JAMES RUSE AHS

GESTA NON VERBA

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 1997

MATHEMATICS

3 UNIT / 4 UNIT COMMON PAPER

Time Allowed - Two Hours
(Plus 5 minutes reading time)

All questions may be attempted

All questions are of equal value

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

Standard integral tables are printed at the end of the examination paper and may be removed for your convenience. Approved silent calculators may be used.

The answers to the seven questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc. Each bundle must show your Candidate's Number.

QUESTION 1 (Start a new page)

(a) Fully factorise $2x^4 - 54x$.

(b) The gradient function of a curve is $\frac{dy}{dx} = x^2 - 1$ and the curve passes through the point $(2, 1)$. Find the equation of the curve.

(c) Differentiate :

(i) $y = e^{[illegible]}$

(ii) $y = \cos^2 x$

(d) Solve $\tan 4\theta = \tan\theta$ for all real θ.

(e) State the DOMAIN and RANGE of $f(x) = 3\cos^{-1}(2x)$

QUESTION 2 (Start a new page)

(a) Find the 8 th term in the expansion of $(2 + 3x)^{12}$.

(b) (i) On the same axes sketch the graphs of
$y - 2x = 0$ and $y + \cos x = 0$ for $-\pi \le x \le \pi$.

(ii) Use the graph to deduce the number of solutions to $2x + \cos x = 0$.

(c) Differentiate $y = \log_e \left(\frac{2x}{(x-1)^2}\right)$.

(d) Use the substitution $u = e^x$ to find $\int \frac{e^x}{1 + e^{2x}}\, dx$

QUESTION 3 (Start a new page)

(a) Use the table of Standard Integrals provided as a guide to find,

$$\int \frac{\sin 2x}{\cos 2x}\, dx$$

(b) Given that $f(x) = 1 + x^2$ for $x \ge 0$. Find an expression for $f^{-1}(x)$, the inverse of $f(x)$.

(c) The surface area of a sphere is increasing at a constant rate of $6\text{cm}^2/\text{sec}$. At what rate is its volume increasing when its radius is 5cm.

(d) Find the exact volume generated when the region bounded by the functions $y = e^x$, $x = \log_e 2$ and the co-ordinate axes is rotated about the x axis.

QUESTION 4 (Start a new page)

(a) The equation of motion of an object moving x metres along a fixed straight line after t seconds is given by $x(t) = 3 + 4\sin(2t)$

(i) Show that its motion is Simple Harmonic.

(ii) Find its speed when it passes through its centre of motion.

(iii) Where is the object when its acceleration is maximum?

(b) Find the exact area bounded by the curve $y = 3\sin^{-1}(2x)$, the x-axis and the line $x = \frac{1}{2}$

(c) A bag contains 8 Red, 7 White and 5 Black marbles. If three marbles are drawn together from the bag, find the probability that they contain exactly two white marbles.

QUESTION 5 (Start a new page)

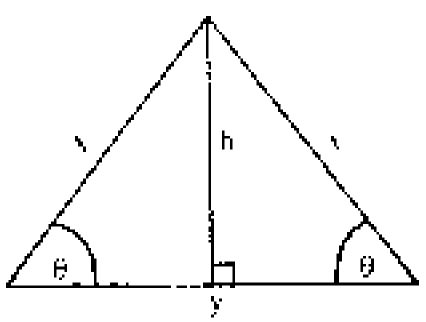
(a) At what points on the curve $y = \cos^{-1}x$ is the gradient $-\frac{2}{\sqrt{3}}$.

(b) How many ways can the letters of the word EQUATION be arranged if,
(i) there are no restrictions.
(ii) the word ATE appears in the arrangement.
(iii) the letters Q,U,E are not together.

(c) It is given that $f(x) = 1 + \log_e(x+1)$ and $g(x) = \sqrt{x}$ for $0 \le x \le 4$.
If $D = f(x) - g(x)$ is the vertical distance between the two curves, find the minimum length of D.

QUESTION 6 (Start a new page)

(a)



The perimeter of the isosceles triangle shown is four times its height. Its sides are x, x and y units long, its height h units and the base angles θ degrees.
Find θ to the nearest degree.

(b) (i) Prove that: $\frac{dv}{dt} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$

(ii) The time, in seconds, for a particle to move x metres along a straight line is given by: $t(x) = \sqrt{x^2 - 1}$ for $x > 0$

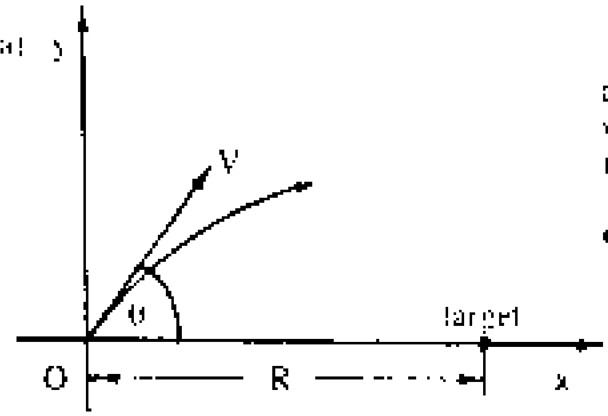
(α) What is its initial position?

(β) Show that its velocity function is given by: $v(x) = \frac{\sqrt{x^2-1}}{x}$.

(γ) Find its acceleration as a function of x.

QUESTION 7 (Start a new page)

(a)



A projectile is fired from O at an angle θ to the horizontal with initial velocity V m/s to strike a target R metres right of O on level ground.
Given the components of its displacement from O after t seconds is

$$x = Vt\cos\theta$$
$$y = -\frac{gt^2}{2} + Vt\sin\theta$$

(i) If the projectile is to hit the target, prove that:
$$\tan^2\theta - \left(\frac{2V^2}{gR}\right)\tan\theta + 1 = 0.$$

(ii) Show that the target will be hit from two angles of projection, say θ_1 and θ_2, if $R < \frac{V^2}{g}$.

(iii) Let the respective times of flight for each path be t_1 and t_2.
By considering the roots of the equation in (i) or otherwise, prove that:
$$t_1^2 + t_2^2 = \frac{4V^2}{g^2}$$

(b) A person makes an investment by depositing \$1 on the first day, \$2x on the second day, \$3x² on the third day and continues the process for 1998 days. The total amount, S, of the investment is

$$S = 1 + 2x + 3x^2 + 4x^3 + \cdots\cdots + 1998x^{1997} \quad \text{for } x > 1$$

The sum can be expressed as:

$$S = \frac{Ax^{B} - Bx^{A} + 1}{(x-1)^2}$$

Find the value of $A + B$.

END of PAPER

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QUESTION 1

(a) $2x(x-3)(x^2+3x+9)$

(b) $y=\frac{1}{3}x^3-x+\frac{1}{3}$

(c) (i) $\sec x\tan x\, e^{\sec x}$

(ii) $-2\sin x\cos x$

(d) $\theta=\frac{n\pi}{3}$, n an integer

(e) D $\{x: -\frac{1}{2}\le x\le\frac{1}{2}\}$

R $\{y: 0\le y\le 3\pi\}$

QUESTION 2

(a) ${}^{12}C_7\, 2^5(3x)^7$

$=55\,427\,328\,x^7$

(b) (i) –

(ii) one solution

(c) $-(x+1)/x(x-1)$

(d) $\tan^{-1}(e^x)+c$

QUESTION 3

(a) $\frac{1}{2}\sec 2x+c$

(b) $f^{-1}(x)=\sqrt{1-x}$

(c) $15\text{ cm}^3/\text{s}$

(d) $3\pi/2\ u^3$

QUESTION 4

(a) (i) $\ddot{x}=-4(x-3)$

(ii) 8 m/s

(iii) $x=-1$ m

(b) $\frac{3}{4}(\pi-2)\ u^2$

(c) $\frac{91}{380}$

QUESTION 5

(a) $\left(-\frac{1}{2},\frac{2\pi}{3}\right),\left(\frac{1}{2},\frac{\pi}{3}\right)$

(b) (i) $8!=40320$

(ii) $3!6!=4320$

(iii) $5.4.3.2.3!=720$

(c) $\ln 2$

QUESTION 6

(a) $\theta\doteqdot 53°$

(b) (i) —

(ii)(α) $x=1$

(β) —

(γ) $\ddot{x}=\frac{-1}{x^3}$

QUESTION 7

(a) (i) —

(ii) —

(iii) —

(b) $S=\frac{1998x^{1999}-1999x^{1998}+1}{(x-1)^2}$

$\therefore A+B=3997$