J.R.A.H.S. TRIAL HSC EXAMINATION 3/4 UNIT MATHEMATICS 1995

QUESTION 1 (Start a new page)

(a) Express 0.373737....... as a proper fraction in simplest form.

(b) Find the size of the acute angle between the lines $2x + y = 5$ and $3x - y = 1$.

(c) Find all solutions to : $\frac{1}{x-2} \leq 4$.

(d) Differentiate with respect to x: $y = \tan^{-1} 2x$.

(e) Solve $2\cos^2 x + 3\sin x - 3 = 0$, for $0 \leq x \leq 2\pi$.

QUESTION 2 (Start a new page)

(a) Prove the identity: $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$.

(b) Given that $\frac{dy}{dx} = \frac{1}{1+x^2}$, and $x=1$ when $y=0$, find y when $x = \sqrt{3}$.

(c) Evaluate : $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3-x^2}}$

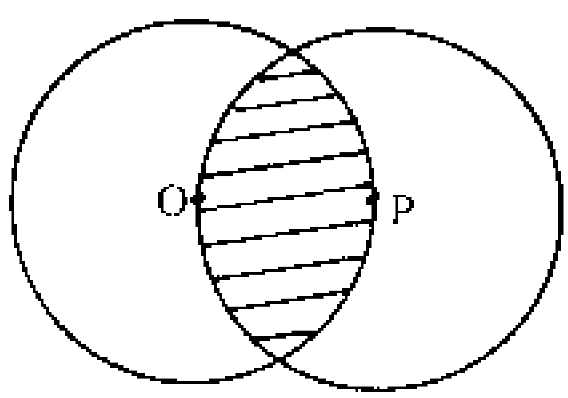
(d) Prove by mathematical induction that $3^{4n} - 1$ is divisible by 80 for all positive integers n.

QUESTION 3 (Start a new page)

(a) Let the equation of motion of an object moving x metres along a straight line after t seconds be: $x(t) = 4\sin 3t - 5\cos 3t$ $(t \geq 0)$. Show that its motion is Simple Harmonic, and find its period of motion.

(b) Evaluate : $\int_0^{\frac{\pi}{8}} \cos^2 2x\,dx$

(c) In the diagram shown, the two circles are of radius 1 metre and pass through centres O and P. Find the area of their intersection (to two decimal places).



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QUESTION 4 (Start a new page)

(a) If 5% of monkeys are colourblind, what is the probability that a random sample of 20 monkeys should contain at least two colourblind monkeys? (Answer to three decimal places.)

(b) A person invests \$1000 at the beginning of each year in a superannuation fund. If interest is paid at 9% per annum, find :

(i) the value of the investment at the end of 30 years.
(ii) how many years would elapse for the investment to be worth \$50,000.

(c) Neatly sketch $y = 3\cos^{-1}\pi x$, and state its domain and range.

QUESTION 5 (Start a new page)

(a) The acceleration of a particle is given by $\frac{d^2x}{dt^2} = 16(1 + x)$, where x cm. is the displacement from the origin. When $t = 0$, $x = 0$ and $v = 4$cm./ sec.

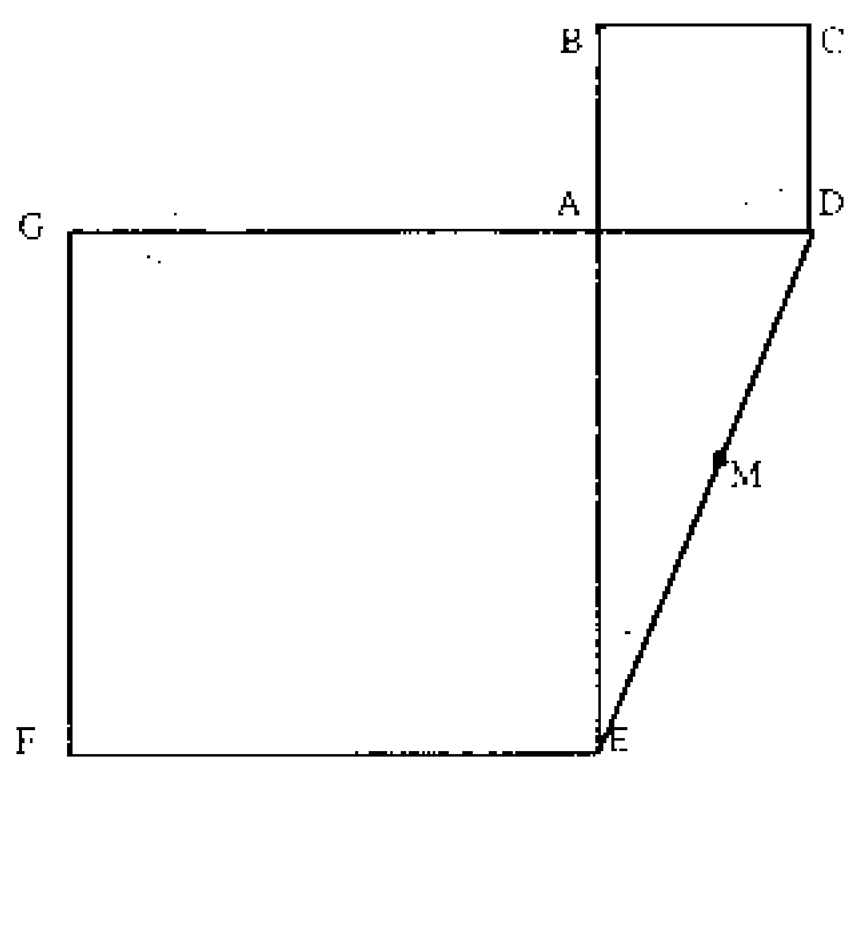
(i) Derive an expression for its velocity in terms of its displacement.
(ii) Deduce that its displacement function is $x(t) = e^{4t} - 1$.

(b) Evaluate $\cot 2\theta$ if $\cot^2\theta - \cot\theta = 1$.

(c) ABCD and AEFG are two squares of different areas, and $GD \perp BE$. M is the mid point of DE.

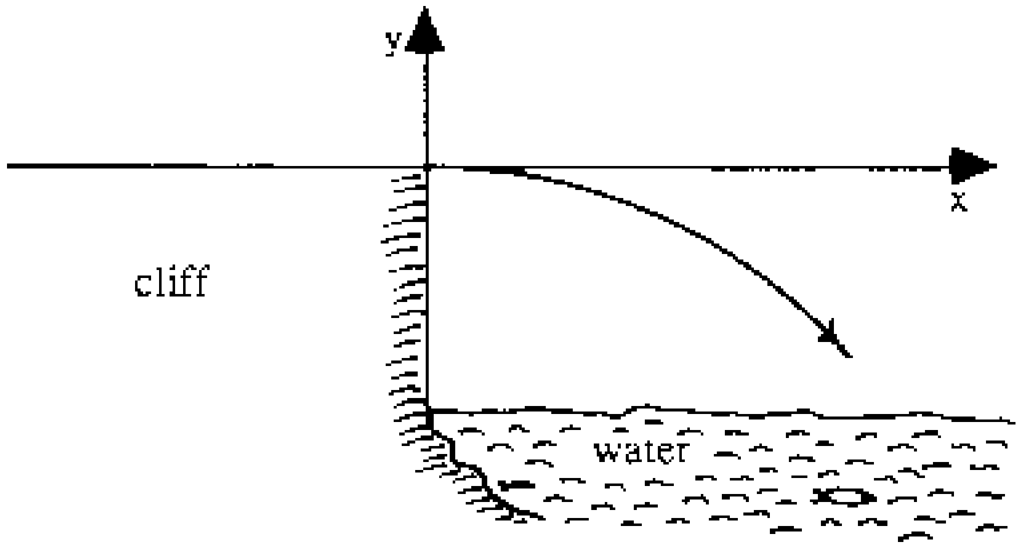
(i) Give a reason why DE is the diameter of the circle with points A,D and E on its circumference.
(ii) Prove that BDEG is a cyclic quadrilateral.
(iii) Prove that $AM \perp BG$.

Copy the diagram below onto your answer sheet.



QUESTION 6 (Start a new page)

(a) An object is projected horizontally from the top edge of a vertical cliff 40 metres above sea level with a velocity of 40m/s. (Take $g = 10\ m/s^2$)



(i) Using the top edge of the cliff as origin, prove that the parametric equations of the path of the object are:

$$x = 40t \qquad y = -5t^2$$

(ii) Calculate when and where the object hits the water.

(iii) Find the velocity (magnitude and direction) of the object the instant it hits the water.

(b) The inside of a vessel used for water has the shape of a solid of revolution obtained by the rotation of the parabola $9y = 8x^2$ about the y - axis. The depth of the vessel is 8 cm.

(i) Prove that a volume of water h cm. from its bottom is $\frac{9}{16}\pi h^2$.

(ii) If water is poured into the vessel at a rate of 20 cm^3/sec., find the rate at which the level of water is rising when the vessel is half full.

QUESTION 7 (Start a new page)

(a) Two parametric points $P(2p,p^2)$ and $Q(2q,q^2)$ lie on the parabola $x^2 = 4y$, and the line through PQ is parallel to the line $y = mx$.

(i) Show that $p + q = 2m$.

(ii) Derive the equation of the normal to the parabola at the point P.

(iii) Find the co–ordinates of N, the point of intersection of the normals from P and Q.

(iv) Determine the locus of N as the line PQ moves parallel to the line $y = mx$. State any restrictions on the locus of N.

(b) A_n and B_n are two series given by :

$$A_n = 1^2 + 5^2 + 9^2 + 13^2 + \ldots\ldots + (4n - 3)^2$$
$$B_n = 3^2 + 7^2 + 11^2 + 15^2 + \ldots\ldots \qquad \text{for } n = 1,2,3,\ldots\ldots$$

(i) Find the nth term of B_n.

(ii) If $S_{2n} = A_n - B_n$, prove that $S_{2n} = -8n^2$.

(iii) Hence, or otherwise, evaluate :

$$101^2 - 103^2 + 105^2 - 107^2 + \ldots\ldots + 1993^2 - 1995^2.$$

<u>END OF PAPER</u>

SOLUTIONS

1 (a) $\frac{37}{49}$

(b) $\pi/4$

(c) $x < 2$, $x \geq 9/4$

(d) $\frac{2}{1+4x^2}$

(e) $\pi/6$, $5\pi/6$, $\pi/2$

2 (a) —

(b) $\pi/12$

(c) $\pi/2$

(d) –

3 (a) $T = 2\pi/3$

(b) $\pi/16 + 1/8$

(c) 1.23 (2D)

4 (a) 0.264 (3D)

(b) (i) \$145 575

(ii) 19 (nearest yr)

(c) $D: -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

$R: 0 \leq y \leq 3\pi$

5 (a) (i) $v^2 = 4(1+x)$

(ii) —

(b) $\frac{1}{2}$

(c) –

6 (a) (i) –

(ii) $80\sqrt{2}$, $t = 2\sqrt{2}$ sec

(iii) $35°$ with speed $20\sqrt{6}$ m/s

(b) " $\frac{20\sqrt{2}}{4\pi}$ cm/s

7 (a) (i) –

(ii) $x + py = p^3 + 2p$

(iii) $N[-pq(p+q), p^2+q^2+pq+2]$

(iv) $y = \frac{x}{2m} + 4m^2 + 2$

Restriction $y > 3m^2 + 2$

$x > -2m^3$

(b) (i) $B_n = (4n-1)^2$

(ii) —

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