QUESTION 1 (START A NEW PAGE)

(a) Differentiate:

(i) $\dfrac{1}{1 + 4x^2}$

(ii) $e^{2x} \log_e 2x$

(b) Write down primitive functions of :

(i) $\sqrt{2x + 1}$

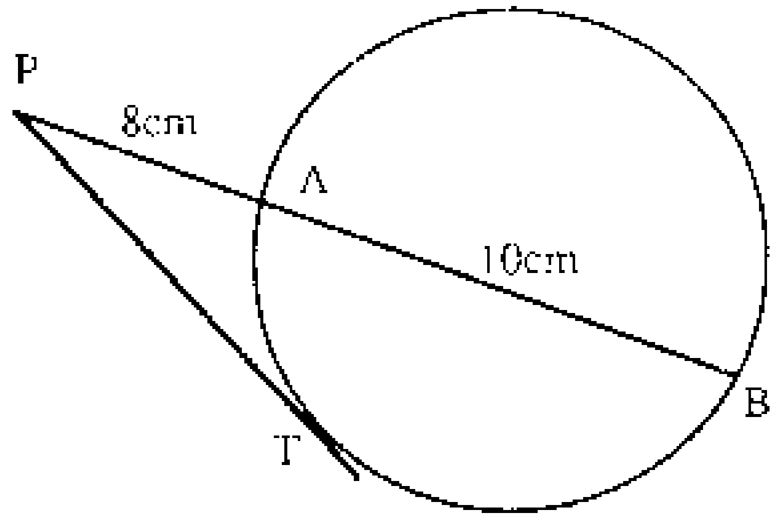
(ii) $\dfrac{1}{1 + 9x^2}$

(c) A and B are the points $(-4, 3)$ and $(2, \ 1)$ respectively. Find the coordinates of the point Q which divides AB externally in the ratio 4 : 5.

(d) Draw the graph of a function $y = f(x)$ for $1 \le x \le 2$ such that $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$ for $1 \le x \le 2$.

QUESTION 2 (Start A New Page)

(a) PT is a tangent to a circle ABT. PAB is a secant intersecting the circle in A and B. PA = 8cm and AB = 10cm. Find the length of PT giving reason(s) for your answers.



(b) Find the gradients of the 2 lines which make angles of 45° with the line whose equation is $2x \quad 3y - 6 = 0$.

(c) A particle moves along a straight line with a displacement $x(t)$ metres from 0 given by $x(t) = t(2t - 3)(t \quad 4)$ where t is measured in seconds.

(i) Draw the displacement time graph $x(t)$ and the velocity-time graph $v(t)$. (**Note**: Co-ordinates of stationary points need not be shown.)

(ii) Describe the motion of the particle for $\dfrac{3}{2} \le t \le 4$.

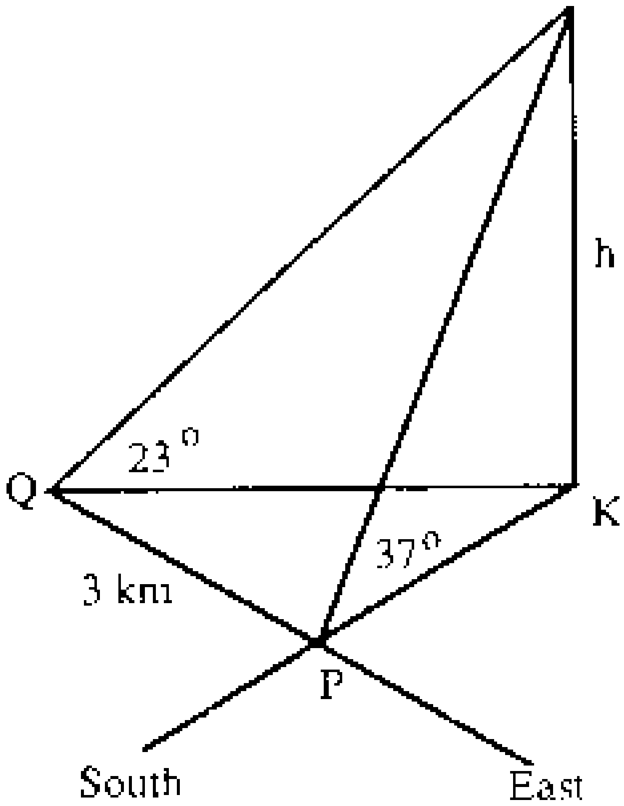
QUESTION 3 (Start a New Page)

(a) The angle of elevation of a hill top from a place P due south of it is 37°. The angle of elevation of this same hill top from a place Q, due west of P, is 23°. The distance of Q from P is 3 kilometres. If the height of the hill is h kilometres:

(i) Prove that PK = h cot 37°.

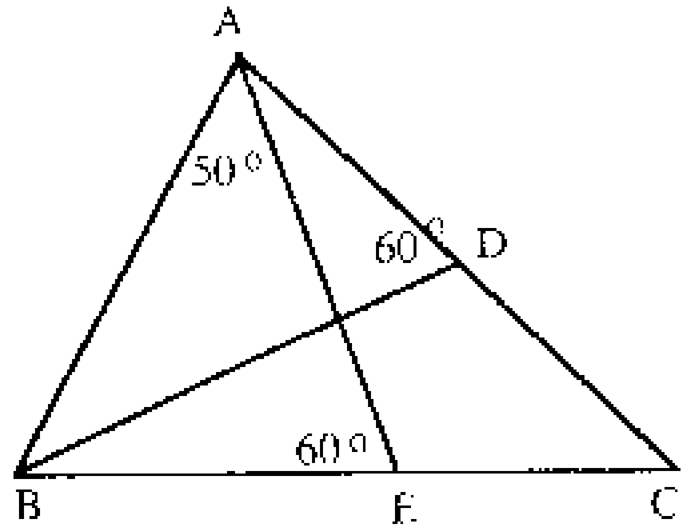
(ii) Find a similar expression of QK.

(iii) Calculate the height of the hill to the nearest 10 metres.



(b) ABC is a triangle. D lies on AC and $\angle ADB = 60°$. E lies on BC and $\angle AEB = 60°$ $\angle BAE = 50°$

Copy this diagram onto your page and find the size of $\angle CDE$ giving full reasons for your answer.



(c) Find the number of ways of arranging 2 men, 2 boys and 2 girls in a circle if:

(i) there is no restriction.

(ii) the two boys sit next to one another.

(d) In solving a problem it is necessary to find a value of r for which $\pi r^2 + 2\pi rh$ is a minimum knowing that $\pi r^2 h = 5$. Write down a problem which could be solved using this information.

Question 4 (Start a new page)

(a) (i) On the same set of axes draw neat sketches of $y = x^2$ and $y = 4x - x^2$ showing the co-ordinates of the points of intersection.

(ii) Find the volume of the solid generated when the region bounded by these two curves is rotated one revolution about the x-axis.

(b) A body is projected with speed 24.5 ~~mm~~$^{-1}$ from the top of a cliff 58.8m high at an angle of α to the horizontal where $\alpha = \tan^{-1}(\frac{4}{3})$. Take the bottom of the cliff as the origin and take the acceleration due to gravity g as $9.8ms^{-2}$

(i) Show that $x = 14.7t$ and $y = 58.8 + 19.6t - 4.9t^2$.

(ii) Find the range of the horizontal plane through the foot of the cliff.

(iii) Find the speed of the body when it reaches this point.

Question 5. (Start a New Page)

(a) Sketch the graph of $y = 2\cos^{-1}x \quad \frac{\pi}{4}$ stating its natural domain and range.

(b) Find the general solution of the trigonometric equation $\cos 3\theta = \cos\theta$.

(c) The rise and fall of the tide at a certain harbour may be taken to be simple harmonic, the interval between successive high tides being 12 hours 30 minutes. The harbour entrance has a depth of 11 metres at high tide and 7 metres at low tide. If low tide occurs at 9.05 a.m. on a certain day find the earliest time thereafter that a ship drawing 10 metres can pass through the entrance.

Question 6 (Start a New Page)

(a) Prove by induction that $9^{n+2} - 4^{n}$ is divisible by 5 for integers $n \geq 1$.

(b) Newton's law of cooling states that the rate at which a body loses heat to its surroundings is proportional to the difference between the temperature T of the body and the temperature S of its surrounding medium. This can be expressed by the differential equation

$$\frac{dT}{dt} = k(T - S)$$

where t is the time in minutes and k is a constant.

(i) Show that $T = S + Be^{-kt}$, where B is a constant, is a solution.

(ii) If the temperature of a beaker of water falls from 90°C to 60°C in 5 minutes at a room temperature of 20°C, find

(α) the time taken for the temperature of the water to cool to 50°C (Give your answer correct to 1 decimal place).

(β) the temperature of the water 15 minutes after reaching 60°C. (Give your answer correct to 1 decimal place.)

QUESTION 7 (Start a New page.)

(a) The chord PQ joining the points $P(2p, p^2)$ and $Q(2q, q^2)$ on $x^2 = 4y$ always passes through the point A(2, 0) when produced.

(i) Show $(p + q) = pq$

(ii) Find the co-ordinates of M, the midpoint of PQ.

(iii) Find the equation of the parabola on which M always lies as P varies. On the same set of axes sketch this parabola and the parabola $x^2 = 4y$ showing co-ordinates of vertices and points of intersection.

(iv) Write down the equation of the locus of M indicating any restriction which exists for the domain.

(b) The structural steel work of a new office building is finished. Across the street 60m from the foot of a freight elevator shaft in the building a spectator is standing, watching the freight elevation ascend at a constant rate of 15m/s. How fast is the angle of elevation of the spectators line of sight to the elevator increasing 6 seconds after his line of sight passed the horizontal? [Give your answer to 2 significant figures in rad./sec.]

THIS IS THE END OF THE PAPER

3U 1992

JRAHS 3 Unit Trial 1992

Question 1.

(a)(i) $\frac{-2x}{(4+x^2)^2}$

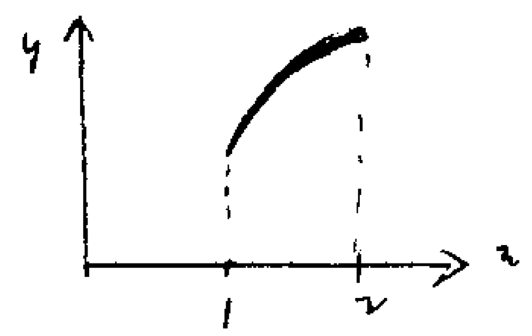
(ii) $e^{2x}\left(2\ln x + \frac{1}{x}\right)$

(b)(i) $\frac{(2x+1)\sqrt{2x+1}}{3} + c$

(ii) $\frac{1}{3}\tan^{-1}3x + c$

(c) $Q(-28, 19)$

(d)



Question 2

(a) $PT = 12$ cm

(b) $m = -\frac{1}{5}, 5$

(c)(i) $v = 6t^2 - 22t + 12$, $a = 12t - 22$

(ii) —

Question 3.

(a)(i) $PK = h\cot 31°$

(ii) $QK = h\cot 37°$

(iii) $h \doteqdot 1540$

(b) $E\hat{D}C = 70°$

(c) (i) $5!$

(ii) $2! \times 4!$

(d) Find the radius which minimises the surface area of a cylinder with one end and fixed volume $5a^3$.

Question 4

(a)(ii) $\frac{5\sqrt{11}}{3}$

(b)(i) $x = 14.7t$, $y = -4.9t^2 + 19.6t + 58.8$

(ii) $t = 6$, $x = 88.2$

(iii) $v \doteqdot 41.9$ m/s

Question 5

(a) Domain $-1 \le x \le 1$

Range $-\pi/4 \le y \le \pi/4$

(b) $\theta = n\frac{\pi}{2}$, n an integer

(c) time = 1.15 pm

Question 6

(a) —

(b)(i) —

(ii)(α) 7.6 min

(β) 27.5°C

(c)

Question 7

(a)(i) —

(ii) $M\left(p+q, \frac{1}{2}(p^2+q^2)\right)$

(iii) $y = \frac{1}{2}(x^2 - 2x)$

(iv) restriction $x \le 0$, $x \ge 4$.

(b) 0.077 rad/sec