International Grammar School

# Year 12
# Mathematics Extension 1
# HSC Trial Examination
# 2014

**General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page of this question paper
- All necessary working should be shown in every question

**Note:** Any time you have remaining should be spent revising your answers.

**Total marks – 70**

**Section I**

**10 marks**

- Attempt Questions 1–10

**Section II**

**60 marks**

- Attempt Questions 11-14
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover

**DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM**

**Section I**

**10 marks**

**Attempt Questions 1 – 10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Question 1 – 10.

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**1.** How many numbers greater than 5 000 can be formed with the digits 2, 4, 5, 6 and 9 if a digit cannot occur more than once in any number?
Hint: The numbers can have 4 or 5 digits.

(A) 72

(B) 120

(C) 144

(D) 192

**2.** Find the derivative of $e^{\cos x}$

(A) $e^{\cos x}$

(B) $e^{\sin x}$

(C) $-\sin x\, e^{\cos x}$

(D) $\sin x\, e^{\cos x}$

**3.** The expansion needed to show $\sin 75^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ is:

(A) $\sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

(B) $\sin(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

(C) $\sin(100^\circ - 25^\circ) = \sin 100^\circ \cos 25^\circ + \cos 100^\circ \sin 25^\circ$

(D) $\tan(45^\circ + 30^\circ) = \dfrac{\tan 45^\circ \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

**4.** The Cartesian equation of the tangent to the parabola $x = t - 3$, $y = t^2 + 2$ at $t = -3$ is:

(A) $6x + y + 25 = 0$

(B) $6x + y + 36 = 0$

(C) $6x - y - 25 = 0$

(D) $6x + 2y - 25 = 0$

**5.** A particle moves in a straight line. Its position at any time *t* is given by

$x = 3\cos 2t + 4\sin 2t.$

The acceleration in terms of *x* is:

(A) $\ddot{x} = -3x$

(B) $\ddot{x} = -4x$

(C) $\ddot{x} = -16x^2$

(D) $\ddot{x} = -6\cos 2x + 8\sin 2x$

**6.** $\displaystyle\int \frac{dx}{\sqrt{1 - 3x^2}} =$

(A) $\left(\sin^{-1} 3x\right) + C$

(B) $\left(\tan^{-1} 3x\right) + C$

(C) $\frac{1}{\sqrt{3}}\left(\tan^{-1} \sqrt{3}\ x\right) + C$

(D) $\frac{1}{\sqrt{3}}\left(\sin^{-1} \sqrt{3}\ x\right) + C$

**7.** What is the domain and range of $y = \frac{1}{2}\cos^{-1}(\frac{x}{2})$ ?

(A) Domain: $-2 \le x \le 2$ Range: $0 \le y \le \pi$

(B) Domain: $-1 \le x \le 1$ Range: $0 \le y \le \pi$

(C) Domain: $-2 \le x \le 2$ Range: $0 \le y \le \frac{\pi}{2}$

(D) Domain: $-1 \le x \le 1$ Range: $0 \le y \le \frac{\pi}{2}$

**8.** Which of the following is an expression for $\int \frac{x}{(2-x^2)^3} dx$ ?

Use the substitution $u = 2 - x^2$.

(A) $\frac{1}{2(2-x^2)^2} + C$

(B) $\frac{1}{4(2-x^2)^2} + C$

(C) $\frac{1}{4(2-x^2)^4} + C$

(D) $\frac{1}{8(2-x^2)^4} + C$

**9.** Find the acute angle between the lines $y = 2x$ and $x + y - 5 = 0$. Answer correct to the nearest degree.

(A) $18^\circ$

(B) $32^\circ$

(C) $45^\circ$

(D) $72^\circ$

**10.** What are the coordinates of the point that divides the interval joining the points $A(-1, 2)$ and $B(3, 5)$ externally in the ratio 3:1?

(A) $(2.5, 4.25)$

(B) $(2.5, 6.5)$

(C) $(5, 4.25)$

(D) $(5, 6.5)$

## Section II

**60 Marks**
**Attempt Questions 11 - 14.**
**Allow about 1 hour 45 minutes for this section.**

Answer each question in a separate writing booklet. Extra writing booklets are available.
In Questions 11 - 14, your responses should include relevant mathematics reasoning and/ calculations.

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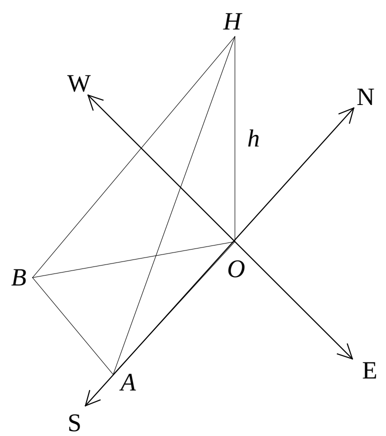
**Question 11** **Marks**

(a) What are the roots of the equation $4x^3 - 4x^2 - 29x + 15 = 0$ given that one root is the difference between the other two roots? **3**

(b) Solve for $x$ the inequality $\dfrac{2x-5}{x-4} \geq x$. **3**

(c) A class consists of 10 boys and 12 girls. How many ways are there of selecting a committee of 3 boys and 2 girls from this class? **2**

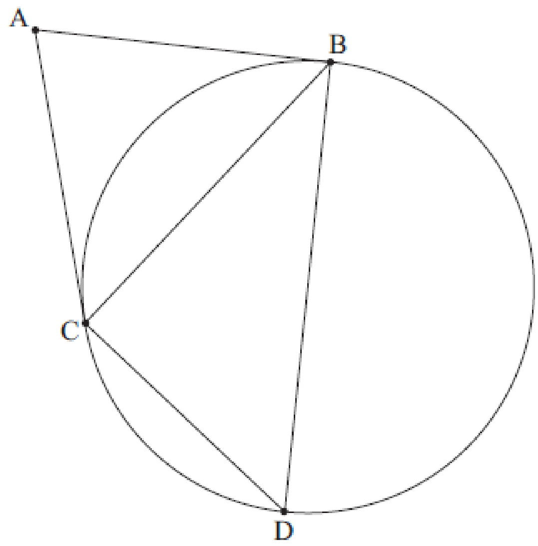
(d) Point $A$ is due south of a hill and the angle of elevation from $A$ to the top of the hill is 35º. Another point $B$ is a bearing 200º from the hill and the angle of elevation from $B$ to the top of the hill is 46º. The distance $AB$ is 220 m.




(i) Express $OA$ and $OB$ in terms of $h$. **2**

(ii) Calculate the height $h$ of the hill correct to three significant figures. **2**

(e) $AB$ and $AC$ are tangents to a circle. $D$ is a point on the circle such that $\angle BDC = \angle BAC$ and $2\angle DBC = \angle BAC$.



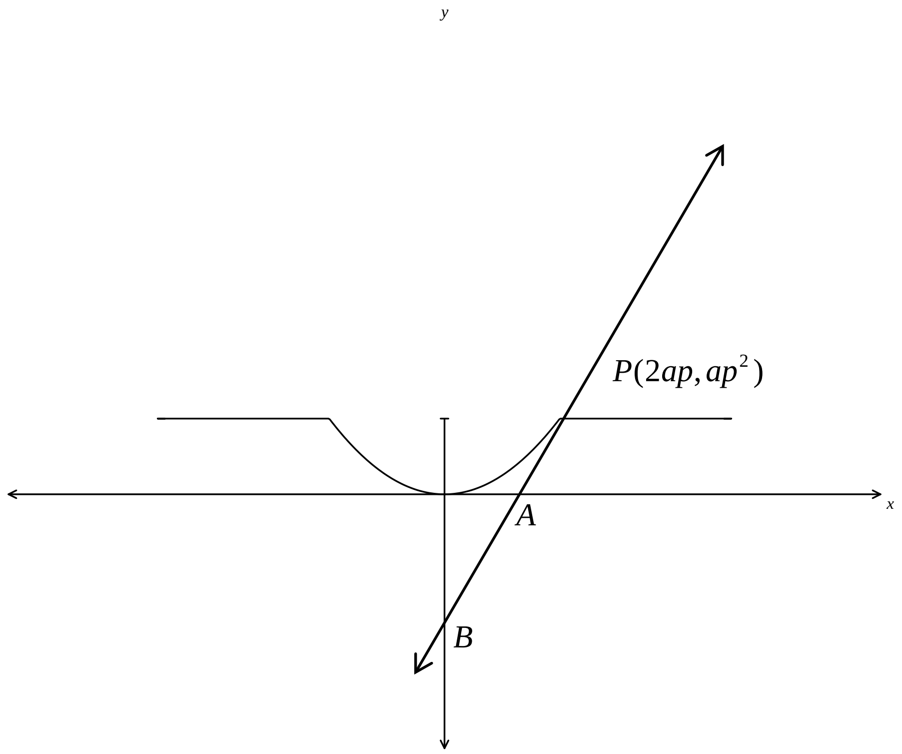

Show that $DB$ is a diameter. **3**

**Question 12 (15 marks)** Use a SEPARATE writing booklet **Marks**

(a) (i) Show that $3\sin x\cos x = \frac{3}{2}\sin 2x$ **1**

(ii) Hence or otherwise, find the exact value of $\int_0^{\frac{\pi}{2}} 9\ \sin^2 x\cos^2 x\,dx$. **2**

(b)




The tangent at the point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ cuts the $x$-axis at $A$ and the $y$-axis at $B$.

(i) Find the coordinates of $A$ and $B$ in terms of $P$. **2**

(ii) Find the coordinates of $M$, the midpoint of $A$ and $B$ in terms of $P$. **1**

(iii) Show that the locus of $M$ is a parabola. **1**

(iv) Find the coordinates of the focus of this parabola and the equation of the directrix. **1**

(c) Greg designs a sky-diving simulator for a video game. He simulates the rate of change of the velocity of the skydivers as they fall by:

$\frac{dV}{dt} = -k(V - P)$, where $k$ and $P$ are constants.

The constant $P$ represents the terminal velocity of the skydiver in the prone position which is 55 m/s.

(i) Show that $V = P + Ae^{-kt}$ is solution of this differential equation. **1**

(ii) Initially the velocity of the skydiver is 0 m/s and the velocity after 10 seconds is 27 m/s. Find values for $A$ and $k$. **2**

(iii) Find the velocity of the skydiver after 17 seconds. **1**

(iv) How long does it take the skydiver to reach a velocity of 50 m/s? **1**

(d) Factorise $P(x) = x^3 + 3x^2 - 9x + 5$. **2**

**Question 13 (15 marks)** Use a SEPARATE writing booklet **Marks**

(a) Find the exact value of $\sin\left[\cos^{-1}\frac{2}{3}+\tan^{-1}\left(-\frac{3}{4}\right)\right]$ **2**

(b) Prove by mathematical induction that **3**

$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = \frac{a\left(r^n - 1\right)}{r-1}$$

for all $a$ and $r$, where $n$ is a positive integer.

(c) For the function $f(x) = x^2 + 6x$:

(i) Find the domain over which $f(x)$ is monotonic increasing. **1**

(ii) Find an expression for the inverse function $f^{-1}(x)$ over this restricted domain. **2**

(iii) Sketch the graph of $y = f^{-1}(x)$, and state the domain and range of $y = f^{-1}(x)$. **3**

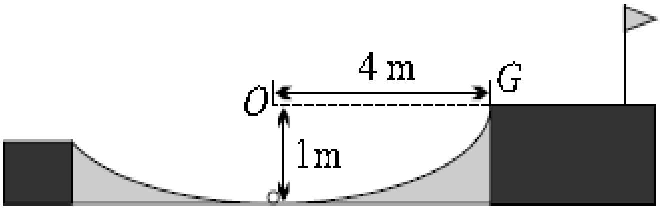
(d) By expressing $\sin x + \cos x$ in the form $r\sin(x+\alpha)$, solve the equation $\sin x + \cos x = 1$ for $0 \le x \le 2\pi$. **4**

**Question 14 (15 marks)** Use a SEPARATE writing booklet **Marks**

(a) State the domain and range of $y = 2\sin^{-1}(3x+2)$ and hence sketch $y = 2\sin^{-1}(3x+2)$. **3**

(b) Find the area bounded by the curve $y = \dfrac{1}{9+x^2}$, the $x$ axis and the lines $x = 0$ and $x = \sqrt{3}$. **2**

(c)




The diagram above shows a golf ball in the middle of a 1metre deep bunker and 4 metres from the edge of the green at $G$. The ball is hit with an initial speed of 12 m/s at an angle of elevation $\alpha$. Putting the origin at $O$, the horizontal and vertical equations of motions are:

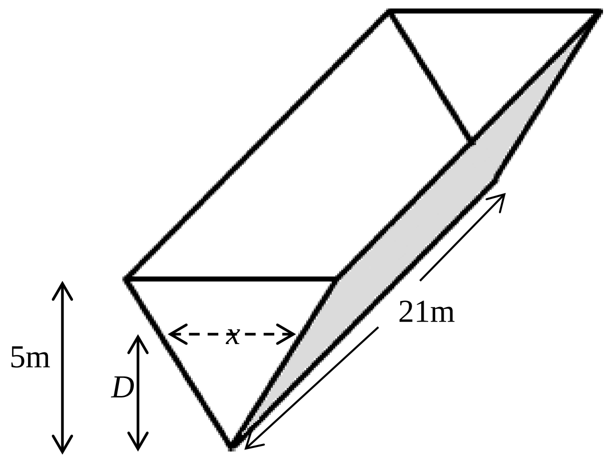
$$x = 12t\cos\alpha \qquad\qquad y = -5t^2 + 12t\sin\alpha - 1$$

(You are NOT required to prove these equations)

(i) Find the maximum height the ball reaches above $G$ when $\alpha = 30^o$ **2**

(ii) Find the range of values $\alpha$ may take so that the ball lands on the green at $G$ or beyond. Give your answers to the nearest degree. **3**

(d) A water trough has a vertical cross section in the shape of an equilateral triangle as shown in the diagram. It is initially empty and is being filled with water at the rate of 4 cubic metres per hour.




(i) By considering an equilateral triangle with side length $x$ and perpendicular height $D$, show that $D^2 = \frac{3}{4}x^2$. **1**

(ii) Given that the trough is $5$ metres high and $21$ metres long, use the result in (i) to show that the volume of water in the trough at depth $D$ metres is given by: **2**

$$V = 7\sqrt{3}D^2$$

(ii) Find the exact rate at which the water level is rising when the water has a depth of $1.5$ metres. **2**

**End of Examination**

## STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \;, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

*Note* $\ln x = \log_e x, \quad x > 0$