TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 1996

MATHEMATICS

3 UNIT / 4 UNIT COMMON PAPER

Time Allowed – Two Hours
(Plus 5 minutes reading time)

All questions may be attempted

All questions are of equal value

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

Standard integral tables are printed at the end of the examination paper and may be removed for your convenience; approved silent calculators may be used.

The answers to the seven questions are to be returned in separate bundles clearly labelled Question 1, Question 2 etc. Each bundle must show your Candidate's Number.

QUESTION 1 (12marks)

a) Differentiate with respect to x:

(i) $x^2 \ln(1 + x^2)$. (ii) $(\tan^{-1} x)^2$.

b) Find the exact value of :

(i) $\displaystyle\int_1^2 \frac{x^2+1}{x}\,dx$ (ii) $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$

c) (i) Write down an expansion for $\sin(x - y)$.

(ii) Hence, prove that $\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$.

QUESTION 2 (12marks)

Start question on a new page

a) Use the substitution $u = 1 - 2x$ to evaluate: $\displaystyle\int_0^{0.5} 2x(1-2x)^4\,dx$.

b) (i) Sketch the graph of the function $f(x) = 3\cos^{-1}(2x)$.

(ii) Write down the natural domain and corresponding range of the function.

c) R is the region bounded by the x-axis, between $x = 2$ and $x = -2$ and the curve with equation $y = \frac{1}{\sqrt{4+x^2}}$. The region R is rotated about the x-axis. Find the exact volume of the solid formed.

QUESTION 3 (12marks)

Start question on a new page

a) (i) Write $\cos x - \sqrt{3}\sin x$ in the form $A\cos(x + \alpha)$, where $A>0$, $0<\alpha<\pi$.

(ii) Hence, or otherwise, solve $\cos x - \sqrt{3}\sin x = 1$ for all values of x.

b) There are 8 green cards, 8 red cards and 8 yellow cards in a pack. 4 cards are selected at random without replacement. Find expressions for the probability that :

(i) 4 green cards are chosen.

QUESTION 4 (12marks)
Start question on a new page

a) The rate of growth of a bacteria colony is proportional to the excess of the colony's population over 5 000 and is given by:

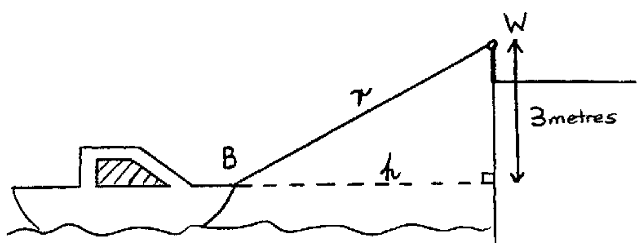
$$\frac{dN}{dt} = k(N - 5\,000).$$

(i) Show that $N = 5\,000 + Ae^{kt}$ is a solution of this differential equation.

(ii) The initial population is 15 000 and reaches 20 000 after 2 days. Find the value of A and k.

(iii) Hence, calculate the expected population after 7 days.

b) A boat is being winched towards a wharf by a rope attached to its bow, B. The winch W is 3 metres above B. The rope is wound in at the rate of 12 metres per minute. Find the rate at which the boat is approaching the wharf when the distance from the bow to the wharf is 5 metres (assume that the rope is stretched tightly in a straight line at all stages of the operation).



QUESTION 5 (12marks)
Start question on a new page

a) (i) Prove that $\cot x + \tan x = 2\operatorname{cosec} 2x$.

(ii) Hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec} 2x \, dx$.

(iii) Using the identity in part (i) above, prove $\cot 15^0 = 2 + \sqrt{3}$.

b) (i) Sketch the graph of $x = -a \cos nt$ where $0 \le t \le \frac{2\pi}{n}$ and a, n are positive constants, clearly marking the intercepts.

(ii) Given the displacement $x(t)$ of a particle is given by $x(t) = -a \cos nt$ prove $\ddot{x} = -n^2 x$.

(iii) Assume that tides rise and fall in simple harmonic motion. A ship needs 9 metres of water to pass down a channel safely. At low tide the channel is 8 metres deep and at high tide it is 12 metres deep. Low tide is at 7.00 am and high tide is at 1.30pm. During what times can the ship proceed safely?

QUESTION 6 (12marks)

Start question on a new page

a) (i) Draw a neat sketch of the region bounded by the line $y = x$ and the curve $y = \frac{3 - x^2}{2}$, marking the coordinates of the points of intersection.

(ii) Calculate the area of the region bounded by this line and the curve.

(iii) α) Given $f(x) = \frac{3 - x^2}{2}$, find the largest possible domain such that this function has an inverse.

β) State the domain and range of this inverse function.

γ) On your diagram, sketch this inverse function.

(i) Using the identity $(1 + x)^8(1 + x)^8 = (1 + x)^{16}$ or otherwise, show that

$$\binom{8}{0}^2 + \binom{8}{1}^2 + \binom{8}{2}^2 + \dots\dots\dots\dots + \binom{8}{8}^2 = \binom{16}{8}$$

(ii) Albert tosses a coin 8 times and quite independently Bruce also tosses a coin 8 times. Calculate the probability that the number of heads obtained by Albert equals the number of heads obtained by Bruce.

QUESTION 7 (12marks)

Start question on a new page

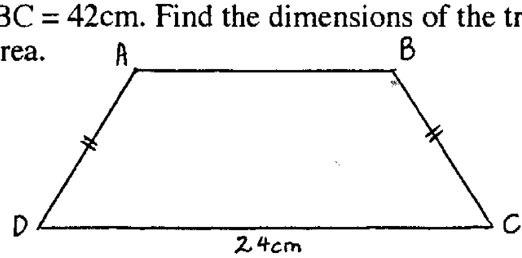
a) A kat on the large planet HEART, sits at a point P on the top of a wall which is 3.6 metres high. It sees a "nouse" on the ground. The "nouse" is exactly 4 metres from the base of the wall. The kat jumps horizontally from the top of the wall with initial velocity of 6 metres per second. Take the acceleration due to gravity g to be 20 metres per second ^{2}, neglect air resistance and assume that the jump of the kat can be modelled by a projectile particle.

(i) Find expressions for $x(t)$ and $y(t)$, the horizontal and vertical displacements of the kat from P after t seconds.

(ii) Find the time taken for the kat to reach the ground.

(iii) Find the distance by which the kat fails to reach the nouse.

b) Isosceles trapezia are drawn with side CD always 24cm, sides AD and BC equal and AD + AB + BC = 42cm. Find the dimensions of the trapezium with greatest possible area.



SOLUTIONS - 3UNIT TRIAL - 1996 - JRAHS (1)

Question 1

a) (i) $\frac{d}{dx}\left(x^2\ln(1+x^2)\right)$

[2] $= x^2 \cdot \frac{2x}{1+x^2} + \ln(1+x^2)\cdot 2x$

$= \frac{2x^3}{1+x^2} + 2x\ln(1+x^2)$

(ii) $\frac{d}{dx}\left(\tan^{-1}x\right)^2$

$= 2\left(\tan^{-1}x\right)\cdot\frac{1}{1+x^2}$ [2]

$= \frac{2\tan^{-1}x}{1+x^2}$

b) (i) $\int_1^2 \frac{x^2+1}{x}\,dx$

$= \int_1^2 \left(x+\frac{1}{x}\right)dx$ [3]

$= \left[\frac{x^2}{2}+\ln x\right]_1^2$

$= 2+\ln 2 - \frac{1}{2} - \cancel{\ln 1}$

$= \ln 2 + \frac{3}{2}$

(ii) $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$

$= \left[\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{3}}$

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}1$

$= \frac{\pi}{3} - \frac{\pi}{2}$ [2]

$= -\frac{\pi}{6}$

c) (i) $\sin(x-y) = \sin x\cos y - \cos x\sin y$ [1]

(ii) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$ [2]

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

Question 2

a) $\int_0^{1/2} 2x(1-2x)^4\,dx$

$u = 1-2x$, $x = \frac{1-u}{2}$, $dx = -\frac{1}{2}du$, $x=0 \Rightarrow u=1$, $x=\frac{1}{2} \Rightarrow u=0$

$= \int_1^0 2\left(\frac{1-u}{2}\right)u^4 \cdot -\frac{1}{2}\,du$

$= \int_0^1 \frac{1}{2}(1-u)u^4\,du$ [1]

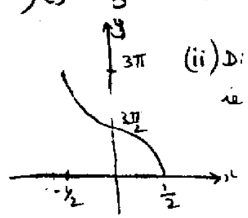
$= \frac{1}{2}\left[\frac{u^5}{5}-\frac{u^6}{6}\right]_0^1$ [1]

$= \frac{1}{2}\left(\frac{1}{5}-\frac{1}{6}\right)$

$= \frac{1}{60}$ [1]

total [5]

b) (i) $\frac{y}{3} = \cos^{-1}2x$ [4]



(ii) D: $-1 \le 2x \le 1$

ie $-\frac{1}{2} \le x \le \frac{1}{2}$

R: $0 \le \frac{y}{3} \le \pi$

ie. $0 \le y \le 3\pi$

(2)

2 c) $V = \pi\int_{-2}^{2}\frac{1}{4+x^2}\,dx$ [1]

$= \frac{\pi}{2}\left[\tan^{-1}\frac{x}{2}\right]_{-2}^{2}$ [1]

$= \frac{\pi}{2}\left[\tan^{-1}1 - \tan^{-1}(-1)\right]$

$= \frac{\pi}{2}\left[\frac{\pi}{4}+\frac{\pi}{4}\right]$

$= \frac{\pi^2}{4}$ [1]

b) (i) $P(4G) = \frac{{}^{8}C_4}{{}^{24}C_4} = \frac{70}{10624} = \frac{5}{759}$

OR

[2] $P(GGGG) = \frac{8}{24}\cdot\frac{7}{23}\cdot\frac{6}{22}\cdot\frac{5}{21}$

$= \frac{5}{759}$

(ii) P(at least 2 Green)

$= P(2G,2O) + P(3G,1O) + P(4G)$

$= \frac{{}^{8}C_2\cdot{}^{16}C_2}{{}^{24}C_4} + \frac{{}^{8}C_3\cdot{}^{16}C_1}{{}^{24}C_4} + \frac{{}^{8}C_4}{{}^{24}C_4}$

$\doteqdot 0.407$ [2]

Alternatively

P(at least 2 green)

$= 1 - \left[P(0\text{ green}) + P(1\text{ green})\right]$

$= 1 - \left[\frac{{}^{16}C_4}{{}^{24}C_4} - \frac{{}^{8}C_1\cdot{}^{16}C_3}{{}^{24}C_4}\right]$

$= 0.407$

Question 3

a) (i) $\cos x - \sqrt{3}\sin x = A\cos(x+\alpha)$

$= A\cos x\cos\alpha - A\sin x\sin\alpha$

$\therefore 1 = A\cos\alpha$, $\sqrt{3} = A\sin\alpha$

$\frac{1}{A} = \cos\alpha$, $\frac{\sqrt{3}}{A} = \sin\alpha$

$\therefore \frac{1}{A^2}+\frac{3}{A^2} = 1$

$4 = A^2$

$2 = A$ [2]

$\tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$

$\therefore \cos\alpha - \sqrt{3}\sin\alpha = 2\cos\left(x+\frac{\pi}{3}\right)$

(ii) Solve $\cos\alpha - \sqrt{3}\sin\alpha = 1$

ie $2\cos\left(x+\frac{\pi}{3}\right) = 1$

$\cos\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$

$x+\frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3}$, $n\in J$

ie. $x = 2n\pi - \frac{2\pi}{3}$ or $2n\pi$, $n \in J$

c) $T_{k+1} = {}^{30}C_k\cdot(2x)^{30-k}\cdot\left(-\frac{1}{x^2}\right)^k$

$= {}^{30}C_k\cdot 2^{30-k}\cdot x^{30-k}\cdot(-x)^{-2k}$

$= {}^{30}C_k\cdot 2^{30-k}\cdot(-1)^k\cdot x^{30-k-2k}$

ie $30-k-2k = 12$

$-3k = -18$

$k = 6$ [3]

$\therefore$ Coefficient of x^{12} term is:

${}^{30}C_6\cdot 2^{24}\cdot(-1)^6$

$= 30.\ .\ 2^{24}$

Question 4

a) (i) $N = 5000 + Ae^{kt}$ [2]

$\frac{dN}{dt} = k.\ Ae^{kt}$
$= k(N - 5000)$

(ii) When $t = 0$, $N = 15000$:

$N = 5000 + Ae^{kt}$
$15000 = 5000 + Ae^{0}$
$\underline{10000 = A}$

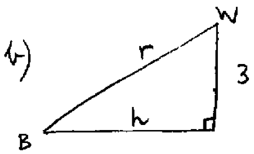
$\therefore N = 5000 + 10000e^{kt}$

When $t = 2$, $N = 20000$:

$20000 = 5000 + 10000e^{k\times 2}$
$\frac{15000}{10000} = e^{2k}$ [3]
$\ln 1{\cdot}5 = 2k$
$\frac{\ln 1{\cdot}5}{2} = k$
$(k \doteqdot 0{\cdot}20273)$

(iii) $N = 5000 + 10000e^{7\times\frac{\ln 1.5}{2}}$
$N = \underline{46335}$ (nearest number)
[2]

b)



$h^2 + 9 = r^2$
When $h = 5$:
$25 + 9 = r^2$
$\underline{\sqrt{34} = r}$

[5]

$\frac{dr}{dt} = 12$ m/s
$\frac{dh}{dt} = ?$ when $h = 5$

$h^2 + 9 = r^2$
$h = \sqrt{r^2 - 9}$

$\frac{dh}{dr} = \frac{2r}{2\sqrt{r^2-9}}$

ie $\frac{dh}{dr} = \frac{r}{\sqrt{r^2-9}}$

When $r = \sqrt{34}$:

$\frac{dh}{dr} = \frac{\sqrt{34}}{5}$

$\therefore$ By Chain Rule:

$\frac{dh}{dt} = \frac{dh}{dr} \cdot \frac{dr}{dt}$
$= \frac{\sqrt{34}}{5} \cdot 12$
$= \frac{12\sqrt{34}}{5}$ m/s
$(\doteqdot 13{\cdot}99$ m/s$)$

Question 5

a) (i) $\cot x + \tan x$

$= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$= \frac{1}{\sin x \cos x}$

$= \frac{2}{\sin 2x}$ [2]

$= \underline{2\operatorname{cosec} 2x}$

(ii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec} 2x\, dx$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} (\cot x + \tan x)\, dx$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} \left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right) dx$

$= \frac{1}{2}\Big[\ln(\sin x) - \ln(\cos x)\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \frac{1}{2}\left[\ln\left(\frac{\sin x}{\cos x}\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ [2]

$= \frac{1}{2}\Big[\ln(\tan x)\Big]_{\pi/6}^{\pi/3}$

$= \frac{1}{2}\left[\ln\left(\tan\frac{\pi}{3}\right) - \ln\left(\tan\frac{\pi}{6}\right)\right]$

$= \frac{1}{2}\left[\ln\sqrt{3} - \ln\left(\frac{1}{\sqrt{3}}\right)\right]$

$= \frac{1}{2}\ln\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$

$= \underline{\frac{1}{2}\ln 3}$

(iii) $\cot x + \tan x = 2\operatorname{cosec} 2x$

$\therefore \cot 15 + \tan 15 = 2\operatorname{cosec} 30°$

$\cot 15 + \frac{1}{\cot 15} = 2 \times 2$

$\cot^2 15 - 4\cot 15 + 1 = 0$

Let $y = \cot 15$:

$y^2 - 4y + 1 = 0$

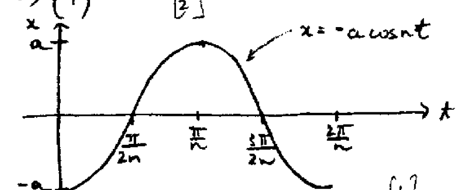
$y = \frac{4 \pm \sqrt{16 - 4.1.1}}{2}$

$y = \frac{4 \pm \sqrt{12}}{2}$ [2]

$y = 2 \pm \sqrt{3}$

$\therefore \underline{\cot 15 = 2 + \sqrt{3}}$ only

b) (i) [1]



[1]

Period $= \frac{2\pi}{n}$

(ii) $x = -a\cos nt$

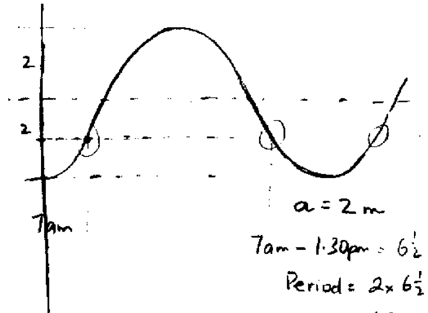
$\dot{x} = +an\sin nt$ [1]

$\ddot{x} = +an^2\cos nt$
$= -n^2(-a\cos nt)$

ie $\ddot{x} = -n^2 x$

$\therefore$ SHM

(iii)



$a = 2\,m$

7am – 1.30pm $= 6\frac{1}{2}$

Period $= 2 \times 6\frac{1}{2}$ hr

$= 13$

ie $\frac{2\pi}{n} = 13$

$\frac{2\pi}{13} = n$

$x = -a\cos nt$

ie $x = -2\cos nt$

When $x = -1$; (ie depth $= 8 + 1 = 9m$)

$-1 = -2\cos\frac{2\pi}{13}t$

$\frac{1}{2} = \cos\frac{2\pi}{13}t$

$\frac{2\pi}{13}t = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$t = \frac{13}{2\pi}\times\frac{\pi}{3}, \frac{13}{2\pi}\times\frac{5\pi}{3}, \frac{13}{2\pi}\times\frac{7\pi}{3}$

$t = 2\frac{1}{6},\ 10\frac{5}{6},\ 15\frac{1}{6}$

Alternatively:

$x = b - a\cos nt$

$9 = 10 - 2\cos\left(\frac{2\pi}{13}t\right)$

$-1 = -2\cos\left(\frac{2\pi}{13}t\right)$

$\therefore$ time boat can proceed is: [4]

$7 + 2\frac{1}{6}$ hr $=$ 9.10am

$7 + 10\frac{5}{6}$ hr $=$ 5.50pm

$\therefore$ 9.10am — 5.50pm

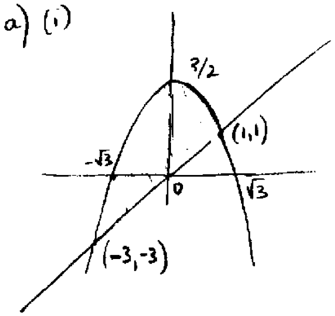
also boat can proceed again at:

$7 + 15\frac{1}{6} =$ 10.10pm

$\therefore$ 10.10 — 12 midnight (same day)

Question 6.

a) (i)



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(ii) $\int_{-3}^{1}\left(\left(\frac{3-x^2}{2}\right) - x\right)\,dx$

$= \frac{1}{2}\int_{-3}^{1}(3 - x^2 - 2x)\,dx$ 2

$= \left[3x - \frac{x^3}{3} - x^2\right]_{-3}^{1}$

$= \left(3 - \frac{1}{3} - 1\right) - (-9 + 9 - 9)$

$= 1\frac{2}{3} + 9$

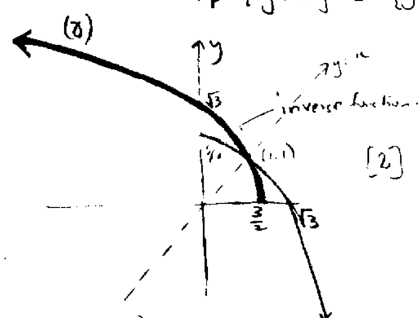
$= 10\frac{2}{3}\,u^2$

(iii) α) $x \geq 0$ Alt Sol^n OR $\{x \leq 0\}$

β) $D_F: \{x \leq \frac{3}{2}\}$ OR $\{x \leq \frac{3}{2}\}$

$R_F: \{y \geq 0\}$ OR $\{y \leq 0\}$

(γ)



[2]

$x = \frac{3-y^2}{2}$

$\Rightarrow 2x = 3 - y^2$

$y^2 = 3 - 2x$

b(i) Using $(1+x)^8(1+x)^8 = (1+x)^{16}$

ie $\left({}^8C_0 + {}^8C_1x + {}^8C_2x^2 + \dots + {}^8C_8x^8\right)\left({}^8C_0 + {}^8C_1x + {}^8C_2x^2 + \dots + {}^8C_8x^8\right) =$

${}^{16}C_0 + {}^{16}C_1x + {}^{16}C_2x^2 + \dots + {}^{16}C_{16}x^{16}$

RHS: Coefficient of x^8 term $= {}^{16}C_8$

LHS: Coefficient of x^8 terms $= {}^8C_0 \cdot {}^8C_8 + {}^8C_1 \cdot {}^8C_7 + {}^8C_2 \cdot {}^8C_6 + \dots$

$\dots + {}^8C_6 \cdot {}^8C_2 + {}^8C_7 \cdot {}^8C_1 + {}^8C_8 \cdot {}^8C_0$

[3] $= ({}^8C_0)^2 + ({}^8C_1)^2 + ({}^8C_2)^2 + \dots + ({}^8C_7)^2 + ({}^8C_8)^2$

as required

(ii) Using the binomial expansion $(q+p)^8$ where $q = p = \frac{1}{2}$ the probability that each of A, B (independently) tosses exactly r heads in 8 tosses of a coin is ${}^8C_r\, q^{8-r}p^r = {}^8C_r\left(\frac{1}{2}\right)^{8-r}\left(\frac{1}{2}\right)^r = {}^8C_r\left(\frac{1}{2}\right)^8$

Since the tossing of r heads by Albert & Bruce are independent events, then the probability that both toss exactly r heads in 8 tosses is:

${}^8C_r\left(\frac{1}{2}\right)^8 \times {}^8C_r\left(\frac{1}{2}\right)^8 = ({}^8C_r)^2 \cdot \left(\frac{1}{2}\right)^{16}$

Hence, the probability that Albert and Bruce both toss exactly:

0 heads: $({}^8C_0)^2\left(\frac{1}{2}\right)^{16}$

1 heads: $({}^8C_1)^2\left(\frac{1}{2}\right)^{16}$

2 heads: $({}^8C_2)^2\left(\frac{1}{2}\right)^{16}$

$\vdots$

8 heads: $({}^8C_8)^2\left(\frac{1}{2}\right)^{16}$

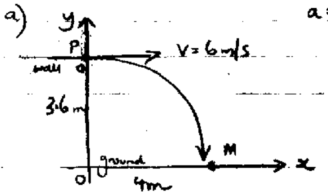
Thus, the probability that they toss the same number of heads (ie both 0 heads or 1 head or 2 heads or ... or 8 heads)

$= ({}^8C_0)^2\left(\frac{1}{2}\right)^{16} + ({}^8C_1)^2\left(\frac{1}{2}\right)^{16} + ({}^8C_2)^2\left(\frac{1}{2}\right)^{16} + \dots + ({}^8C_8)^2\left(\frac{1}{2}\right)^{16}$

$= \left(\frac{1}{2}\right)^{16}\left[({}^8C_0)^2 + ({}^8C_1)^2 + ({}^8C_2)^2 + \dots + ({}^8C_8)^2\right]$

$= \left(\frac{1}{2}\right)^{16} \cdot {}^{16}C_8$ (from i) [2]

Question 7

a) 

$a = 20\,m/s^2 \quad g = 20$

When $t=0$: $x=0, y=0, \dot{x}=6, \ddot{x}=0, \dot{y}=0, \ddot{y}=-20$

(i) $\ddot{x} = 0 \qquad \ddot{y} = -20$

$\dot{x} = 6 \qquad \dot{y} = -20t$

$x = 6t \qquad y = -10t^2 + 3.6$ [2]

(ii) Find t if $y = 0$.

$0 = -10t^2 + 3.6$ [2]

$10t^2 = 3.6$

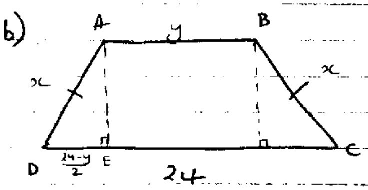
$t^2 = 0.36$

$t = 0.6$ $\quad \therefore$ time taken is 0.6 seconds

(iii) when $t = 0.6$, $x = 6 \times 0.6 = 3.6$ m

$\therefore 4 - 3.6 = 0.4$ [2]

$\therefore$ Cat misses the mouse by 0.4 m

b) 

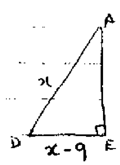
$2x + y = 42$

$y = 42 - 2x \quad \text{---(1)}$ [6]

Now: $DE = \dfrac{24-y}{2}$

$= \dfrac{24 - (42-2x)}{2}$ (from (1))

$\therefore DE = x - 9$



Also: $AE = \sqrt{x^2 - (x-9)^2}$

$= \sqrt{x^2 - x^2 + 18x - 81}$

$\therefore AE = \sqrt{18x - 81}$

Let Area of Trapezium be A

$\therefore A = \frac{1}{2}(y + 24) \times AE$

$= \frac{1}{2}(42 - 2x + 24)\sqrt{18x - 81}$

$= \frac{1}{2}(66 - 2x)\sqrt{18x - 81}$

$= (33 - x)\sqrt{18x - 81}$

For max Area, $\dfrac{dA}{dx} = 0$:

$\dfrac{dA}{dx} = (33-x)\cdot\frac{1}{2}(18x-81)^{-\frac{1}{2}}\cdot 18 + (18x-81)^{\frac{1}{2}}\cdot(-1)$

$= (18x-81)^{-\frac{1}{2}}\left[9(33-x) - (18x-81)\right]$

$= (18x-81)^{-\frac{1}{2}}\left[378 - 27x\right]$

$= \dfrac{378 - 27x}{\sqrt{18x-81}}$

ie $\dfrac{dA}{dx} = 0$ when $\dfrac{378-27x}{\sqrt{18x-81}} = 0$

$378 - 27x = 0$

$x = 14$

Check $x = 14$ is a max:

x	12	14	16
$\frac{dA}{dx}$	4.6	0	-3.7
	/	—	\

$\therefore$ relative max at $x = 14$

As this is a continuous function, then $x = 14$ is the absolute maximum.

Dimensions of Trapezium with greatest area is: