2016

Assessment Task 4
Trial HSC Examination

Mathematics Extension 1

Examiners ~ Mr G. Huxley, Mr J. Dillon, Mrs P. Biczo, Mr G. Rawson.

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen.
- Board approved calculators and mathematical templates and instruments may be used.
- This examination booklet consists of 11 pages including a multiple choice answer sheet.
- A Reference Sheet is provided for your use in this examination.

Total marks 70

Section I

Total marks 10

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided on the last page of this question booklet.
- Allow about 15 minutes for this section

Section II

Total marks 60

- Attempt questions 11–14
- Answer each question in the writing booklets provided.
- Start a new booklet for each question with your student number (or name) and question number at the top of the page.
- All necessary working should be shown for every question
- Allow about 1hour and 45 minutes for this section

Name and Student Number : ______________________________

Teacher : ______________________________

Section I

10 marks
Attempt Questions 1 – 10

Use the multiple choice answer sheet for Questions 1 – 10.

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1 What is the acute angle between the lines $y = 2x$ and $x + y - 5 = 0$?
Answer correct to the nearest degree.

(A) 18° (B) 32°
(C) 45° (D) 72°

2 When the polynomial $P(x)$ is divided by $(x + 1)(x - 2)$ its remainder is $18x + 17$.

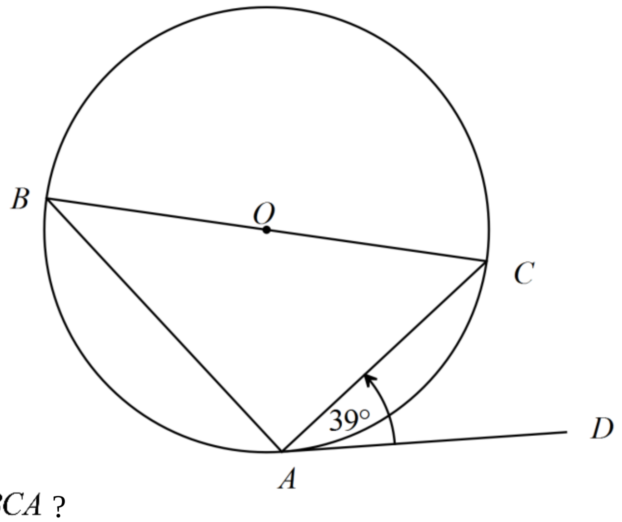
What is the remainder when $P(x)$ is divided by $(x - 2)$?

(A) $18x + 15$ (B) -19
(C) 35 (D) 53

3 In the circle, centre O, BC is a diameter.

AD is a tangent to the circle with A being the point of contact.

$\angle DAC = 39°$.



What is the size of $\angle BCA$?

(A) 39° (B) 51°
(C) 78° (D) 89°

4 What is the middle term in the expansion of $(2x-4)^4$?

(A) 81 (B) $216x^2$

(C) $384x^2$ (D) $-96x^3$

5 At a dinner party, the host, hostess and their six guests sit at a round table.
In how many ways can they be arranged if the host and hostess are separated?

(A) 720 (B) 1440

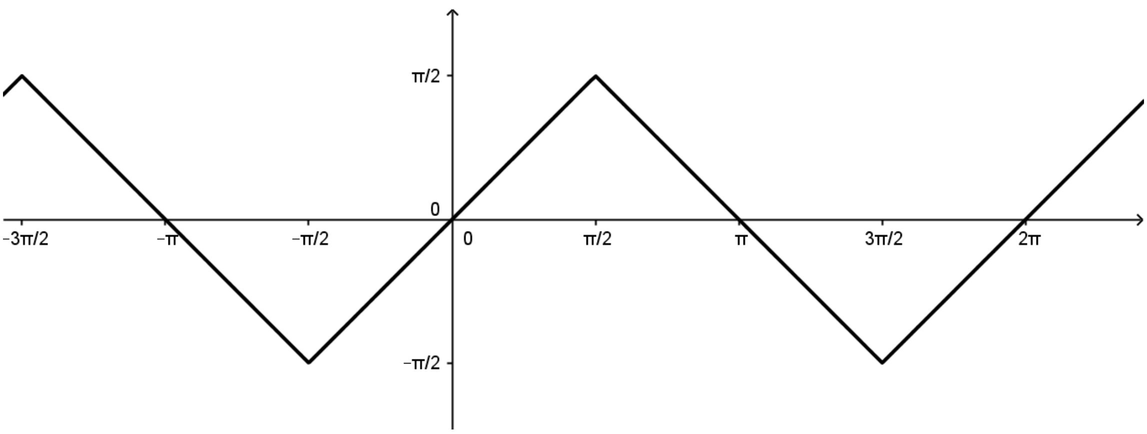
(C) 3600 (D) 5040

6 Which of the following expressions give the solution to the inequality: $\frac{3}{x} < 1$?

(A) $x < 0$ only (B) $x < 0$ or $x > 3$

(C) $0 < x < 3$ (D) $x > 3$ only

7

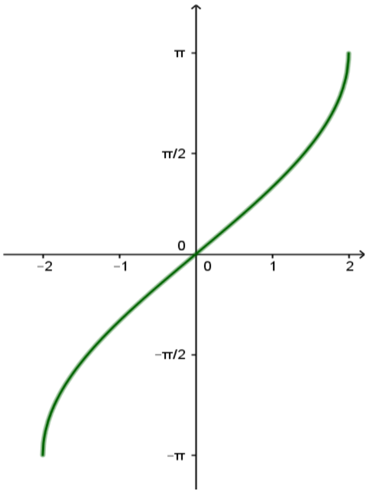


The equation of the graph shown is which of the following?

(A) $y = \sin\left(\sin^{-1} x\right)$ (B) $y = \cos\left(\cos^{-1} x\right)$

(C) $y = \sin^{-1}\left(\sin x\right)$ (D) $y = \cos^{-1}\left(\cos x\right)$

8



The graph shown is in the form $y = a\sin^{-1} bx$. What are the values of a and b?

(A) $a = 2,\ b = 2$

(B) $a = 2,\ b = \frac{1}{2}$

(C) $a = 1,\ b = 2$

(D) $a = 1,\ b = \frac{1}{2}$

9 When using the substitution $x = u - 2$ which of the following will be equivalent to

$\int x(x+2)^8\,dx$?

(A) $\int u^9\,du$

(B) $\int (u-2)^8\,du$

(C) $\int u^9 - 2u^8\,du$

(D) $\int u^8\,du$

10 $\int \frac{dx}{25+16x^2} = ?$

(A) $\frac{1}{20}\tan^{-1}\left(\frac{5x}{4}\right)+c$

(B) $\frac{1}{4}\tan^{-1}\left(\frac{4x}{5}\right)+c$

(C) $\frac{1}{5}\tan^{-1}\left(\frac{5x}{4}\right)+c$

(D) $\frac{1}{20}\tan^{-1}\left(\frac{4x}{5}\right)+c$

End of Section I ~

Section II

60 marks

Attempt Questions 11 - 14

Answer each question in a separate writing booklet.

All necessary working should be shown in every question.

Question11 (15 marks) **Marks**

Start a new answer booklet.

(a) Evaluate: $\int 2(x+1)(x^2+2x-3)^2\,dx$ **1**

(b) (i) Find $\int \frac{x}{\sqrt{x^2+1}}\,dx$ using the substitution: $u=\sqrt{x^2+1}$ **3**

(ii) Hence, or otherwise, determine: $\int_0^3 \frac{x}{\sqrt{x^2+1}}\,dx$ **1**

(c) Evaluate the area between the curve $y=\frac{1}{\sqrt{25-x^2}}$ and the x-axis from $x=-4$ to $x=4$.

Give your answer correct to 3 significant figures. **2**

(d) (i) Use the substitution $u=\sin x$ to evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\,dx$ **2**

(ii) Hence, or otherwise, evaluate: $\int_0^{\pi} \frac{\cos x}{1+\sin^2 x}\,dx$ **1**

(e) Use the substitution $u=e^{4x}+9$ to give the exact value of:

$\int_0^{\ln 2} \frac{3e^{4x}}{\sqrt{e^{4x}+9}}\,dx$ **2**

(f) Find the exact volume of the solid formed if the curve

$y=\cos x+1$ from $x=0$ to $x=\frac{\pi}{2}$ is rotated about the x-axis **3**

Question12 (15 marks) **Marks**

Start a new answer booklet.

(a) In how many ways can a committee of 2 men and 3 women be chosen from a group of 7 men and 9 women? **1**

(b) Find the term independent of x in the expansion of $\left(3x^2+\dfrac{2}{x}\right)^6$ **3**

(c) A sphere is being heated so that its surface area is expanding at a constant rate of $0{\cdot}025\ \text{cm}^2$ per second.
Find the rate of change of the volume of the sphere with respect to time when the radius is 5 cm. **3**

(d) Evaluate $\lim\limits_{x\to 0}\dfrac{1-\cos^2 2x}{x^2}$ **2**

(e) Consider the function $f\left(x\right)=\dfrac{x}{4-x^2}$

(i) Show that $f'\left(x\right)>0$ for all x in the domain of $f\left(x\right)$. **2**

(ii) Sketch the graph of $y=f\left(x\right)$, showing all asymptotes. **2**

(iii) Hence, or otherwise, find the values of k for which the equation $kx=\dfrac{x}{4-x^2}$ has exactly one real solution. **2**

Question 13 (15 marks)

Marks

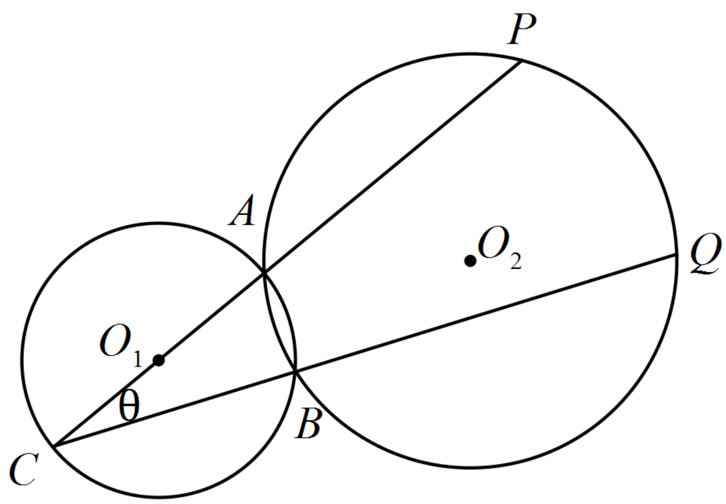
Start a new answer booklet.

(a) Prove by mathematical induction that for all integers $n > 1$,

$$12^n > 7^n + 5^n$$ **3**

(b) What are the roots of the equation $x^3 + 6x^2 - x - 30 = 0$ given one root is the sum of the other two roots? **3**

(c) Two circles with centres O_1 and O_2 intersect at points A and B as shown in the diagram.



AC is the diameter in circle centre O_1 and it intersects the other circle at A and P.

The chord CB produced intersects the second circle again at Q.

Let $\angle ACB = \theta$

Copy or trace the diagram into your writing booklet

(i) Prove that AQ is a diameter of the circle with centre O_2 **2**

(ii) Show that $\angle ABO_1 = (90 - \theta)^0$. **1**

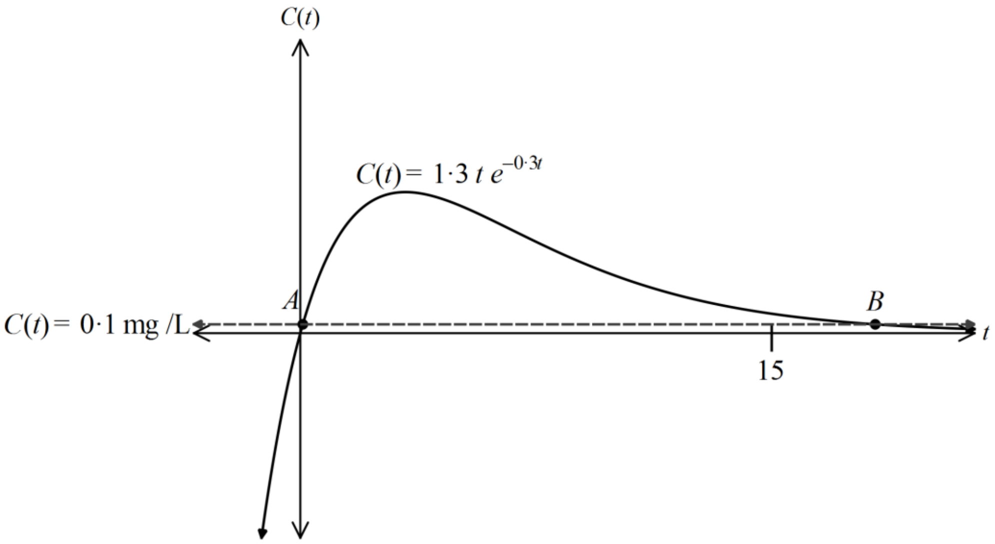
Question 13 continues on the next page.

(d) A patient was administered with a drug.
The concentration of the drug in the patient's blood followed the rule:

$$C(t) = 1.3t\, e^{-0.3t}$$

where time, t, is measured in hours and $C(t)$ is measured in mg/L.

This rule is graphed below.



The doctor left instructions that the patient must not receive another dose of the medicine until the concentration of the drug had dropped to below 0.1 mg/L.

(i) Using $t = 15$ as a first approximation, use one application of Newton's method to find approximately when the concentration of the drug in the blood of the patient reaches 0.1 mg/L. **2**

(ii) Would it be appropriate to use your answer in (i) as the time when the drug would next be administered? Explain your answer. **2**

(e) The roots of the quadratic equation $x^2 + 4x + 2 = 0$ are α and β.

Show that the value of $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3}$ is -5. **2**

Question14 (15 marks) **Marks**

Start a new answer booklet.

(a) The point $P\left(4p,\ 2p^2\right)$ lies on the parabola $x^2=8y$.

The tangent to the parabola at P meets the x axis at T.

The normal at P meets the y axis at N.

(i) Draw a diagram showing this information **1**

(ii) Show that the coordinates of T and N are $\left(2p,0\right)$ and $\left(0,4+2p^2\right)$ respectively. **2**

(iii) The point G divides NT externally in the ratio 2:1.

Show that the coordinates of G are $\left(4p,-4-2p^2\right)$. **2**

(iv) Find the equation of the locus of G **1**

(b) Consider the function $f\left(x\right)=\tan^{-1}\left(x^2\right)$.

(i) Find the domain and range of $y=f\left(x\right)$. **2**

(ii) Find the derivative of $y=f\left(x\right)$ and show that there is a minimum turning point at $x=0$ on the curve. **2**

(iii) Hence, sketch the curve $y=f\left(x\right)$. **1**

(c) (i) By writing the general solution for the equation $\sin x=\cos x$,

show that the first positive value of x that satisfies the equation is $\dfrac{\pi}{4}$. **2**

(ii) Calculate the acute angle between the curves $y=\sin x$ and $y=\cos x$ at this point.

Give your answer to the nearest degree. **2**

End of Section II ~

Multiple Choice:

1.D	2.D	3.B	4.C	5.C
6.B	7.C	8.B	9.C	10.D

Question No. 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE6 determines integrals by reduction to a standard form through a given substitution

Outcome	**Solutions**	**Marking Guidelines**
HE6 (a)	$\frac{\left(x^2+2x-3\right)^3}{3}$ by reverse chain rule.	(a) **1 mark** : correct answer
(b) (i)	Let $u=\sqrt{x^2+1}$ $\frac{du}{dx}=\frac{x}{\sqrt{x^2+1}}$ $\int\frac{x}{\sqrt{x^2+1}}=\int du$ $=u+c$ $=\sqrt{x^2+1}+c$	(b) (i) **3 marks**: correct solution including answer as a function of *x*. **2 marks**: Significant progress towards solution. **1 mark**: Some relevant progress.
(ii)	$\int_0^3\frac{x}{\sqrt{x^2+1}}\,dx=\left.\sqrt{x^2+1}\right]_0^3$ $=\sqrt{10}-1$	(ii) **1 mark:** correct answer.
(c)	$A=\int_{-4}^{4}\frac{1}{\sqrt{25-x^2}}\,dx=2\int_0^4\frac{1}{\sqrt{25-x^2}}\,dx$ $=2\left[\sin^{-1}\frac{x}{5}\right]_0^4$ $=2\left(\sin^{-1}\frac{4}{5}-\sin^{-1}0\right)$ $=1\cdot 85\,\text{units}^2$	(c) **2 marks:** Correct solution. **1 mark:** Considerable progress.
(d)(i)	$\int_0^{\frac{\pi}{2}}\frac{\cos x}{1+\sin^2 x}\,dx=\int_0^1\frac{du}{1+u^2}$ $=\left[\tan^{-1}u\right]_0^1$ $=\tan^{-1}1-\tan^{-1}0$ $=\frac{\pi}{4}$	(d)(i) **2 marks:** Correct solution. **1 mark:** Considerable progress.

(ii)

HE6

$$\int_0^{\pi}\frac{\cos x}{1+\sin^2 x}\,dx=\int_0^{\frac{\pi}{2}}\frac{\cos x}{1+\sin^2 x}\,dx+\int_{\frac{\pi}{2}}^{\pi}\frac{\cos x}{1+\sin^2 x}\,dx$$

$$=\left(\tan^{-1}1-\tan^{-1}0\right)+\left(\tan^{-1}0-\tan^{-1}1\right)$$

$$=\frac{\pi}{4}-\frac{\pi}{4}=0$$

Alternately, we can see that the curve will rotate below the x-axis because $\cos\left(\pi-x\right)=-\cos(x)$

(d) (ii) 1 mark: Correct solution, using an appropriate method. (Note: It is not valid to perform the integral with boundaries 0 and 0)

(e)

$$\int_0^{\ln 2}\frac{3e^{4x}}{\sqrt{e^{4x}+9}}\,dx=\frac{3}{4}\int_0^{\ln 2}\frac{4e^{4x}}{\sqrt{e^{4x}+9}}\,dx$$

$$=\frac{3}{4}\int_{10}^{25}\frac{du}{\sqrt{u}}$$

$$=\frac{3}{4}\Big[2\sqrt{u}\Big]_{10}^{25}$$

$$=\frac{3}{2}\left(5-\sqrt{10}\right)$$

(e) 2 marks: Correct solution.
1 mark: Considerable progress.

(f)

$$V=\pi\int_0^{\frac{\pi}{2}}\left(\cos x+1\right)^2dx$$

$$=\pi\int_0^{\frac{\pi}{2}}\frac{1}{2}+\frac{1}{2}\cos 2x+2\cos x+1\,dx$$

$$=\pi\left[\frac{3x}{2}+\frac{1}{4}\sin 2x+2\sin x\right]_0^{\frac{\pi}{2}}$$

$$=\pi\left(\frac{3\pi}{4}+0+2\right)-\left(0+0+0\right)$$

$$=\pi\left(\frac{3\pi}{4}+2\right)$$

(f) 3 marks: Correct solution, involving changing the integral, evaluating the primitive and evaluating.
2 marks: Considerable progress.
1 mark: Some progress.

Year 12	Mathematics Extension 1 2016	TRIAL EXAM
Question No. 12	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

HE3 - Uses a variety of strategies to investigate mathematical models and applications.

Part / Outcome	Solutions	Marking Guidelines
(a)	$^7C_2 \times {}^9C_3 = 1764$	**1 mark** – Correct solution
(b)	$\left(3x^2 + \frac{2}{x}\right)^6$ has general term $= \binom{6}{r}\left(3x^2\right)^{6-r}\left(\frac{2}{x}\right)^r$ $= \binom{6}{r} 3^{6-r}.x^{12-2r}.2^r.x^{-r}$ $= \binom{6}{r} 3^{6-r}.2^r.x^{12-3r}$ term independent of x has $12 - 3r = 0$ $3r = 12$ $r = 4$ term is: $\binom{6}{r} 3^{6-r}.2^r.x^{12-3r}$ $= \binom{6}{4} 3^{6-4}.2^4.x^0$ $= \binom{6}{4}.3^2.2^4 = 15.9.16 = 2160$	**3 marks** – Correct solution **2 marks** – Substantially correct **1 mark** – Partial progress towards correct solution
(c)	$\frac{dS}{dt} = 0\cdot 025;$ when $r = 5, \frac{dV}{dt} = ?$ $\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} \quad \Rightarrow \quad$ need $\frac{dV}{dS}$ $S = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3 \qquad \Rightarrow \qquad \frac{dV}{dS} = \frac{dV}{dr}.\frac{dr}{dS}$ $\frac{dS}{dr} = 8\pi r \qquad \frac{dV}{dr} = 4\pi r^2 \qquad\qquad = 4\pi r^2.\frac{1}{8\pi r}$ $= \frac{r}{2}$ now, $\frac{dV}{dt} = \frac{dS}{dt}.\frac{dV}{dS}$ $= 0\cdot 025 \times \frac{r}{2}$ $= 0\cdot 025 \times \frac{5}{2}$ $= 0\cdot 0625 \text{ cm}^3/\text{s}$	**3 marks** – Correct solution **2 marks** – Substantially correct **1 mark** – Partial progress towards correct solution

Question 12 continued…

(d)

$$\begin{aligned}\lim_{x\to0}\frac{1-\cos^2 2x}{x^2}&=\lim_{x\to0}\frac{\sin^2 2x}{x^2}\\&=4\lim_{x\to0}\left(\frac{\sin 2x}{2x}\right)^2\\&=4\end{aligned}$$

2 marks – Correct solution

1 mark – Substantially correct

(e)

(i) $f\left(x\right)=\dfrac{x}{4-x^2}$

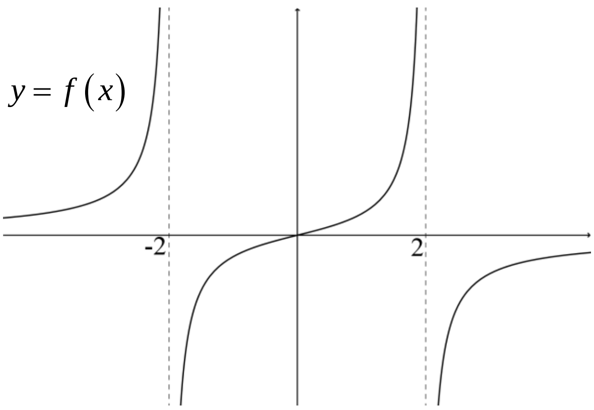
$$\begin{aligned}f'\left(x\right)&=\frac{1.\left(4-x^2\right)-\left(-2x\right).x}{\left(4-x^2\right)^2}\\&=\frac{4+x^2}{\left(4-x^2\right)^2}>0\end{aligned}$$

since $4+x^2>0$, and $\left(4-x^2\right)^2>0 \quad \left(x\neq0\right)$

2 marks – Correct solution

1 mark – Substantially correct

(ii)



2 marks – Correct solution

1 mark – Substantially correct

(iii) $0\leq k\leq f'\left(0\right)$

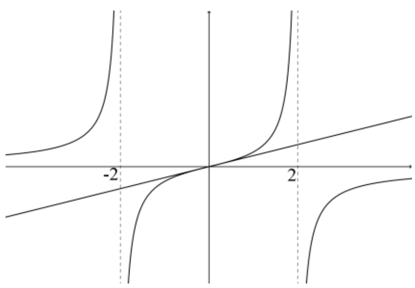
$$0\leq k\leq\frac{4+0^2}{\left(4-0^2\right)^2}$$

$$0\leq k\leq\frac{1}{4}$$

(For $kx=\dfrac{x}{4-x^2}$ to have exactly one real solution, the graphs of $y=kx$ and $y=\dfrac{x}{4-x^2}$ intersect once. So $y=kx$ has a gradient between (and including) 0 and tangent of $f(x)$ at $x=0$.)

2 marks – Correct solution

1 mark – Substantially correct



Also note, that trying to solve this cubic equation, dividing through by x, and then using the discriminant on the remaining quadratic has changed the question into an easier (and incorrect) question. Many tried this. All cubics have at least on solution, and therefore the remaining quadratic factor should have *no* solutions. Using this understanding was better, but still missed two possibilities…

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

HE2 uses inductive reasoning in the construction of proofs

HE3 uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay

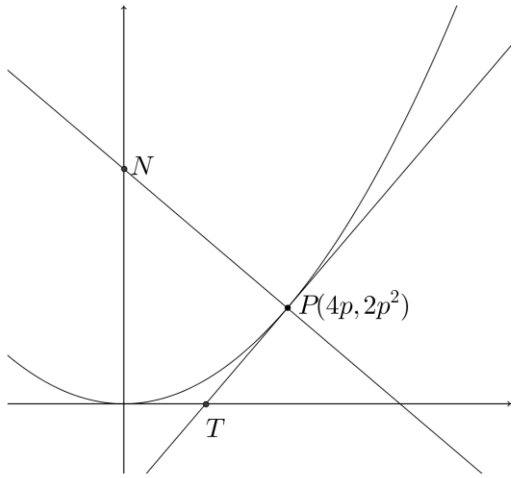
Outcome	Solutions	Marking Guidelines
(a) HE2	Show true for $n=2$, $LHS = 12^2 = 144$ $RHS = 7^2 + 5^2 = 74$ $\therefore LHS > RHS$ $\therefore$ True for $n=2$ Assume true for $n=k$, i.e. $12^k > 7^k + 5^k$ Prove true for $n=k+1$, $12^{k+1} = 12.12^k$ $> 12\left(7^k + 5^k\right)$ $= 12.7^k + 12.5^k$ $= 7^{k+1} + 5^{k+1} + 5.7^k + 7.5^k$ $> 7^{k+1} + 5^{k+1}$ $\therefore$ True for $n=k+1$ $\therefore$ True for all $n \geq 2$, by Mathematical Induction	**Award 3** for correct solution **Award 2** for proving the result true for $n=2$ and attempting to use the result of $n=k$ to prove the result for $n=k+1$. **Award 1** for proving the result true for $n=2$.
(b) PE3	Let the roots be α, β and γ. With $\alpha = \beta + \gamma$ $\alpha + \beta + \gamma = 2\alpha = -\frac{6}{1} = -6$ $\therefore \alpha = -3$ ..... (1) $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{-1}{1} = -1$ $\therefore \alpha(\beta+\gamma) + \beta\gamma = -1$ $\therefore -3(\beta+\gamma) + \beta\gamma = -1$ .....(2) $\alpha\beta\gamma = -\frac{-30}{1} = 30$ $\therefore -3\beta\gamma = 30$ $\therefore \beta\gamma = -10$ .....(3) $(3) \to (2)$ $-3(\beta+\gamma) + -10 = -1$ $\therefore \beta + \gamma = -3$ .....(4) $(4) \to \gamma = -3 - \beta$ .....(5) $(5) \to (3)$ $\beta(-3-\beta) = -10$ $\therefore \beta^2 + 3\beta - 10 = 0$ $\therefore (\beta+5)(\beta-2) = 0$ $\therefore \beta = -5$ or 2 $\therefore \gamma = 2$ or -5 $\therefore$ Roots are $-3, 2$ and -5.	**Award 3 marks** for correct roots **Award 2 marks** for substantial progress towards determining the roots **Award 1 mark** for limited progress towards determining the roots

(c) (i) PE3	$\angle ABC = 90^0$ $\begin{pmatrix}\text{The angle at the circumference} \\ \text{in a semi-circle is } 90^o\end{pmatrix}$ $\angle ABC = \angle APQ$ $\begin{pmatrix}\text{The exterior angle of a cyclic quadrilateral} \\ \text{equals the opposite (or remote) interior angle}\end{pmatrix}$ $= 90^0$ $\therefore AQ$ is a diameter $\begin{pmatrix}\text{A right angle at the circumference subtends} \\ \text{a diameter}\end{pmatrix}$	**Award 2 marks** for correct solution **Award 1 mark** for substantial progress towards solution
(ii) PE3	$OC = OB$ (radii) $\therefore \angle O_1CB = \angle O_1BC = \theta$ $\begin{pmatrix}\text{equal angles are opposite} \\ \text{equal sides in } \Delta O_1CB\end{pmatrix}$ $\therefore \theta + \angle ABO_1 = 90^0$ $\begin{pmatrix}\text{angle sum of right angle} \\ \angle\text{ABC (from (i))}\end{pmatrix}$ $\therefore \angle ABO_1 = (90-\theta)^0$	**Award 1 mark** for correct solution
(d) (i) HE3	$C(t) = 1.3te^{-0.3t}$ We want $0.1 = 1.3te^{-0.3t}$ Let $f(t) = 1.3te^{-0.3t} - 0.1$ $\therefore f'(t) = 1.3e^{-0.3t} - 0.39te^{-0.3t}$ $t_2 = 15 - \dfrac{f(15)}{f'(15)} = 15 - \dfrac{0.1166254325}{-0.05054593425}$ $= 17.3073158$ hours ≈ 17 hours 18 minutes	**Award 2 marks** for correct solution **Award 1 mark** for substantial progress towards solution
(ii) HE3	After 17.3073158 hours the concentration is above 0.1 mg/L $(C(17.3073158) \approx 0.123)$. $\therefore$ It would be necessary to use more applications to find a more appropriate time.	**Award 2 marks** for correct solution with correct reasoning supplied **Award 1 mark** for substantial progress towards solution
(e) PE3	$\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} = \dfrac{\alpha^3 + \beta^3}{(\alpha\beta)^3}$ But $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= (\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta)$ $= (-4)((-4)^2 - 3.2)$ $= -40$ $\therefore \dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} = \dfrac{\alpha^3+\beta^3}{(\alpha\beta)^3} = \dfrac{-40}{(2)^3} = -5$ (as required)	**Award 2 marks** for correct solution **Award 1 mark** for substantial progress towards solution

Year 12 Trial Higher School Certificate Extension 1 Mathematics Examination 2016
Question No. 14 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 solves problems involving polynomials and parametric representations
PE5 determines derivatives which require the application of more than one rule of differentiation
HE4 Uses the relationship between functions, inverse functions and their derivatives.
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form.

Outcome	Solutions	Marking Guidelines
PE3	(a)(i) 	1 mark : correct diagram
PE3	(ii) Tangent to $x^2 = 8y$ has gradient p. $\therefore$ equation is $y - 2p^2 = p(x - 4p)$. Cuts x axis when $y = 0$. $\therefore -2p^2 = px - 4p^2,\ px = 2p^2$ $x = 2p$. Normal has gradient $\dfrac{-1}{p}$. $\therefore$ equation is $y - 2p^2 = \dfrac{-1}{p}(x - 4p)$. Cuts y axis when $x = 0$. $\therefore y - 2p^2 = 4$, $\therefore y = 4 + 2p^2$. $\therefore T$ and N are the points $(2p, 0)$ and $(0, 4+2p^2)$ respectively.	2 marks : correct solution for T and N. 1 mark : significant progress towards correct solution
PE3	(iii) If G divides NT externally, ratio is $2:-1$. Dividing $N(0, 4+2p^2)$, $T(2p, 0)$, where G is the point $\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$, G is $\left(\dfrac{2\times 2p - 1\times 0}{2-1}, \dfrac{2\times 0 - 1\times(4+2p^2)}{2-1}\right)$, $= (4p, -4-2p^2)$.	2 marks : correct solution 1 mark : significant progress towards correct solution

<table>
<tr>
<td>PE3</td>
<td>

(iv) At G, $\begin{cases} x = 4p & [1] \\ y = -4 - 2p^2 & [2] \end{cases}$

From $[1]$, $p = \frac{x}{4}$.

Substituting in $[2]$, $y = -4 - 2\left(\frac{x}{4}\right)^2$,

$y = -\frac{x^2}{8} - 4$ or $x^2 = -8(y+4)$.

</td>
<td>1 mark : correct solution</td>
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<tr>
<td>H6, PE5</td>
<td>

(b) For $f(x) = \tan^{-1}(x^2)$,

as $\tan^{-1} x$ can be found for all values of x,
Domain of $f(x) = \tan^{-1}(x^2)$ is all real x.

For $\tan^{-1} x$, range is $\frac{-\pi}{2} < y < \frac{\pi}{2}$. But for $y = \tan^{-1}(x^2)$, since the values for which tan inverse will be calculated give positive or zero only, it is only these corresponding y values that will be in the range.

Range : $0 \le y < \frac{\pi}{2}$.

</td>
<td>2 marks : both domain and range correct
1 mark : one of above correct.</td>
</tr>
<tr>
<td></td>
<td>

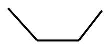
(ii) $f(x) = \tan^{-1}(x^2)$

$$f'(x) = \frac{\frac{d(x^2)}{dx}}{1 + (x^2)^2}$$

$$= \frac{2x}{1 + x^4}$$

Testing $x = 0$, for $f'(x) = \frac{2x}{1+x^4}$

<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td></tr>
<tr><td>$f'(x)$</td><td>-1</td><td>0</td><td>1</td></tr>
</table>

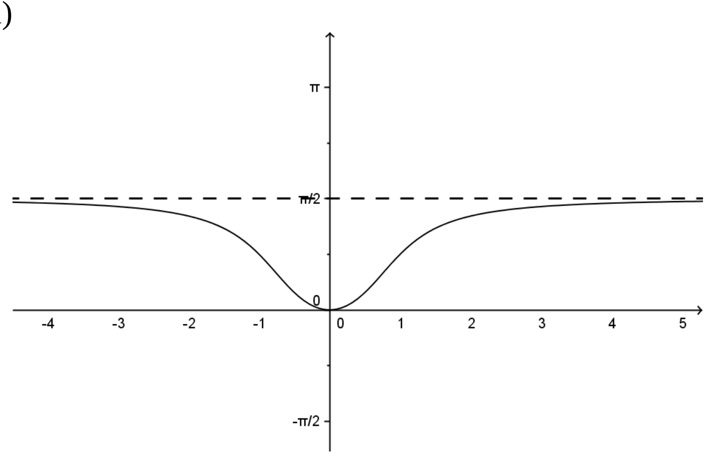


$\therefore$ minimum turning point at $x = 0$.

</td>
<td>2 marks : correct solution
1 marks : substantial progress towards correct solution</td>
</tr>
<tr>
<td>PE5, HE7

HE7</td>
<td>

(iii)



</td>
<td>1 mark : correct graph</td>
</tr>
</table>

(c) (i) Given $\sin x = \cos x$,

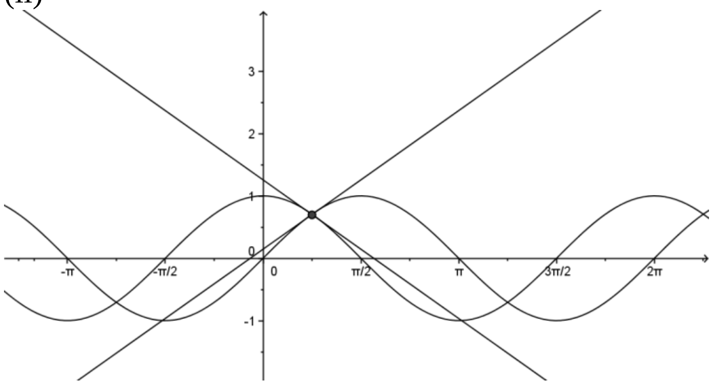
$$\frac{\sin x}{\cos x} = 1 \text{ and } \tan x = 1.$$

$$x = n\pi + \frac{\pi}{4} \text{ for } n\text{, an integer.}$$

When $n = 0$, $x = \frac{\pi}{4}$.

2 marks : correct solution
1 mark : substantial progress towards correct solution

(ii)



For the acute angle between the curves need the gradient of the tangents at the point of intersection.

For $y = \sin x$, $\frac{dy}{dx} = \cos x$.

At $x = \frac{\pi}{4}$, $\frac{dy}{dx} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

For $y = \cos x$, $\frac{dy}{dx} = -\sin x$.

At $x = \frac{\pi}{4}$, $\frac{dy}{dx} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$.

Using $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$,

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)}{1 + \frac{1}{\sqrt{2}} \times \left(-\frac{1}{\sqrt{2}}\right)}\right|$$

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)}{1 + \frac{1}{\sqrt{2}} \times \left(-\frac{1}{\sqrt{2}}\right)}\right|$$

$$\tan\theta = 2\sqrt{2}$$

$\theta = 71°$ (to the nearest degree)

2 marks : correct solution
1 mark : significant progress towards correct solution