STUDENT NAME: ____________________________

TEACHER: ___________________________________

HURLSTONE AGRICULTURAL

HIGH SCHOOL

2020 **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics - Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 – 14)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

1. There are eight questions in a multiple-choice test. Each question has four possible answers, only one of which is correct.

What is the probability of answering exactly five questions correctly by chance alone, correct to 3 significant figures?

A. 0.0000153

B. 0.000412

C. 0.000977

D. 0.0231

2. What is the vector projection of $\underset{\sim}{p} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ onto $\underset{\sim}{q} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$?

A. $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

B. $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

C. $\begin{pmatrix} -2\sqrt{5} \\ 4\sqrt{5} \end{pmatrix}$

D. $\begin{pmatrix} 2\sqrt{5} \\ -4\sqrt{5} \end{pmatrix}$

3. What is the value of $\sin 2x$ given that $\sin x = \frac{\sqrt{3}}{2}$ and x is obtuse?

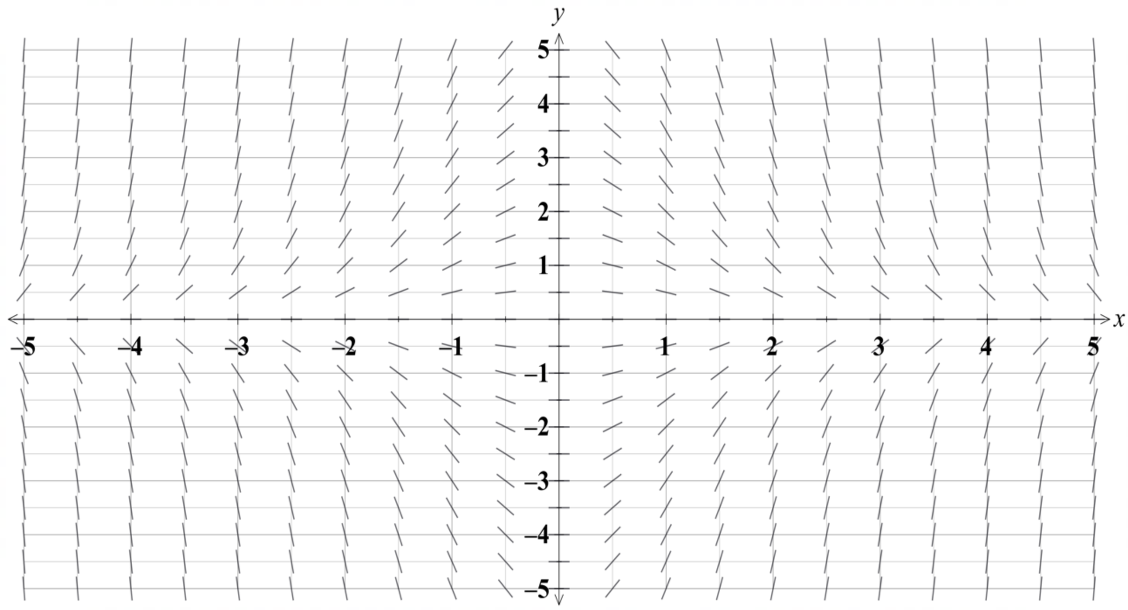
A. $-\frac{\sqrt{3}}{4}$

B. $-\frac{\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{4}$

D. $\frac{\sqrt{3}}{2}$

4. Which of the following differential equations could be represented by the slope field diagram below?



A. $y' = -xy$

B. $y' = xy$

C. $y' = -x^2y$

D. $y' = x^2y$

5. What is the derivative of $\tan^{-1}(2x-1)$?

A. $\dfrac{1}{4x^2-4x+2}$

B. $\dfrac{2x-1}{2x^2-2x+1}$

C. $\dfrac{2}{2x^2-2x+1}$

D. $\dfrac{1}{2x^2-2x+1}$

6. The polynomial $P(x)=x^3-12x^2+21x+k=0$ has a double root.
What are the possible values of k?

A. $k=-1$ or $k=-7$

B. $k=1$ or $k=7$

C. $k=10$ or $k=-98$

D. $k=-10$ or $k=98$

7. A curve C has parametric equations $x=\cos^2 t$ and $y=4\sin^2 t$ for $t \in R$.
What is the Cartesian equation of C?

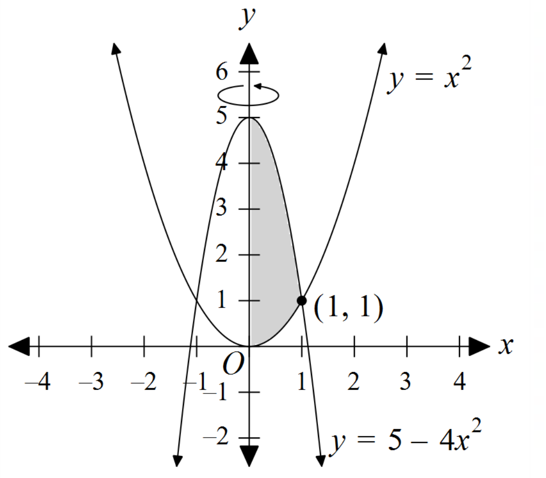
A. $y=1-x$ for $0 \le x \le 1$

B. $y=4-4x$ for $x \in R$

C. $y=4-4x$ for $0 \le x \le 1$

D. $y=1-x$ for $x \in R$

8. A solid of revolution is to be formed by rotating the shaded area shown between the graphs of $y = x^2$ and $y = 5 - 4x^2$ about the y-axis.



Which integral would give the volume of the solid of revolution?

A. $V = \pi \int_0^1 \left[\left(5 - 4x^2\right)^2 - x^4 \right] dx$

B. $V = \pi \int_0^5 \left(\frac{5}{4} - \frac{5y}{4} \right) dy$

C. $V = \pi \left(\int_0^1 x^4 \, dx + \int_1^5 \left(5 - 4x^2\right)^2 dx \right)$

D. $V = \pi \left(\int_0^1 y \, dy + \frac{1}{4} \int_1^5 (5 - y) \, dy \right)$

9. A body of still water has suffered an oil spill and a circular oil slick is floating on the surface of the water.

The area of the oil slick is increasing by 0.1 m^2/minute

At what rate is the radius increasing when the area is 0.3 m^2?

A. 0.0098 m/min

B. 0.03 m/min

C. 0.0515 m/min

D. 0.0531 m/min

10. A four-digit security code is to be created for a building alarm, using any selection of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The code must be entered in the correct order to disarm the alarm when entering the building.

Digits ***may*** be repeated.

It has been decided that the code will contain exactly two different digits, for example 4224 or 7177.

If an intruder, who knew about this restriction, tried to guess the alarm code, what is the probability they would get it correct?

A. $\frac{1}{10000}$

B. $\frac{1}{5040}$

C. $\frac{1}{2100}$

D. $\frac{1}{630}$

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) *Use a separate writing booklet.* **Marks**

(a) Solve $\dfrac{x+2}{x-5} \geq 0$. **2**

(b) Find the polynomial $Q(x)$ that satisfies $x^3 - x^2 - 3x - 5 = (x-3)Q(x) + 4$. **2**

(c) Use the substitution $u = 1 + 2\tan x$ to evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{(1+2\tan x)^2 \cos^2 x}\,dx$. **2**

(d) A heated metal ball is dropped into a liquid.

As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by:

$T = 400e^{-0.05t} + 25, \quad t \geq 0$

(i) Find the temperature of the ball as it enters the liquid. **1**

(ii) Find the value of t if $T = 300$. Answer correct to 3 significant figures. **1**

(iii) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures. **2**

(iv) Using the equation for temperature T in terms of t given above, explain why the temperature of the ball can never fall to 20°C. **1**

Question 11 continues on page 8…

(e) The continuous random variable, X, has the following probability density function:

$$f(x) = \begin{cases} ax^2(4-x) & 1 \le x \le 4 \\ 0 & x < 1 \text{ or } x > 4 \end{cases}$$

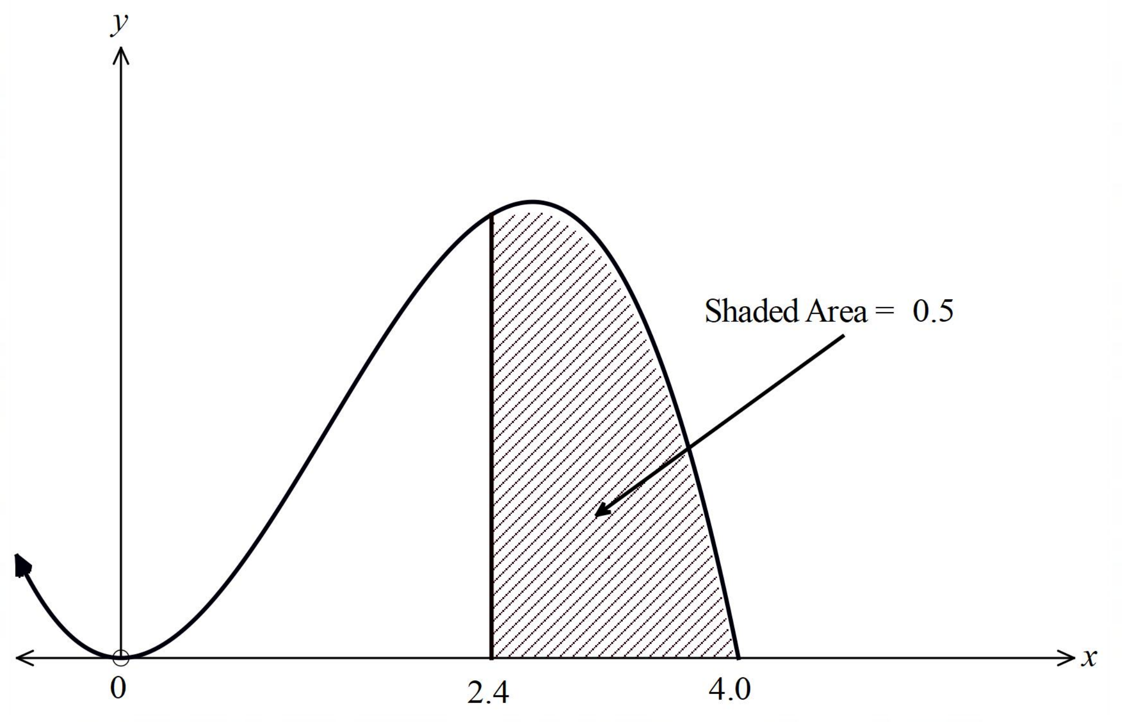
(i) Find the value of a. **2**

(ii) Write an expression that could be used to correctly calculate $P(3 < x < 4)$. **1**

(Do not evaluate your expression)

(iii) The graph of the probability density function for $f(x)$ is shown below. **1**

The line $x = 2.4$ creates an area of 0.5 square units to the right of the line and under the curve, as shown.



Explain what measure the value of $x = 2.4$ represents in relation to $f(x)$.

End of Question 11

Question 12 (15 marks) *Use a separate writing booklet.* **Marks**

(a) Consider vectors $\underset{\sim}{\mathbf{a}} = 4\underset{\sim}{\mathbf{i}} - 5\underset{\sim}{\mathbf{j}}$ and $\underset{\sim}{\mathbf{b}} = -2\underset{\sim}{\mathbf{i}} + 4\underset{\sim}{\mathbf{j}}$.

(i) Find the magnitude and direction (to the nearest minute) of $\underset{\sim}{\mathbf{a}} + \underset{\sim}{\mathbf{b}}$. **2**

(ii) Find the resultant vector of $2\underset{\sim}{\mathbf{b}} - \underset{\sim}{\mathbf{a}}$. **2**

(iii) Calculate the dot product $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}}$ **1**

(iv) Find the angle (to the nearest minute) between the vectors $\underset{\sim}{\mathbf{a}}$ and $\underset{\sim}{\mathbf{b}}$. **2**

(b) A state-wide housing study found that 36% of adults in NSW have a mortgage.

(i) A random sample of 25 adults in NSW is to be taken to determine the proportion of those who have a mortgage. **2**

Show that the mean and standard deviation for the distribution of sample proportions of such random samples are 0.36 and 0.096 respectively.

(ii) Part of a table of $P(Z \le z)$ values, where Z is a standard normal variable, is shown. **2**

z	$+0.00$	$+0.01$	$+0.02$	$+0.03$	$+0.04$	$+0.05$	$+0.06$	$+0.07$	$+0.08$	$+0.09$
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

Of a random sample of 25 adults in NSW, use the table to estimate the probability that at most three will have a mortgage.

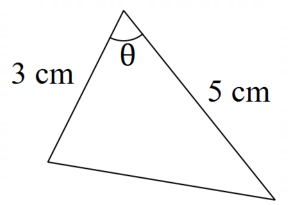
Give your answer correct to four decimal places.

(iii) If a random sample of 25 adults in NSW is taken, find the probability that the sample proportion is equal to the population proportion. **2**
Give your answer correct to four decimal places.

Question 12 continues on page 10...

Marks

(c) A triangle has sides of length 3 cm and 5 cm. The included angle, θ, is decreasing at a rate of 0.05 radians per second. **2**



Find the rate at which the area of the triangle is decreasing when θ is $\frac{\pi}{4}$

End of Question 12

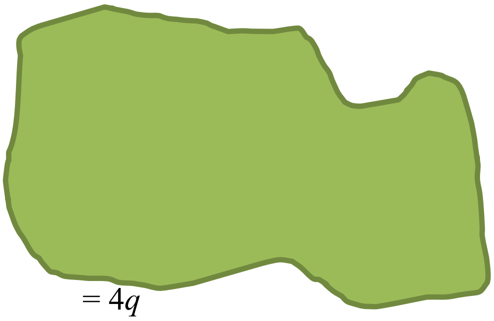
Question 13 (15 marks) *Use a separate writing booklet.* **Marks**

(a) Melinda aims to prove that $7^n - 3^n$ is divisible by 4 for all positive integral values of *n*.
Part of her proof is shown below.

Assume true for $n = k$ i.e $7^k - 3^k = 4p$, where p is a positive integer.

Show that $7^{k+1} - 3^{k+1} = 4q$, where q is a positive integer.

LHS = $7^{k+1} - 3^{k+1}$



$= 4q$

$\therefore$ if true for $n = k$ also true for $n = k + 1$
therefore by induction it is also true for all positive integral values of *n*.

(i) Melinda spilled some ink on her work. **2**
Write what you think Melinda might have written in those unreadable lines.

(ii) Apart from the obscured lines, another step has been left out of the proof. **1**

Write the missing step.

(b) (i) Show that $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$. **1**

(ii) Hence, if α, β and γ are the angles of ΔABC and $\sin\gamma = 2\sin\alpha\cos\beta$, **2**
show that ΔABC has two equal angles.

(c) (i) Use the substitution $t = \tan\frac{x}{2}$ to show that $\operatorname{cosec} x + \cot x = \cot\frac{x}{2}$. **2**

(ii) Hence evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\operatorname{cosec} x + \cot x)\,dx$ **3**

Question 13 continues on page 12…

Question 13 continued…

Marks

(d) The probability that the 7:30 a.m. train arrives on time is 0.85. **2**

Find an expression for the probability that the train is on time at least five days during one week

(e) Show that if 100 unique numbers are in a set, then at least 17 of them have the same remainder when divided by 6. **2**

End of Question 13

Question 14 (15 marks) *Use a separate writing booklet.*

Marks

(a) A tank contains 2,500 litres of water and 25 kg of dissolved salt. Fresh water enters the tank at a rate of 20 litres per minute. The solution is thoroughly mixed at all times and is drained from the tank at a rate of 15 litres per minute.

(i) Using y for the amount of salt in the tank in kilograms (as a function of time), and t for time in minutes, show that the concentration of salt in the tank (kg/L) at time t can be given by **2**

$$C = \frac{y}{2500 + 5t}$$

(ii) Explain why the rate of change of salt in the tank can by given by $y' = -15C$ **1**

(iii) Find y, the amount of salt in the tank as a function of t. **4**

(iv) If the tank has a capacity of 5,000 litres, how much salt is in the tank when it overflows? **2**

(b) For the differential equation $y' = \dfrac{6}{5x^2 + 4x - 1}$:

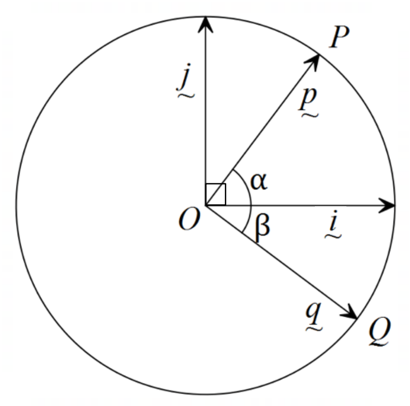
(i) Show that $y' = \dfrac{5}{5x-1} - \dfrac{1}{x+1}$. **1**

(ii) Find the solution to the differential equation; given that when $x = \dfrac{1}{2}$, $y = 3$. **2**

Question 14 continues on page 14…

Marks

(c) Use the unit circle and the vectors in the diagram to derive the expansion of $\cos(\alpha + \beta)$. **3**



End of Paper

HAHS Maths Extension 1 Trial Examination
Marking Guidelines

Outcomes Assessed in this paper:

ME11-1: uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

ME11-2: manipulates algebraic expressions and graphical functions to solve problems

ME11-4: applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change

ME11-5: uses concepts of permutations and combinations to solve problems involving counting or ordering

ME12-1: applies techniques involving proof or calculus to model and solve problems

ME12-2: applies concepts and techniques involving vectors and projectiles to solve problems

ME12-3: applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

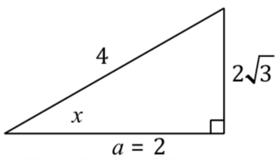
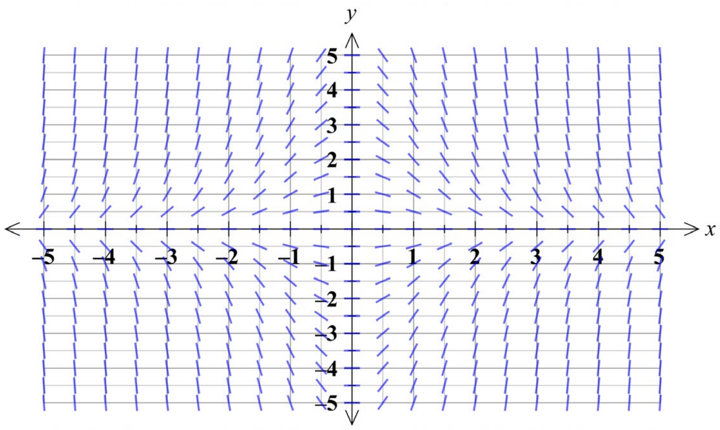
ME12-4: uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution

ME12-5: applies appropriate statistical processes to present, analyse and interpret data

***ME12-7**: evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms*

Section I

No	Working	Answer
1. ME 12-5	$P(exactly\ 5 \text{ correct}) = {}^8\mathbf{C}_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^3$ $= \frac{189}{8192}$ $= 0.02307128906$ ≈ 0.0231	**D**
2. ME 12-2	$\text{Proj}_{\underset{\sim}{q}}(\underset{\sim}{p}) = \frac{\underset{\sim}{p}.\underset{\sim}{q}}{\lvert\underset{\sim}{q}\rvert^2} \cdot \underset{\sim}{q}$ $\underset{\sim}{p} = \begin{pmatrix}4\\-3\end{pmatrix}$ onto $\underset{\sim}{q} = \begin{pmatrix}-1\\2\end{pmatrix}$. $\underset{\sim}{p}.\underset{\sim}{q} = 4\times -1 + \ -3\times 2 = -10$ $\lvert\underset{\sim}{q}\rvert^2 = (-1)^2 + 2^2 = 5$ $\text{Proj}_{\underset{\sim}{q}}(\underset{\sim}{p}) = \frac{-10}{5} \cdot \begin{pmatrix}-1\\2\end{pmatrix}$ $= -2\begin{pmatrix}-1\\2\end{pmatrix}$ $= \begin{pmatrix}2\\-4\end{pmatrix}$	**B**

No	Working	Answer
3. ME 12-3	$4^2 = \left(2\sqrt{3}\right)^2 + a^2$ $a^2 = 16 - 12$ $a = 2$ $\sin 2x = 2\sin x\cos x$ $= 2 \times \frac{2\sqrt{3}}{4} \times -\frac{2}{4} = -\frac{\sqrt{3}}{2}$  	**B**
4. ME 12-4	 The slope in the 1st and 3rd quadrants is always negative. The slope in the 2nd and 4th quadrants is always positive. This rules out options B, C and D.	**A**
5. ME 12-1	$\frac{d}{dx}(\tan^{-1}f(x)) = \frac{f'(x)}{1+(f(x))^2}$ $f(x) = 2x - 1$ and $f'(x) = 2$ $\frac{d}{dx}(\tan^{-1}(2x-1)) = \frac{2}{1+(2x-1)^2}$ $= \frac{2}{4x^2-4x+2}$ $= \frac{1}{2x^2-2x+1}$	**D**
6. ME 11-1	For $P(x)$ to have a double root, its derivative must have a root in common with $P(x)$. $P'(x) = 3x^2 - 24x + 21 = 0$ $3(x^2 - 8x + 7) = 0$ $3(x-7)(x-1) = 0$ So the root is either $x = 7$ or $x = 1$. $P(7) = 7^3 - 12(7)^2 + 21(7) + k = 0$ so $k = 98$ $P(1) = 1^3 - 12(1)^2 + 21(1) + k = 0$ so $k = -10$	**D**

No	Working	Answer
7. ME 11-2	The parametric equations are: $x = \cos^2 t$ (1) $y = 4\sin^2 t$ (2) $\frac{(2)}{4}$ gives $\frac{y}{4} = \sin^2 t.$ (3) (1) + (3) and using $\cos^2 t + \sin^2 t = 1$ gives $x + \frac{y}{4} = 1 \Rightarrow 4x + y = 4.$ $0 \le \cos^2 t \le 1$ and so $0 \le x \le 1.$ Therefore, $y = 4 - 4x$ for $0 \le x \le 1.$	**C**
8. ME 12-4	If we are to rotate about the y-axis, we must integrate with respect to y. Thus, we must solve the functions for x. $y = x^2 \rightarrow x = \sqrt{y}$ $y = 5 - 4x^2 \rightarrow x = \frac{\sqrt{5-y}}{2}$ We would then integrate with respect to y, using two integrals to break the calculation up at the intersection where $y = 1$. $V = \int_0^1 \pi\left(\sqrt{y}\right)^2 dy + \int_1^5 \pi\left(\frac{\sqrt{5-y}}{2}\right)^2 dy = \pi\left(\int_0^1 y\,dy + \frac{1}{4}\int_1^5 (5-y)\,dy\right)$	**D**

No	Working	Answer
9. ME 11-4	$\frac{dA}{dt} = 0.1\ \text{m}^2/\text{min}$ $A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ $\therefore \quad \frac{dr}{dA} = \frac{1}{2\pi r}$ $\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA}$ $= 0.1 \times \frac{1}{2\pi r}$ $= \frac{0.05}{\pi r}$ $When\ A = 0.3$ $\pi r^2 = 0.3$ $r = \sqrt{\frac{0.3}{\pi}}$ $\frac{dr}{dt} = \frac{0.05}{\pi\sqrt{\frac{0.3}{\pi}}}$ $= 0.05150322694$ ≈ 0.0515 m/min	**C**
10. ME 11-5	We have $^{10}\mathbf{C}_2$ ways of choosing the pair of numbers to use in the code. With the restriction, we have 14 ways of arranging any pair of digits. Eg. If we used 0 and 1 We could have 2 of each digit: 0011 0110 0101 1100 1001 1010 (6 arrangements) We could have 3 x 0 and 1 x 1 0001 0010 0100 1000 (4 arrangements) Or we could have 3 x 1 and 1 x 0 1110 1101 1011 0111 (4 arrangements) So we have $^{10}\mathbf{C}_2 \times 14 = 630$ arrangements with exactly 2 digits. Probability of guessing the correct code is $\frac{1}{630}$.	**D**

Trial HSC Examination 2020
Mathematics Extension 1

Name ______________________ Teacher ________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

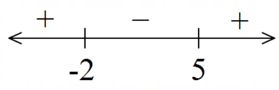
A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ correct C ○ D ○

	A	B	C	D
1.	○	○	○	●
2.	○	●	○	○
3.	○	●	○	○
4.	●	○	○	○
5.	○	○	○	●
6.	○	○	○	●
7.	○	○	●	○
8.	○	○	○	●
9.	●	○	○	○
10.	○	○	○	●

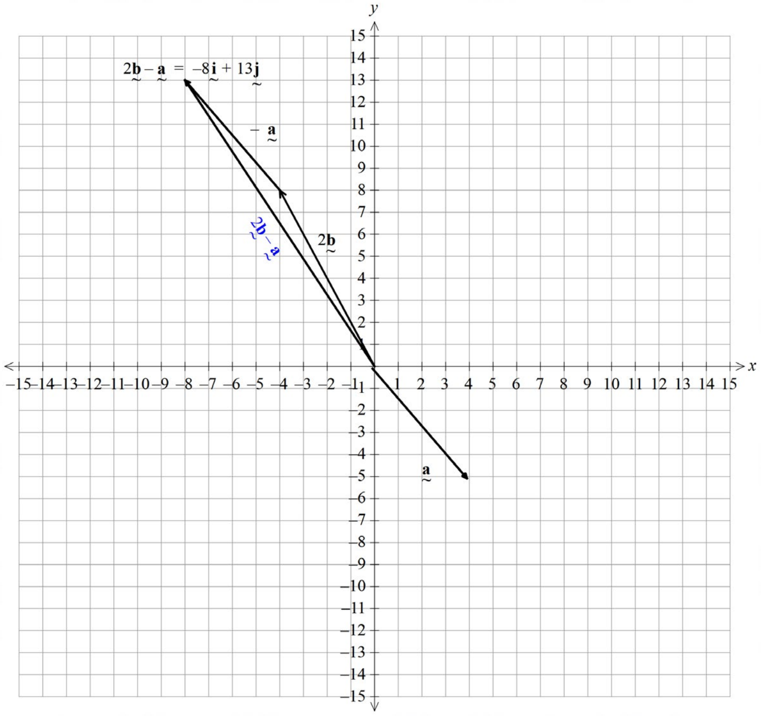
Question 11	2020

Q11	Solutions	Marking guidelines
(a) ME 11-2	The expression is 0 when $x=-2$ and undefined when $x=5$. We can do a sign analysis: When $x=-3$, $\dfrac{x+2}{x-5}>0$ When $x=0$, $\dfrac{x+2}{x-5}<0$ When $x=6$, $\dfrac{x+2}{x-5}>0$  So the solution is $x\le -2$, and $x>5$ (Note $x\neq 5$)	2 marks for correct solution 1 mark for attempt with a minor error
(b) ME 11-2	$x^3-x^2-3x-5=(x-3)Q(x)+4$ (subtract 4 from both sides) $x^3-x^2-3x-9=(x-3)Q(x)$ (divide both sides by $x-3$) $Q(x)=$ $\quad x^2+2x+3$ $x-3\,\overline{)\,x^3-x^2-3x-9}$ $-(x^3-3x^2)$ $2x^2-3x$ $-(2x^2-6x)$ $3x-9$ $-(3x-9)$ 0 $Q(x)=x^2+2x+3$	2 marks for correct solution 1 mark for a attempt with a minor error

Q11	Solutions	Marking guidelines
(c) ME 12-1	Let $u = 1 + 2\tan x$, $\frac{du}{dx} = 2\sec^2 x = \frac{2}{\cos^2 x}$ $dx = \frac{\cos^2 x}{2}\, du$ $x = 0 \;\; u = 1$ and when $x = \frac{\pi}{4}, \;\; u = 3$ $\int_0^{\frac{\pi}{4}} \frac{1}{(1+2\tan x)^2 \cos^2 x}\, dx = \int_1^3 \frac{1}{2u^2}\, du$ $= -\left[\frac{1}{2u}\right]_1^3$ $= -\left(\frac{1}{6} - \frac{1}{2}\right)$ $= \frac{1}{3}$	**2 marks** for correct answer with valid working **1 mark** finding an expression for the integral in terms of u or equivalent
(d) ME 12-4	(i) Ball enters the liquid when $t = 0$ $T = 400e^{-0.05t} + 25$ $= 400e^{-0.05 \times 0} + 25 = 425°C$	1 Mark: Correct answer.
	(ii) $300 = 400\, e^{-0.05t} + 25$ $e^{-0.05t} = \frac{275}{400}$ $-0.05t = \ln\left(\frac{11}{16}\right)$ $t = 7.4938 \approx 7.49$ min	1 Mark: Correct answer.
	(iii) $T = 400\, e^{-0.05t} + 25$ $\frac{dT}{dt} = -20e^{-0.05t}$ $= -20e^{-0.05 \times 50}$ $\approx -1.64°C$ $\therefore$ rate of decrease is $1.64°C$ per minute	2 Marks: Correct answer. 1 Mark: Differentiates correctly to find the rate of change.
ME 12-4 & *ME 12-7*	(iv) When t approaches infinity then $e^{-0\cdot 05t} \to 0$ $\therefore T > 25$ and can never fall to 20°C.	1 Mark: Correct answer.

Q11	**Solutions**	Marking guidelines
(e) ME 12-5	i) $P(1 < x < 4) = 1$ $\int_1^4 ax^2(4-x)\,dx = 1$ $a\int_1^4 4x^2 - x^3\,dx = 1$ $a\left[\frac{4}{3}x^3 - \frac{x^4}{4}\right]_1^4 = 1$ $a \times \frac{81}{4} = 1$ $a = \frac{4}{81}$	2 marks for correct answer with valid working 1 mark for using total probability equals 1 to attempt to find *a*.
	ii) $P(3 < x < 4) = 1 - P(X < 3)$ $= 1 - \frac{4}{81}\int_1^3 x^2(4-x)\,dx$ *or* $P(3 < x < 4) = \frac{4}{81}\int_3^4 x^2(4-x)\,dx$	Any correct expression acknowledging the difference
ME 12-5 & *ME 12-7*	iii) Since it is a probability density function, the area below the curve is 1 square unit. The area to the right of $x = 2.4$ equals 0.5. So $x = 2.4$ divided the total area in half, so **it is the median.**	Correct answer and valid explanation of finding half of the area.

Question 12	**2020**

Q12	Solution	Marking Guidelines
(a) ME 12-2	i) $4\underset{\sim}{\mathbf{i}} - 5\underset{\sim}{\mathbf{j}} + \left(-2\underset{\sim}{\mathbf{i}} + 4\underset{\sim}{\mathbf{j}}\right) = 2\underset{\sim}{\mathbf{i}} - \underset{\sim}{\mathbf{j}}$ magnitude of $a + b = \sqrt{2^2 + (-1)^2}$ $= \sqrt{5}$ direction $=$ α where $\tan\alpha = \left(-\frac{1}{2}\right)$ $\alpha = \tan^{-1}\left(-\frac{1}{2}\right)$ $= -26.56505118$ $\approx -26°34'$	2 marks for both direction and magnitude correct 1 mark for one answer correct
	ii) Graphically  Algebraically $2\underset{\sim}{\mathbf{b}} - \underset{\sim}{\mathbf{a}} = 2 \times (-2\boldsymbol{i} + 4\mathbf{j}) - (4\boldsymbol{i} - 5\mathbf{j})$ $= -4\boldsymbol{i} + 8\mathbf{j} - 4\boldsymbol{i} + 5\mathbf{j}$ $= -8\boldsymbol{i} + 13\mathbf{j}$	2 marks for correct result using a diagram or algebraically 1 mark for attempt with a minor error
	iii) $\underset{\sim}{\mathbf{a}} \,.\, \underset{\sim}{\mathbf{b}} = x_1 x_2 + y_1 y_2$ $= 4 \times -2 + \ -5 \times 4$ $= -8 - 20$ $= -28$	1 mark for correct answer

Q12	Solution	Marking Guidelines
(a) ME 12-2	iv) $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} = \lvert\underset{\sim}{\mathbf{a}}\rvert\lvert\underset{\sim}{\mathbf{b}}\rvert \cos\theta$ where θ is the angle between the vectors. $\lvert\underset{\sim}{\mathbf{a}}\rvert = \sqrt{4^2 + (-5)^2} = \sqrt{41}$ $\lvert\underset{\sim}{\mathbf{b}}\rvert = \sqrt{(-2)^2 + 4^2} = \sqrt{20}$ $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} = \sqrt{41} \times \sqrt{20} \cos\theta$ $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} = -28$ (from iii)) $\therefore \sqrt{820}\cos\theta = -28$ $\cos\theta = -\frac{28}{\sqrt{820}}$ $\theta = \cos^{-1}\left(-\frac{28}{\sqrt{820}}\right)$ $\theta = \cos^{-1}(-0.9778)$ $= 167° \, 54'$ (nearest minute)	2 marks for correct solution 1 mark for equating the expressions for scalar product of vectors but with an error in algebra or calculation
(b) ME 12-5	(i) $E(\hat{P}) = p = 0.36$ $sd(\hat{P}) = \sqrt{\frac{0.36 \times 0.64}{25}} = 0.096$	2 Marks: For correct mean and standard deviation 1 Mark: For correct mean or standard deviation
	ii) Transforming to a standard normal variable Z, gives: $P\left(Z < \frac{0.12 - 0.36}{0.096}\right) = P(Z < -2.5)$ $= 1 - P(Z < 2.5)$ $= 1 - 0.9938$ $= 0.0062$	2 Marks: correct solution 1 Mark: calculates $z = \frac{0.12 - 0.36}{0.096}$ or Uses the table correctly with an incorrect value for z
	iii) The number of adults in the sample who have a mortgage is $25 \times 0.36 = 9$. Let X represent the number of adults who have a mortgage and $X \sim \text{Bin}(25, 0.36)$. $P(X = 9) = \binom{25}{9}(0.36)^9(0.64)^{16}$ $= 0.1644$	2 Marks: gives correct solution 1 Mark: attempts to find $P(X = 9)$ where $X \sim \text{Bin}(25, 0.36)$

Q12	Solution	Marking Guidelines
(c) ME 12-1	Area of triangle: $A = \frac{1}{2}ab\sin C$ $= \frac{1}{2} \times 3 \times 5 \times \sin\theta = \frac{15}{2}\sin\theta$ Differentiating using chain rule: $\frac{dA}{dt} = \frac{15}{2}\cos\theta \times \frac{d\theta}{dt}$ Substituting: $\frac{dA}{dt} = \frac{15}{2}\cos\frac{\pi}{4} \times 0.05 = \frac{3}{8\sqrt{2}}$ or $\frac{3\sqrt{2}}{16}$	2 marks for correct solution 1 mark for attempt with a minor error

Question 13	**2020**

Q13	Solution	Marking Guidelines
(a) ME 12-1	i) LHS $= 7^{k+1} - 3^{k+1}$ $= 7^k \times 7 - 3^k \times 3$ $= 4 \times 7^k + 3 \times 7^k - 3 \times 3^k$ $= 4 \times 7^k + 3(7^k - 3^k)$ $= 4 \times 7^k + 3(4p)$ $= 4(7^k + 3p)$ $= 4q$	2 marks for any correct working to prove LHS = $4q$ 1 mark for working with a minor error, or incomplete **A few students were poor in Mathematical Induction**
	ii) When $n = 1$, $7^k - 3^k = 7^1 - 3^1$ $= 7 - 3$ $= 4$ Which is divisible by 4 $\therefore$ true for $n = 1$	1 mark for correct step (the first step in induction) **A few students did not read the question properly.**
(b) ME 12-3	i) LHS $= \sin(\alpha+\beta) + \sin(\alpha-\beta)$ $= \sin\alpha\cos\beta + \sin\beta\cos\alpha + \sin\alpha\cos\beta - \sin\beta\cos\alpha$ $= 2\sin\alpha\cos\beta$ $=$ RHS	1 mark for correct solution **Some students forgot to (refer to Reference Sheet)**
	ii) $\alpha + \beta + \gamma = \pi$ (Angle sum of a triangle) $\alpha + \beta = \pi - \gamma$ Using the given statement and part (a) $\sin\gamma = 2\sin\alpha\cos\beta$ $= \sin(\alpha+\beta) + \sin(\alpha-\beta)$ $= \sin(\pi-\gamma) + \sin(\alpha-\beta)$ $= \sin\gamma + \sin(\alpha-\beta)$ $\alpha - \beta = 0, \pi, 2\pi,$ $\alpha = \beta$ (angles in a triangles are less than 180) $\therefore \Delta ABC$ is an isosceles triangle as two sides are equal.	2 Marks: Correct answer. 1 Mark: Uses the expression in part (a) **Most students forgot to apply:** $\sin\theta = \sin(180^\circ - \theta)$
(c) ME 12-3	i) LHS $= \operatorname{cosec}x + \cot x$ $= \dfrac{1+t^2}{2t} + \dfrac{1-t^2}{2t}$ $= \dfrac{1+t^2+1-t^2}{2t}$ $= \dfrac{1}{t}$ $= \cot\dfrac{x}{2}$ $=$ RHS	2 Marks: Correct answer. 1 Mark: Writes $\operatorname{cosec}x$ and $\cot x$ in terms of t. **(refer to Reference Sheet)**

Q13	Solution	Marking Guidelines
	ii) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\text{cosec}x + \cot x)dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\cot\frac{x}{2}dx$ $= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{0.5\cos\frac{x}{2}}{\sin\frac{x}{2}}dx$ $= 2\left[\ln\left(\sin\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= 2\left[\ln\left(\sin\frac{\pi}{4}\right) - \ln\left(\sin\frac{\pi}{6}\right)\right]$ $= 2\left(\ln\frac{1}{\sqrt{2}} - \ln\frac{1}{2}\right)$ $= 2\ln\left(2^{-\frac{1}{2}} \div 2^{-1}\right)$ $= 2\ln 2^{\frac{1}{2}}$ $= \ln 2$	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Finds the primitive function. **Many students used a wrong substitution in finding the integration and made careless mistakes in calculating the integration.**
(d) ME 12-5	Let p be the probability of the train being on time $p = 0.85$, $n = 7$ $P(X = x) = {}^7C_x\, 0.85^x 0.15^{7-x}$ Expression is: $P(X \geq 5) = P(X = 5) + P(X = 6) + P(X = 7)$ $= {}^7C_5\, 0.85^5 0.15^2 + {}^7C_6\, 0.85^6 0.15^1 + {}^7C_7\, 0.85^7 0.15^0$	2 Marks: Correct answer. 1 Mark: Shows some understanding. **Most students were able to write the expression in full while a few students forgot to write the coefficients.**
(e) ME 11-5	When dividing a number by 6, there are 6 possible remainders (0 through 5). $100 \div 6 = 16 \text{ r}4$ Thus, by the pigeonhole principle, there are at least 17 numbers with the same remainder when divided by 6.	2 Marks: Correct answer. 1 Mark: Shows some understanding. **Most students have not justified their answer of 16r4 Some students were weak in applying pigeonhole and did not mention all possible remainders.**

Question 14	2020

Q14	Solution	Marking Guidelines
(a)(i) ME 12-1	(i) The amount of liquid in the tank at time t is a linear function of time. Water is entering the tank at 20 L/min and leaving the tank at 15 L/min. $A = 2500 + 20t - 15t = 2500 + 5t$ The concentration of the solution is the amount of salt in the tank, y, divided by the amount of liquid. $C = \frac{y}{2500+5t}$	2 marks: Correct solution, including the formula for volume at any time, as well as a definition for how concentration is determined. 1 mark: One component answered satisfactorily.
(ii) ME 12-1 & ME 12-7	(ii) The rate of change of salt is the concentration of salt per litre multiplied by the amount of liquid, in litres, leaving the tank, or $y' = C\,\frac{\text{kg}}{\text{L}} \times -15\,\frac{\text{L}}{\text{min}} = -15C\,\frac{\text{kg}}{\text{min}}$	1 mark: correct reference to change in volume, **as well as** it's relationship to concentration of the solution. 0 marks: Only referring to the loss of 15L.
(iii) ME 12-4	(iii) We have $y' = -\frac{15y}{2500+5t}$. This is a separable differential equation. $\frac{dy}{dt} = -\frac{15y}{2500+5t}$ (divide both sides by y and "multiply" dt) $\frac{1}{y}dy = -\frac{15}{2500+5t}dt$ (integrate both sides) $\int \frac{1}{y}dy = -15\int \frac{1}{2500+5t}dt$ $\ln\|y\| = -15 \times \frac{1}{5}\ln\|2500+5t\| + D$ (y and t are positive, so we can eliminate the absolute values. Using D as a constant.) $\ln y = \ln(2500+5t)^{-3} + D$ (raise to sides to the power of e) $y = (2500+5t)^{-3} \times e^D$ (e^D is also an arbitrary constant. We can replace it with a new constant.) $y = \frac{A}{(2500+5t)^3}$ Applying the initial condition: when $t = 0$, $y = 25$. $25 = \frac{A}{2500^3} \rightarrow A = 25 \times 2500^3$ $y = \frac{25\times2500^3}{(2500+5t)^3}$	4 marks: All steps of solution correct. 3 marks: Almost complete satisfaction of solution. 2 marks: Significant progress. 1 mark: Some relevant progress.
(iv) ME 12-4	(iv)The tank overflows when $A = 5000$: $5000 = 2500 + 5t \rightarrow t = 500$ $y = \frac{25\times2500^3}{(2500+5\times500)^3} = 3.125$ kg	2 marks: Both steps correct, CFPA in (iii), but only if answer is <25 kg. 1 mark: Relevant progress.

Q14	Solution	Marking Guidelines
(b) ME 12-4	i) $\dfrac{5}{5x-1} - \dfrac{1}{x+1} = \dfrac{5x+5-5x+1}{5x^2+4x-1}$ $= \dfrac{6}{5x^2+4x-1}$	1 mark: Adequate steps to show equality *Note: Partial fractions method would have wasted time.*
	ii) $y = \displaystyle\int \dfrac{6}{5x^2+4x-1}\,dx = \int \dfrac{5}{5x-1} - \dfrac{1}{x+1}\,dx$ $= \ln(5x-1) - \ln(x+1) + C$ $= \ln\left(\dfrac{5x-1}{x+1}\right) + C$ when $x = \dfrac{1}{2}, y = 3$ $3 = \ln\left(\dfrac{\frac{5}{2}-1}{\frac{1}{2}+1}\right) + C$ $3 = \ln\left(\dfrac{\frac{3}{2}}{\frac{3}{2}}\right) + C$ $3 = \ln(1) + C$ $3 = 0 + C$ $\therefore C = 3$ $y = \ln\left(\dfrac{5x-1}{x+1}\right) + 3$	2 marks: correct answer with valid working 1 mark: Relevant progress. *Note: The solution shown is simpler than an expression showing the difference between 2 separate logs*
(c) ME 12-1 *&* *ME 12-7*	We know that $\underset{\sim}{u} \cdot \underset{\sim}{v} = \lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert \cos\theta$ where θ is the angle between the vectors. Thus $\cos\theta = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert}$ Applying this to $\underset{\sim}{p}$ and $\underset{\sim}{q}$ (which have length 1): $\cos(\alpha+\beta) = \underset{\sim}{p}\cdot\underset{\sim}{q}$ In order to find $\underset{\sim}{p}\cdot\underset{\sim}{q}$ we will find their components: $\underset{\sim}{p} = \cos\alpha\,\underset{\sim}{i} + \sin\alpha\,\underset{\sim}{j}$ $\underset{\sim}{q} = \cos(-\beta)\,\underset{\sim}{i} + \sin(-\beta)\,\underset{\sim}{j} = \cos\beta\,\underset{\sim}{i} - \sin\beta\,\underset{\sim}{j}$ (vector $\underset{\sim}{q}$ is at $-\beta$ on the unit circle) $\underset{\sim}{p}\cdot\underset{\sim}{q} = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ Hence, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.	3 marks: Correct solution with explanation. 2 marks: considerable progress. 1 mark: Relevant progress. 0 marks: Using the result to prove the result; or Only quoting formulae from the reference sheet.