. Award 1 for attempting to use an appropriate process.
(c) (i)	Let $\angle PAO = \theta$ $\angle APB = 90^0$ (angle in a semi-circle) $\therefore \angle ABP = (90 - \theta)^0$ (angle sum of ΔABP) $\therefore \angle BMO = \theta$ (angle sum of ΔBMO) $\therefore \angle OAP = \angle BMO$ $\therefore AOPM$ is a cyclic quadrilateral (OP subtends equal angles at A and M) $\therefore A, O, P,$ and M are concyclic	Award 2 for correct solution. Award 1 for substantial progress towards solution.
(ii)	$\angle OPA = \theta$ (angles opposite equal sides OP and OA) $\angle BMO = \theta$ (above) $\therefore \angle OPA = \angle OMB$	Award 2 for correct solution. Award 1 for substantial progress towards solution.
(d) (i)	$P(1) = 1 - 1 - 1 - 1 = -2$ $P(2) = 8 - 4 - 2 - 1 = 1$ $\therefore$ Since $P(x)$ is a continuous function and the sign changes, there must be a root between 1 and 2.	Award 1 for correct solution.
(ii)	$P'(x) = 3x^2 - 2x - 1$ $P(2) = 1$ and $P'(2) = 7$ $\therefore x_2 = 2 - \frac{1}{7} = 1\frac{6}{7}$	Award 2 for correct solution. Award 1 for substantial progress towards solution.
(iii)	$P'(1) = 0$ This means that $x = 1$ is a stationary point $\therefore$ Newton's method will fail.	Award 1 for correct solution

Year 12 Mathematics Extension 1 Task 4 Trial HSC 2009	
Question No. 2	Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

Outcome	Solutions	Marking Guidelines
Q2.		
H5	(a) $\frac{1}{5}$	1 mark correct answer
H5	(b) (i) $x=\int(4-2t)\,dt$ $=4t-t^2+C$ when $x=1, t=0 \Rightarrow C=1$ $x=4t-t^2+1$	1 mark correct answer
	(ii) The particle is at rest when $v=0$ $0=4-2t$ $0=2(2-t)$ $t=2$	1 mark for correct time particle is at rest
	$a=\frac{dv}{dt}=-2$ $\therefore$ The particle is at rest after 2 seconds and has constant acceleration of $-2m/s^2$.	1 mark for correct acceleration at that time.
H5	(c) (i) At $t=3$ $\frac{dA}{dt}=(3)^2-2.(3)+1$ $=4$ The rate at which the disc is increasing is $4cm^2/s$	1 mark for correct answer
	(ii) $A=\int(t^2-2t+1)dt$ $=\frac{t^3}{3}-t^2+t+C$ When $t=0, A=10 \Rightarrow C=10$	1 mark awarded for partial correct solution
	$A=\frac{1}{3}t^3-t^2+t+10$ At $t=3$, $A=\frac{1}{3}(3)^3-(3)^2+3+10$ $=13cm^2$	2 marks awarded for complete correct solution
H5	(d) $LHS=\tan\left(\frac{\pi}{4}+x\right)$ $=\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}$ $\left[\text{since, } \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}\right]$	1 mark awarded for partial correct solution
	$=\frac{1+\tan x}{1-\tan x}$	2 marks awarded for further partial correct solution
	$=\frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}$ $\left[\text{since, } \tan x=\frac{\sin x}{\cos x} \text{ \& } 1=\frac{\cos x}{\cos x}\right]$ $=\frac{\cos x+\sin x}{\cos x-\sin x}$ $=RHS$ $\therefore \tan\left(\frac{\pi}{4}+x\right)=\frac{\cos x+\sin x}{\cos x-\sin x}$	3 marks awarded for complete correct solution
H5	(e) $f(x)=\log_e[(x+2)(x-4)]$  $(x+2)(x-4)>0$ $\therefore x<-2, x>4$	1 mark awarded for partial correct solution 2 marks awarded for complete correct solution

Year 12 Question No. 4 — Extension 1 Mathematics — Solutions and Marking Guidelines — Trial HSC 2009

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

	Solutions	Marking Guidelines
(a) (i)	$y=\frac{x^2}{4}$ $y'=\frac{x}{2}$ At $(2t,\ t^2)$ $y'=t=m_{\text{tangent}}$ $\therefore$ Equation of tangent is $y-t^2=t(x-2t)=tx-2t^2$ $\therefore\ y=tx-t^2$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.
(ii)	Equation of tangent at P is $y=px-p^2$ $\therefore$ At point of intersection $px-p^2=tx-t^2$ $(p-t)x=p^2-t^2=(p-t)(p+t)$ $x=p+t\ (\because p\neq t)$ Sub. into $y=px-p^2$ $y=p(p+t)-p^2=pt$ $\therefore$ Intersect @ $(p+t,\ pt)$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.
(iii)	If $\angle TKP=90^0$ $m_{TK}\times m_{PK}=-1$ $\therefore t\times p=-1\Rightarrow pt=-1$ $\therefore$ Locus of K is $x=p+t,\ y=-1$ $\therefore$ Locus of K is the straight line $y=-1$.	**Award 1** for correct solution.
(b)	$\frac{x}{x-3}<4$ $x(x-3)<4(x-3)^2$ $(x-3)(x-4(x-3))<0$ $(x-3)(12-3x)<0$ $(x-3)(4-x)<0$ $\therefore x<3$ or $x>4$	**Award 3** for correct solution. **Award 2** for substantial progress towards solution. **Award 1** for limited progress towards solution
(c) (i)	$P(-1)=0\Rightarrow 6(-1)^3-7(-1)^2+(-1)a+b=0$ $\therefore b-a=13$ (1) Roots are $-1,\alpha,\frac{1}{\alpha}\Rightarrow -1\times\alpha\times\frac{1}{\alpha}=-\frac{b}{6}$ $\therefore\frac{b}{6}=1\Rightarrow b=6$ Substitute into (1) $\Rightarrow a=-7$	**Award 2** for correct solution. **Award 1** for substantial progress towards solution.
(ii)	$P(x)=6x^3-7x^2-7x+6$ $-1+\alpha+\frac{1}{\alpha}=-\frac{-7}{6}=\frac{7}{6}$ $\alpha+\frac{1}{\alpha}=\frac{13}{6}$ $\alpha^2+1=\frac{13}{6}\alpha$ $6\alpha^2-13\alpha+6=0$ $(3\alpha-2)(2\alpha-3)=0$ $\therefore\alpha=\frac{2}{3}$ or $\alpha=\frac{3}{2}$ $\therefore$ Zeros are $-1,\frac{2}{3},\frac{3}{2}$.	**Award 2** for correct solution. **Award 1** for substantial progress towards solution.

Year 12 Mathematics Extension 1 Task 4 Trial HSC 2009

Question No. 5 Solutions and Marking Guidelines

Outcomes Addressed in this Question

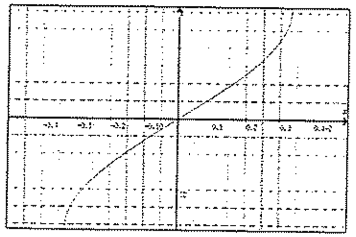
P7	determines the derivative of a function through routine application of the rules of differentiation
PE2	uses multi-step deductive reasoning in a variety of contexts
H8	uses techniques of integration to calculate areas and volumes
HE6	determines integrals by reduction to a standard form through a given substitution

Outcome	Solutions	Marking Guidelines
PE2	**5.** (a) Let S_n: $13\times 6^n+2$ is divisible by 5 i.e. S_n: $13\times 6^n+2=5K$ where K,n are positive integers. For $n=1$, $LHS=13\times 6^1+2=80$ $RHS=5K=5\times 16=80$ $\therefore S_n$ is true for $n=1$ Assume S_n is true for $n=k$ $\therefore S_k$: $13\times 6^k+2=5K$ $\therefore$ $S_{k+1}=13\times 6^{k+1}+2$ $=13\times 6^k\times 6+2$ $=(13\times 6^k)6+2$ $=(5K-2)6+2$ (from $13\times 6^k+2=5K$) $=30K-12+2$ $=30K-10$ $=5(6K-2)$ $=5M$ where M is an integer, since K is a positive integer $\therefore S_n$ is true for $n=k+1$ Since S_n ius true for $n=1$, it is true for $n=2$, $\therefore$ it is true for $n=3$, and so on $\therefore S_n$ is true for all positive integers.	1 mark awarded for partial correct solution 2 marks awarded for further partial correct solution 3 marks for complete correct solution
P7	(b) $\dfrac{d(x.e^{\sin x})}{dx}=x.e^{\sin x}.\cos x+e^{\sin x}.1$ $=e^{\sin x}(x\cos x+1)$	1 mark awarded for partial correct solution 2 marks for complete correct solution
H8	(c) $A=\int_0^{\frac{\pi}{4}}\left(\sin 2x-\dfrac{4x}{\pi}\right)dx$ $=\left[\dfrac{-\cos 2x}{2}-\dfrac{4x^2}{2\pi}\right]_0^{\frac{\pi}{4}}$ $=\left[\dfrac{-\cos 2x}{2}-\dfrac{2x^2}{\pi}\right]_0^{\frac{\pi}{4}}$ $=\left(\dfrac{-\cos 2\left(\frac{\pi}{4}\right)}{2}-\dfrac{2\left(\frac{\pi}{4}\right)^2}{\pi}\right)-\left(\dfrac{-1}{2}-0\right)$ $=0-\dfrac{\pi}{8}+\dfrac{1}{2}$ $=\dfrac{1}{2}-\dfrac{\pi}{8}$	1 mark awarded for partial correct solution 2 marks for complete correct solution
H8	(d) $y=\log_e x$, $e^y=x$, $x^2=e^{2y}$ $V=\pi\int_0^3 x^2\,dy$ $=\pi\int_0^3 e^{2y}\,dy$ $=\pi\left[\dfrac{1}{2}e^{2y}\right]_0^3$ $=\pi\left\{\left(\dfrac{1}{2}e^{2(3)}\right)-\dfrac{1}{2}e^{2(0)}\right\}$ $=\dfrac{1}{2}\pi(e^6-1)$	1 mark awarded for partial correct solution 2 marks for complete correct solution
HE6	(e) $u=\sqrt{x}=x^{\frac{1}{2}}$ $\dfrac{du}{dx}=\dfrac{1}{2}x^{\frac{-1}{2}}$ $\dfrac{du}{dx}=\dfrac{1}{2\sqrt{x}}$ $du=\dfrac{1}{2\sqrt{x}}dx$ $2du=\dfrac{1}{\sqrt{x}}dx$ **When** $x=0, u=0$ $x=3, u=\sqrt{3}$ $\therefore \int_0^3\dfrac{dx}{\sqrt{x}(1+x)}=\int_0^{\sqrt{3}}\dfrac{2du}{1+u^2}$ $=2[\tan^{-1}u]_0^{\sqrt{3}}$ $=2(\tan^{-1}\sqrt{3}-\tan^{-1}0)$ $=2\times\dfrac{\pi}{3}$ $=\dfrac{2\pi}{3}$	1 mark awarded for partial correct solution 2 marks awarded for further partial correct solution 3 marks for complete correct solution

Year 12	Mathematics Extension 1	HSC Trial 2009
Question No. 6	Solutions and Marking Guidelines	

Outcomes Addressed in this Question:

HE4 uses the relationship between functions, inverse functions and their derivatives

Outcome	Solutions	Marking Guidelines
HE4 (a)	$\int_0^1 \frac{dx}{\sqrt{2-x^2}} = \sin^{-1}\frac{x}{\sqrt{2}}\Big]_0^1 = \frac{\pi}{4}$	**2 marks** • Correct solution **1 mark** • Substantial progress.
(b)	Domain: $-\frac{1}{3} \le x \le \frac{1}{3}$ Range: $-\pi \le y \le \pi$ 	**3 marks** • Correct solution (All 3 components correct) **2 marks** • Substantial progress towards solution **1 mark** • Partial progress towards solution
(c)	$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \times \frac{-1}{x^2}$ $= \frac{1}{x^2\sqrt{\frac{x^2-1}{x^2}}}$ $= \frac{1}{x\sqrt{x^2-1}}$	**2 marks** • Correct solution **1 mark** • Substantial progress.
(d)	(i) $f'(x) = -1\left(e^{-x}+e^{x}\right) = -1\left(\frac{1}{e^x}+e^x\right)$ As $e^x > 0$ for all values of x, then the expression inside the brackets is always positive. Multiplication by -1 gives a negative derivative for all values of x. This means that $f(x)$ is decreasing for all values of x.	**1 mark** • Correct derivative as well as indicating the negative derivative for all values of x.
	(ii) $f^{-1}(x)$: $x = e^{-y} - e^{y} = \frac{1}{e^y} - e^y$ $\rightarrow e^{2y} + xe^y - 1 = 0$ Using quadratic formula: $e^y = \frac{-x \pm \sqrt{x^2+4}}{2}$ For all values of x, $x < \sqrt{x^2+4}, \therefore -x + \sqrt{x^2+4} > 0$ But $-x - \sqrt{x^2+4} < 0$ for all values of x Therefore only solution is $e^y = \frac{-x+\sqrt{x^2+4}}{2}$ And given solution follows from this assertion.	**3 marks** • Correct solution, including justification of positive and negative values of e^y.(You can't just assume that 1 solution will be negative. It's not always true for a quadratic.) **2 marks** • Substantial progress towards solution **1 mark** • Partial progress towards solution
	(iii) Solution is found by substituting $x = 6$ into the solution given for $f^{1}(x)$ in part (ii). Calculator answer = $-1\cdot 82$	**1 mark** • Correct solution.

Year 12 Mathematics Extension 1 Task 4 2009 Trial HSC

Question No. 7 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5: applies appropriate techniques from the study of geometry and trigonometry to solve problems

H8: uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines

Q7(a)(i) H5

$$\cos 2x = 2\cos^2 x - 1$$
$$\therefore \cos^2 x = \frac{1}{2}(\cos 2x + 1)$$
$$\int_0^{\frac{\pi}{4}} \cos^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{4}}(\cos 2x + 1)\,dx$$
$$= \frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right]_0^{\frac{\pi}{4}}$$
$$= \frac{1}{2}\left\{\frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{4}\right\}$$
$$= \frac{1}{2}\left\{\frac{1}{2} + \frac{\pi}{4}\right\}$$
$$= \frac{\pi + 2}{8}$$

Marking Guidelines:
2 marks: Fully correct solution
1 mark: Uses double angle result for $\cos 2x$ and integrates.

Q7(a)(ii) H8

$$y = \cos x + \sec x$$
$$y^2 = \cos^2 x + \sec^2 x + 2$$
$$V = \pi\int_0^{\frac{\pi}{4}} y^2\,dx$$
$$= \pi\int_0^{\frac{\pi}{4}} \cos^2 x\,dx + \pi\int_0^{\frac{\pi}{4}} (\sec^2 x + 2)\,dx$$
$$= \pi.\frac{\pi + 2}{8} + \pi.\left[\tan x + 2x\right]_0^{\frac{\pi}{4}}$$
$$= \pi\left\{\frac{\pi + 2}{8} + 1 + \frac{\pi}{2}\right\}$$
$$= \pi\left\{\frac{\pi + 2 + 8 + 4\pi}{8}\right\}$$
$$= \frac{5\pi(\pi + 2)}{8}$$

Marking Guidelines:
3 marks
Fully correct solution.
2 marks
Obtains correct simplified expression for volume before substitution of limits
1 mark
Obtains correct simplified expression for volume before integration.

Q7(b)(i) H5

In $\triangle TOM$, $\sin\theta = \frac{OM}{l}$

$OM = l\sin\theta$

Similarly, $MT = l\cos\theta$

Marking Guidelines:
2 marks: Correct results for both OM and MT.
1 mark: Correct result for one of OM or MT

(ii)

In $\triangle PTM$, $\tan\alpha = \frac{TM}{PM}$

$$PM = \frac{l\cos\theta}{\tan\alpha}$$
$$= l\cos\theta\cot\alpha$$

Marking Guidelines:
2 marks: Justification of result.
1 mark: Sufficient progress towards result

(iii)

Now in $\triangle OPM$, $PM^2 = OP^2 + OM^2$

$$l^2\cos\theta^2\cot^2\alpha = d^2 + l^2\sin^2\theta$$
$$l^2\cos\theta^2\cot^2\alpha - l^2\sin^2\theta = d^2$$
$$l^2\left\{\cos^2\theta\cot^2\alpha - \sin^2\theta\right\} = d^2$$
$$l^2 = \frac{d^2}{\left\{\cos^2\theta\cot^2\alpha - \sin^2\theta\right\}}$$

Marking Guidelines:
2 marks: Justification of result.
1 mark: Sufficient progress towards result

(iv)

$$l^2 = \frac{25^2}{\left\{\cos^2 20\cot^2 24 - \sin^2 20\right\}}$$
$$l = 12m$$

Marking Guidelines:
1 mark: Correct solution