J.R.A.H.S . TRIAL HSC EXAMINATION 3 / 4 UNIT MATHEMATICS 1993

QUESTION 1

(a) Find all real solutions to $x + 6 > x^2$.

(b) Differentiate $f(x) = \cos^{-1}(\sin x)$.

(c) Find k if $x^{k+3} = e^{7\ln x}$, where $x > 0$.

(d) Prove that $\dfrac{1 + \cos 2\theta}{1 - \cos 2\theta} = \cot^2 \theta$.

(e) A firm manufacturing fuses finds that 2 % of them are defective.
From a random sample of 8 fuses, find the probability that the sample contains 3 defective fuses. Give your answer correct to one significant figure.

QUESTION 2 (START A NEW PAGE)

(a) Use the substitution $u = 1 + 3x^3$ to evaluate $\displaystyle\int_0^1 x^2 \sqrt{1+3x^3}\, dx$.

(b) Integrate $\displaystyle\int \frac{x^2 + 1}{x^2 + 4}\, dx$.

(c) Prove by Mathematical Induction that :

$$1 + 4 + 16 + \ldots\ldots + 4^n = \frac{1}{3}[4^{n+1} - 1] \text{ for } n = 0, 1, 2, \ldots$$

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QUESTION 3 (START A NEW PAGE)

(a) A particle moving in a straight line x centimetres from the origin O, after t seconds, is given by $x(t) = 3 - 5\cos 2t$.

(i) Show that its acceleration is given by : $\ddot{x} = -4(x-3)$.

(ii) Find its period of motion.

(b) There are 5 girls and 6 boys in a group.

(i) How many ways could they be arranged in a line such that each girl stands between 2 boys ?

(ii) How many arrangements are possible if two boys A and B stand at each end of a line ?

(c) (i) Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ for $R > 0$ and α acute.

(ii) Hence, or otherwise, find all solutions to $\cos x - \sqrt{3}\sin x = 2$.

QUESTION 4 (START A NEW PAGE)

(a) (i) Neatly sketch the graph of $y = \sin^{-1} x$ and state its domain and range.

(ii) By considering the graph in (i) or otherwise, find the exact value of:

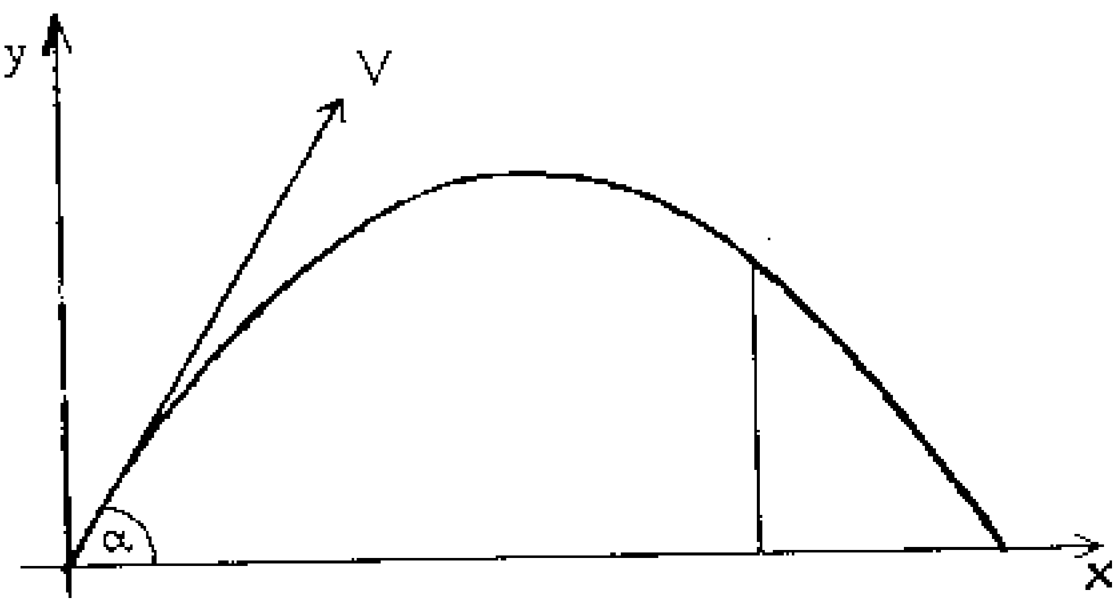
$$\int_0^{\frac{1}{2}} \sin^{-1} x \, dx \, .$$

(b) (i) Show that $\frac{d}{dx}\tan^3 x = 3\sec^4 x - 3\sec^2 x$.

(ii) Using (i) or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$.

QUESTION 5 (START A NEW PAGE)

(a) A particle is projected from a fixed point O on a horizontal plane at an angle of elevation α with a speed of V m/s. After a time t, the horizontal and vertical components of its velocity are: $\dot{x} = V \cos \alpha$ $\quad \dot{y} = V \sin \alpha - gt$



(i) Show that the position P(x,y) of the particle at any time as it moves along its path is given by:

$$y = x \tan \alpha - \frac{gx^2}{2V^2}(1 + \tan^2 \alpha)$$

(ii) If the particle is projected from the origin at an angle of 30^0, find the speed required for it to just clear a vertical wall 4 metres high and 12 metres away from the origin. (Take $g = 10$ m/s^2)
Give your answer correct to two decimal places.

(b) (i) Write down the term T_{r+1} of the expansion $(x + \frac{1}{x^2})^{12}$ for $0 \le r \le 12$.

(ii) Evaluate the term independent of x in the expansion of $(x + \frac{1}{x^2})^{12}$.

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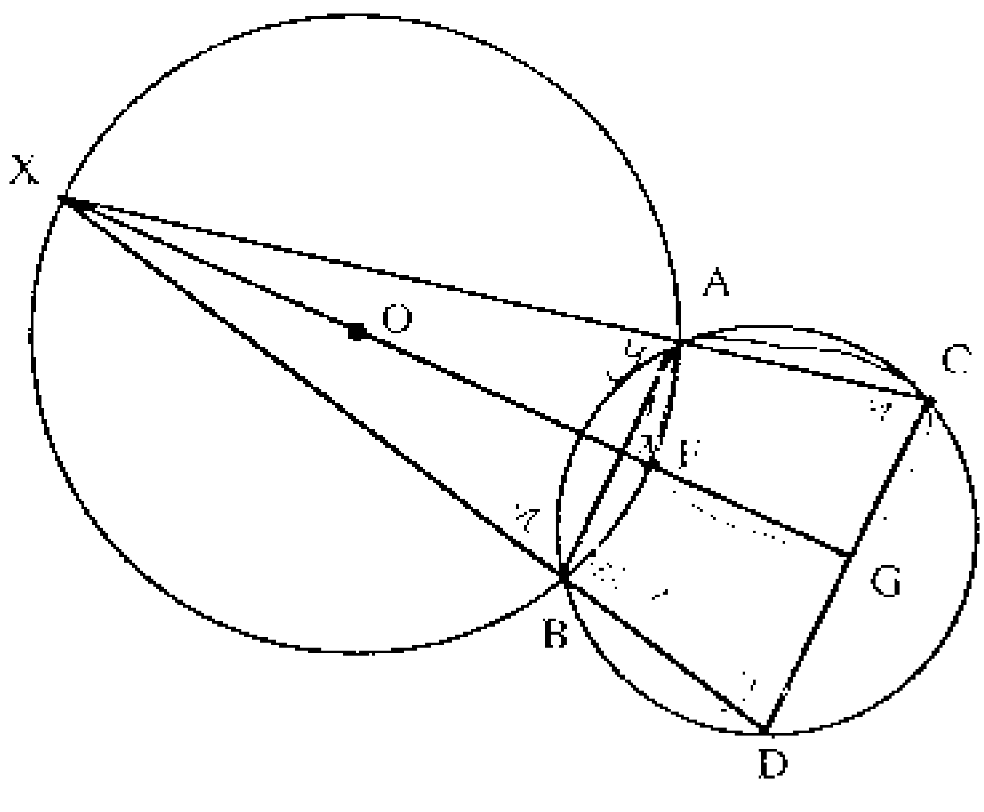
QUESTION 6 (START A NEW PAGE)

(a) Prove that $\dfrac{d^2x}{dt^2} = \dfrac{d\left(\frac{1}{2}v^2\right)}{dx}$.

(b) The acceleration of a particle x metres from origin O at time t seconds is given by $\ddot{x} = -\frac{1}{2}e^{x}$. If its velocity v, is 1 metre / second when $x = 0$, find its velocity when $x = 4$.

(c) Two circles cut at A and B. X is on the circle with centre O and XA, XB cut the other circle at C, D respectively. XO extended cuts the circle XAB at F and chord CD at G.



(i) Neatly copy the diagram onto your answer sheet.

(ii) Prove that ACGF is a cyclic quadrilateral.

(iii) Prove that XG is perpendicular to CD.

QUESTION 7 (START A NEW PAGE)

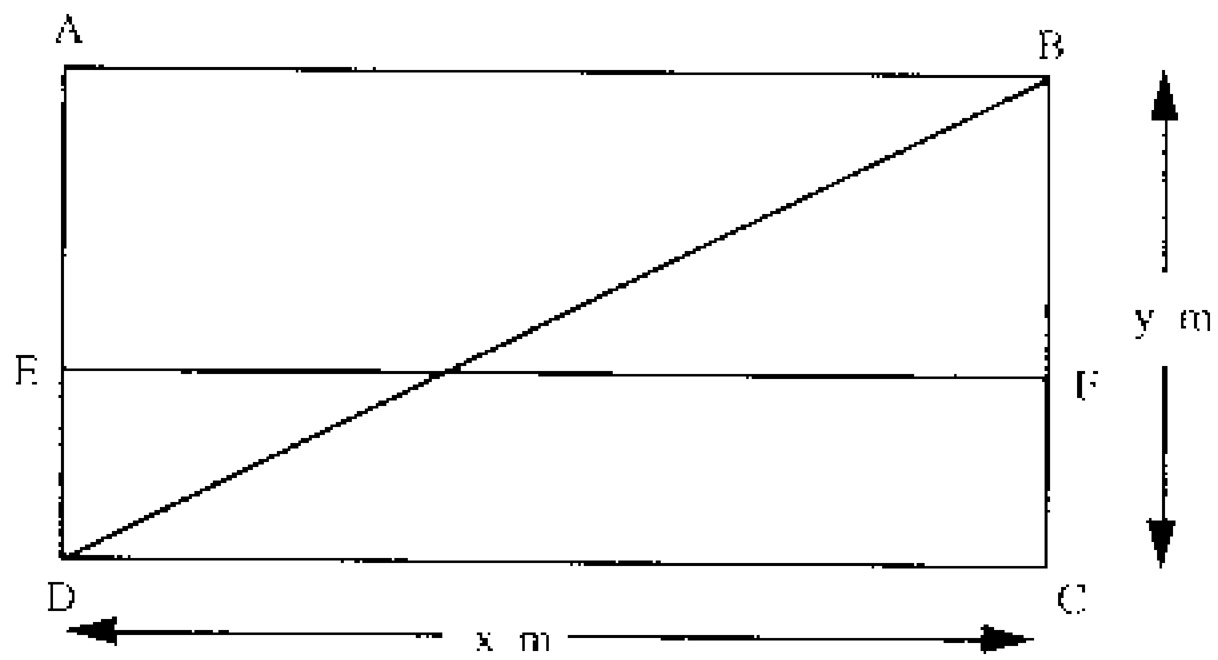
(a) The points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ and $p + q = 2$.

(i) Find the angle which PQ makes with the x axis.

(ii) Find the equation of the locus of M, the mid-point of PQ as it moves about the parabola.

(iii) Clearly state any restrictions on the locus of M.

(b) A rectangle ABCD with sides of length x metres and y metres has an area of $9\ m^2$. Two metal construction strips, one a diagonal BD and the other EF parallel to sides AB and CD are required to strengthen the rectangle.



(i) Show that the total length L, of BOTH strips is :

$$L = x + \frac{\sqrt{x^4 + 81}}{x} \text{ metres.}$$

(ii) Find the dimensions of the rectangle which will minimise the total length L of the strips.

THIS IS THE END OF THE PAPER

1993 3U Answers

Q 1

(a) $-2 < x < 3$

(b) -1, $\cos x > 0$

1, $\cos x < 0$

Undefined $\cos x = 0$

(c) $k = 4$

(d) –

(e) 0.0004

Q 2

(a) $\frac{10}{27}$

(b) $x - \frac{3}{2}\tan^{-1}\frac{x}{2} + c$

(c) –

Q 3 (a)(i) –

(ii) $T = \pi$

(b) (i) 56 40°

(ii) 725 76°

(c) (i) $R = 2$, $\alpha = \pi/3$

(ii) $x = 2n\pi - \pi/3$

4 a (i) D $-1 \le x \le 1$

R $-\frac{\pi}{2} \le y \le \pi/2$.

(ii) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

(b) (i) –

(ii) 4/3

5 (a) (i) –

(ii) 18.11 m/s

(b) (i) $\binom{12}{r} x^{-3r+2}$

(ii) $T_5 = 495$

6 (a) –

(b) $V = \pm\frac{1}{e^{4}}$ m/s

(c) –

7 (a) (i) 1

(ii) $x = 2a$

(iii) $y \geq a$

(b) (ii) $x = 3^{3/4}$, $y = 3^{5/4}$