J.R.A.H.S. TRIAL HSC EXAMINATION 3/4 UNIT MATHEMATICS 1994

QUESTION 1

(a) Solve $2x^2 > 2 - 3x$.

(b) Differentiate $\frac{1}{4 - x^2}$

(c) Find the exact value of $\int_0^{\frac{\pi}{4}} \cos x \sin^2 x \, dx$.

(d) A committee of 5 is to be chosen from 6 boys and 3 girls. Find the probability that the committee contains a particular boy X and a particular girl Y.

(e) Find the acute angle between the lines $y = 3x - 2$ and $x - 2y = 5$

Question 2 (START A NEW PAGE)

(a) (i) Draw a sketch of $y=2\sin^{-1}x$. State the domain and range.

(ii) A region R is bounded by the curve $y=2\sin^{-1}x$, the x-axis and the line $x = 1$.

Find the exact area of the region R.

(b) Find $\int_0^1 x^2(x^3 + 1)^4 \, dx$ using the substitution $u = x^3 + 1$

QUESTION 3 (START A NEW PAGE)

(a) A point moves along the curve $y = \ln x^2$. The x co-ordinate of the point changes at the rate of 4 units per second. At what rate is the y co-ordinate increasing when $x = 3$?

(b) Find the term independent of x in $(x - \frac{1}{x})^6$.

(c) When A and B play chess, the probability of either winning a game is always $\frac{1}{4}$ and the probability of the game being drawn is always $\frac{1}{2}$. Find the probability of A winning at least four games out of five.(Answer correct to four decimal places)

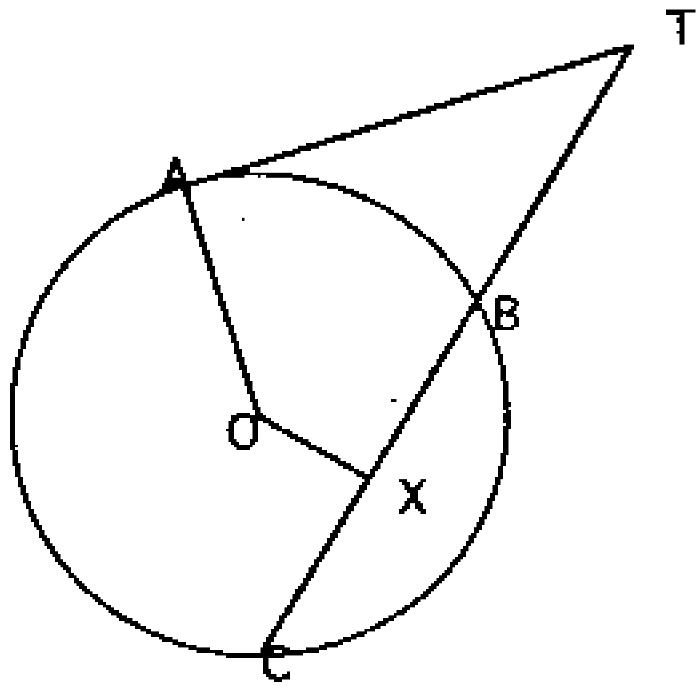
(d) Find the general solution to the equation $\tan\theta = \sin 2\theta$

QUESTION 4 (START A NEW PAGE)

(a) A,B,C are three points on a circle,centre O. The tangent at A meets CB produced at T. X is the mid point of BC .Prove that:-

(i) AOXT is a cyclic quadrilateral

(ii) $\angle AOT = \angle AXT$.



(b) $S_n = a + ar + ar^2 + \ldots\ldots ar^{n-1}$.

(i) Using mathematical induction or otherwise,prove that $S_n = \dfrac{a(1 - r^n)}{1 - r}$

(ii) Write down an expression for the limiting sum of a G.P.

(iii) State the values of r for which it exists.

(iv) If θ is not a multiple of $\frac{\pi}{2}$ and if p and q are given as sums of the following infinite geometric series:-

$$p = 1 + \cos^2\theta + \cos^4\theta + \ldots\ldots,$$

$$q = 1 + \sin^2\theta + \sin^4\theta + \ldots,$$

prove that $p + q = pq$.

QUESTION 5(START A NEW PAGE)

(a) A projectile travels in a parabolic path . The angle of projection is 60° and the velocity at which it is projected is 500 m/sec .

(i) Derive the equations of motion for the projectile in flight. (Air resistance is to be neglected and the acceleration due to gravity is to be taken as $10ms^{-2}$)

(ii) Find the range

(iii) Find the greatest height reached.

(b) P $(2ap,ap^2)$ and Q $(2aq,aq^2)$ are two points on the parabola $x^2 = 4ay$.

(i) Write down the equations of the normals at P and Q.

(ii) Find the co-ordinates of R,the point of intersection of the normals,in terms of p and q.

(iii) If $pq = -2$ find the cartesian equation of the locus of R.

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QUESTION 6 (START A NEW PAGE)

(a) A particle moving with simple harmonic motion makes 100 complete oscillations per minute. Its maximum speed is 10m per sec.Find:-

(i) the exact period of the motion

(ii) the amplitude of the motion.

(iii) the time taken to move from the centre of the oscillation to a point which is distant two-thirds of the amplitude from the centre (Give answer in secs to two significant figures)

(b) (i) Sketch the curve $y = 2\cos x - 1$ for $-\pi \le x \le \pi$. Mark clearly where the graph crosses each axis.

(ii) Find the volume generated by the rotation through a complete revolution about the x-axis of the region between the x-axis and that part of the curve $y = 2\cos x - 1$ for which $|x| \le \pi$ and $y \ge 0$.

Question 7 (START A NEW PAGE)

(a) The constant acceleration of a train is 1metre per sec per sec and its constant retardation is 3 metres per sec per sec.

i) Sketch a velocity-time graph assuming the train starts from rest accelerates in a straight line and immediately decelerates in a straight line until it is again at rest.

ii) Find the time taken for a journey of 1Km.given the journey described in (i)

(b) The integers a,b,d are connected by the relation $a = b + d$.

(i) Use the binomial expansion of $(b + d)^n$, where n is a positive integer, to show that $a^n - b^{n-1}(b + nd)$ is divisible by d^2.

(ii) In the result of part (i) replace b by $a - d$. Hence show that if a is the first term,d the common difference and l the nth term of an arithmetic progression, then $a^n - l\,(a-d)^{n-1}$ is divisible by d^2.

(iii) Deduce that $5^{682} - 2^{692}$ is divisible by 9.

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94 3U Answers

1(a) $x < -2,\ x > \frac{1}{2}$

(b) $\frac{2x}{(4-x^2)^2}$

(c) $\frac{1}{6\sqrt{2}}$

(d) $\frac{\binom{7}{3}}{\binom{9}{5}}$

(e) 45°

Q2 (a)(i) $D: -1 \le x \le 1$

$R: -\pi \le y \le \pi$

(ii) $\pi - 2$

(b) $\frac{31}{15}$

Q3 (a) $8/3$ u/s

(b) $C_4 = -20$

(c) 0.0156

(d) $\theta = n\pi \pm \pi/4$

4 (a) –

(b) (i) –

(ii) $\frac{a}{1-r}$

(iii) $|r| < 1$

(iv)

Q5 (a)(i) –

(ii) $12500\sqrt{3}$ m

(iii) 9375 m

(b)(i) –

(ii) $\left(-apq(p+q),\ a(p^2+pq+q^2+2)\right)$

(iii) $x^2 = 4a(y-4a)$

6(a) (i) 0.6 A

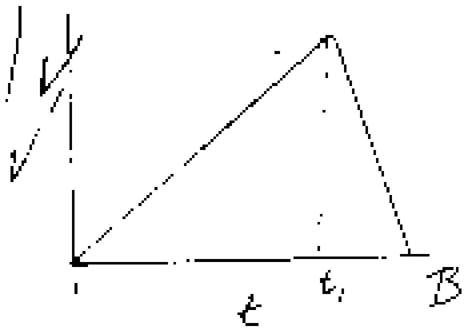
(ii) $a = \frac{3}{\alpha}$ m

(iii) 0.070 rad

(b) (i) –

(ii) $-3\sqrt{3}\pi + 2\pi^2$

7a (i) ✓



(ii) $\frac{2\sqrt{3}}{3}$ h

(b) —