

QUESTION 1: (Start a new page)

- (a) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$
- (b) Express $\log\left(\frac{x^3 y^2}{\sqrt{z}}\right)$ in terms of a , b and c if $\log x = a$, $\log y = b$, and $\log z = c$.
- (c) For the function $y = 2\sin^{-1}\left(\frac{x}{3}\right)$ state the:
- Domain
 - Range
 - Draw a neat sketch of the curve: $y = 2\sin^{-1}\left(\frac{x}{3}\right)$
- (d) Find all values of θ (in radians) if $\sqrt{3} \sin \theta = \cos \theta$

QUESTION 2: (Start a new page)

- (a) If $y = 10^x$, find $\frac{dy}{dx}$ when $x = 1$
- (b) Evaluate in terms of π $\int_0^1 \frac{dx}{x^2 + 3}$
- (c) From eight teachers and six pupils a committee of seven is to be formed. How many committees can be selected if both teachers and pupils are represented and the teachers are in the majority?
- (d) Given that $y = \sin^{-1}(x^2)$, find $\frac{d^2y}{dx^2}$

QUESTION 3: (Start a new page)

- (a) A manufacturer produces computer components of which 85% are found to be satisfactory. From a sample of 10 components:
- Find the probability that at most 1 fails to meet the specification.
 - At least 2 are unsatisfactory.
- Give your answers to part (i) and (ii) correct to 2 decimal places.
- (b) A vessel is being filled at a variable rate $\frac{dV}{dt} = k(A - V)$ where k and A are constants.
- Show that $V = A(1 - e^{-kt})$ is a solution of the differential equation above.
 - Find the capacity of the vessel.
 - Find the value of k if $\frac{1}{8}$ of the vessel is filled in 6 minutes.
 - Find the fraction of the vessel filled in the next 6 minutes.

QUESTION 4: (Start a new page)

- (a) The velocity (m s^{-1}) of a body moving in a straight line is given by $V = e^t - e^{-t}$ where t is the time in seconds. If its initial position is at the origin:
- Find the equation relating x (the displacement from O) and t .
 - Find the initial acceleration.
 - Show that the body does not have a maximum velocity.
 - Find the time taken to reach a point 3m to the right of the origin. Give your answer correct to 1 decimal place.
- (b) The region enclosed by the curve $y = \tan x$, the x -axis and the ordinate $x = \frac{\pi}{4}$ is rotated about the x -axis. Using Simpson's Rule with 5 function values, find an approximate value (to 1 dec. pl) for the volume of the solid formed.

QUESTION 5: (Start a new page)

- (a) A spherical balloon is being inflated. When the radius of the balloon is 6cm its volume is increasing at the rate of $100\text{cm}^3/\text{sec}$. Find the rate at which its surface area is then increasing.
- (b) (i) Find the equation of the tangent to the curve $y = \frac{x+1}{x^2+3}$ at the point where the curve cuts the x axis
(ii) Show that the tangent meets the curve again at a point where the function has a stationary point.

QUESTION 6: (Start a new page)

- (a) Use the substitution $u = \sin x$ to evaluate the integral $\int \cos^3 x \, dx$.
- (b) When $(3+2x)^n$ is expanded as a polynomial in x , the coefficients of x^5 and x^6 have the same value. Find the value of n .
- (c) Prove by induction that $\cos(x+n\pi) = (-1)^n \cos x$ for integer $n \geq 1$

QUESTION 7: (Start a new page)

- (a) Ron put \$500 savings into a Bank for 2 years, where it earned interest at 6% p.a., paid twice a year. He then changed to a Credit Union and his money earned 8% p.a., paid quarterly. If he withdrew all his savings, and had \$633.75, how long was the money kept in the Credit Union?
- (b) A boy throws a ball vertically and it just reaches a height of 40 metres. What is the greatest distance that he is able to throw it on a horizontal plane? (Let $g = 10\text{ms}^{-2}$)

END OF PAPER

QUESTION 1

(a) $3/2$

(b) $3a + 2b - \frac{1}{2}c$

(c) (i) D. $-3 \leq x \leq 3$

(ii) R $-\pi \leq y \leq \pi$

(iii) -

(d) $n\pi + \frac{\pi}{6}$, n an integer

QUESTION 2

(a) $10 \ln 10$

(b) $\frac{\pi}{0.3}$

(c) ${}^8P_4 + {}^8P_5 + {}^8P_6$

(d) $\frac{2(1+x^4)}{(1-x^2)^{3/2}}$

QUESTION 3

(a)(i) = 0.54

(ii) = 0.46

(b)(i) -

(ii) max. capacity = A

(iii) $-t \ln \frac{7}{8} = 0.022$

(iv) $\frac{7}{8}y$.

QUESTION 4

(a)(i) $x = e^t + e^{-t} - 2$

(ii) $\dot{x} = e^t - e^{-t}$

(iii) $t \rightarrow \infty, v \rightarrow \infty$

no max. speed

(iv) $t = \ln\left(\frac{5+\sqrt{21}}{2}\right)$

≈ 1.6

(b) $v = 0.7 u^2$

QUESTION 5

(a) $33\frac{1}{3} \text{ cm}^2/\text{s}$

(b)(i) $x - 4y + 1 = 0$

(ii) ~~(-1, 0)~~ ^{double the x} ~~(-1, 0)~~ ^(-1, 0)

QUESTION 6

(a) $\sin x - \frac{1}{3} \sin^3 x + C$

(b) $n = 14$

(c) -

QUESTION 7

(a) $n = 1.5 \text{ gms}$

(b) 80 m.