Name:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2011

MATHEMATICS EXTENSION 1

Time Allowed – 2 Hours
(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators and templates are permitted
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate *stapled* bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

Question 1 (12 Marks) **Marks**

(a) Find $\frac{d}{dx}(\tan 4x)$. **2**

(b) Find the co-ordinates of the point that divides the interval joining $A(7,2)$ and $B(11,6)$ externally in the ratio 3:5. **2**

(c) Evaluate $\lim_{x \to 0} \frac{3\sin x \cos x}{4x}$. **2**

(d) Solve $\cos 2x = -\frac{1}{2}$ for $0 \le x \le 2\pi$. **2**

(e) If $x = 1 + \cos\theta$ and $y = 2 - \sin\theta$ find a relationship between x and y only. **2**

(f) Evaluate $\int_0^{2\sqrt{3}} \frac{dx}{4+x^2}$. **2**

Question 2 **START A NEW PAGE** (12 Marks) **Marks**

(a) Using all the letters of the word MATHEMATICS, how many different arrangements can be made. **2**

(b) The temperature, $T°$ centigrade, of a pie t minutes after being placed in an oven is given by the formula $T = 180 + Be^{kt}$. Initially the temperature of the pie is $5° C$ and after 15 minutes the temperature has risen to $40° C$.

(i) Find the value of the constant B. **1**

(ii) Find the exact value of the constant k. **2**

(iii) Find the temperature of the pie one hour after being placed in the oven. Give your answer correct to the nearest degree. **3**

(c) (i) On the same set of co-ordinate axes draw neat sketches of the graphs $y = x$ and $y = \frac{2}{x-1}$. **2**

(ii) Hence or otherwise solve $x > \frac{2}{x-1}$. **2**

Question 3 START A NEW PAGE (12 Marks) **Marks**

(a) A district squad of 9 netball players is chosen from 3 netball teams (A, B and C). There are 8 players in each of the teams A, B and C.

(i) If 4 players are chosen at random from team A, 3 from team B and 2 from team C, in how many ways can the district squad be formed? **2**

(ii) Find the probability that Janice from team B and Sarah from team C will be chosen as members of the district squad. **2**

(b) Solve $\sec^2 x + \tan x - 7 = 0$ for $0° \le x \le 360°$. Give your answers correct to the nearest minute. **3**

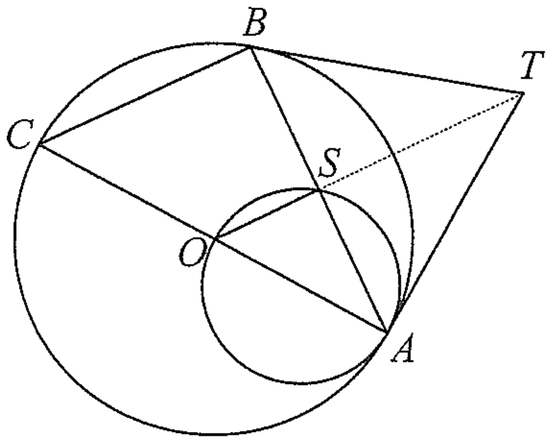
(c) (i) By equating coefficients, find the values of P and Q in the identity $P(2\sin x + \cos x) + Q(2\cos x - \sin x) \equiv 7\sin x + 11\cos x$. **2**

(ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \frac{7\sin x + 11\cos x}{2\sin x + \cos x}\, dx$. **3**

Question 4 START A NEW PAGE (12 Marks) **Marks**

(a) Evaluate $\int_0^1 \frac{x}{(2x+1)^2}\, dx$ using the substitution $u = 2x + 1$. **4**

(b) Two circles touch at point A. The small circle passes through the centre O of the large circle. AB is a chord of the large circle and cuts the small circle at S. AC is a diameter of the large circle. AT and BT are tangents to the large circle. (See diagram)



(i) Prove that CB is parallel to OS. **2**

(ii) Hence prove that $BS = SA$. **2**

(iii) Find the size of $\angle OSA$. **1**

(iv) Prove that the points O, S and T are collinear. **3**

Question 5 START A NEW PAGE (12 Marks) **Marks**

(a) Given that A, B, C and D are the vertices of a cyclic quadrilateral, find the value of $\cos A + \cos B + \cos C + \cos D$. 2

(b) Use the Principle of Mathematical Induction to prove that $11^n - 2^{2n}$ is divisible by 7 for all integers $n \geq 1$. 4

(c) The arc of the curve $y = \sin^{-1} x$ that lies in the positive quadrant is rotated one revolution about the y-axis to form the surface of a container.

(i) If the container is filled to a depth of h metres, show that the volume, $V\ m^3$, of water in the container is given by: $V = \dfrac{\pi}{4}(2h - \sin 2h)$. 3

(ii) The container is being filled at a rate of $6\,m^3/hr$. Calculate the rate at which the depth of water is increasing when the depth is $\dfrac{\pi}{6}\,m$. 3

Question 6 START A NEW PAGE (12 Marks) **Marks**

(a) In a small rural community two hobby farms provide eggs for the local grocer. The grocer makes up cartons containing one dozen eggs, always using 8 eggs from farm A and 4 eggs from farm B. Some of the eggs contain two yolks (called a "double-yolker" egg). Eggs from farm A have an 18% probability of being a double-yolker while the probability for farm B is 24%.

(i) If an egg is chosen at random from one of the cartons, show that there is a 20% probability that it will be a double-yolker. 2

(ii) Find the probability that a carton chosen at random will have exactly three double-yolker eggs. Give your answer correct to the nearest percent. 2

(iii) Find the probability that a carton chosen at random will have at least three double-yolker eggs. Give your answer correct to the nearest percent. 2

(b) Masses are placed at two points A and B which are 1 metre apart. A 1 kg mass (M) is placed at a point P between A and B. The mass M experiences forces of attraction towards both the points A and B. The force (in Newtons) of the attraction towards A is equal to four times the distance AP while the force of attraction towards point B is equal to the square of the distance PB.
Take the origin of the motion at point A and the positive direction of motion in the direction of the ray AB.

(i) The mass M at point P is initially x metres from the origin A. Briefly explain why the acceleration, $\ddot{x}$ m/s, of the mass M is given by: $\ddot{x} = x^2 - 6x + 1$. 1

(ii) If the mass M now starts from rest halfway between A and B, in which direction will it begin to move? Briefly explain you answer. 2

(iii) Find the speed of the mass M when it first reaches point A. 3

Question 7 START A NEW PAGE (12 Marks) **Marks**

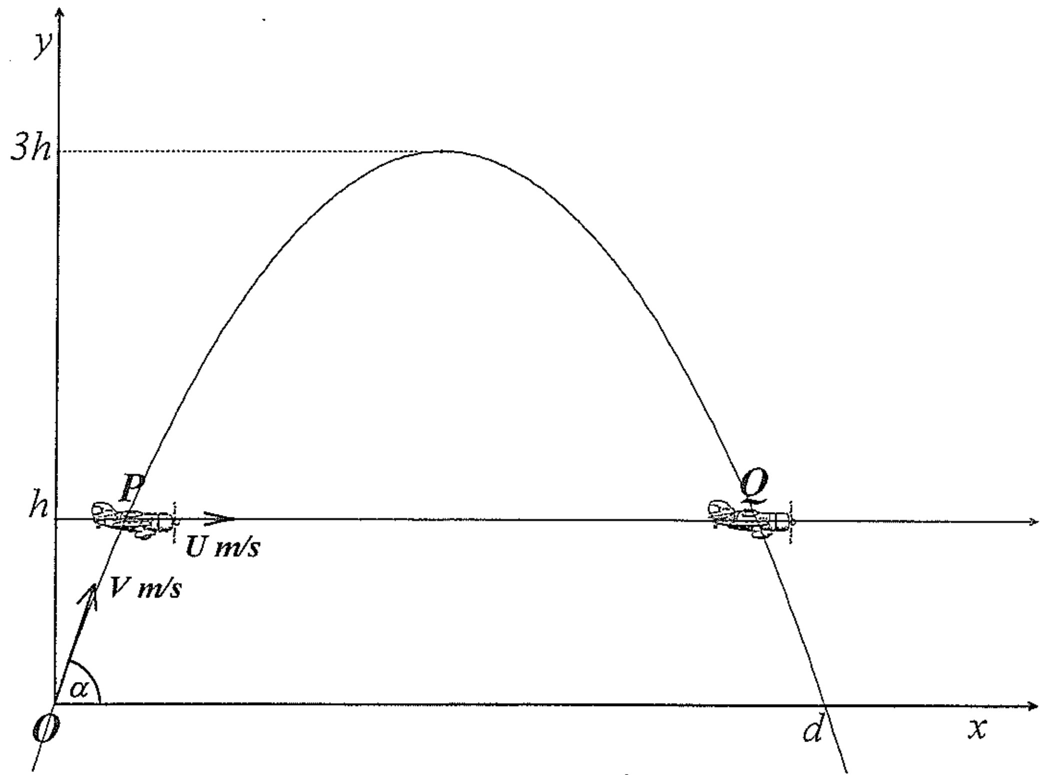
(a) Find the value of the constant term in the expansion of $\left(2y-\frac{1}{y^3}\right)^{20}$. **3**

(b) An enemy plane is flying horizontally at height h metres with speed U m/s.

When it is at point P a ground rocket is fired towards it from the origin O with speed V m/s and angle of elevation α.

The rocket misses the plane, passing too late through the point P. However, it goes on to reach a maximum height of $3h$ metres and then on its descent strikes the plane at Q.

With the axes as shown in the diagram, you may assume that the position of the rocket is given by: $x = Vt\cos\alpha$ and $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$, where t is the time in minutes after firing and g is the acceleration due to gravity.



(i) Show that the initial vertical velocity component $(V\sin\alpha)$ of the rocket's speed equals $\sqrt{6gh}$. **2**

(ii) If the rocket had not struck the plane at Q, it would have returned to the x-axis at a distance d metres from O. **2**

Show that the horizontal component $(V\cos\alpha)$ of the rocket's speed equals $\frac{gd}{2\sqrt{6gh}}$.

(iii) Show that the equation of the path of the rocket is $y = \frac{12hx}{d}\left(1-\frac{x}{d}\right)$. **2**

(iv) If the horizontal component of the rocket's speed is $100\left(3+\sqrt{6}\right)$ m/s, find the time taken by the rocket to strike the plane at Q, in terms of d. **2**

(v) Find the speed of the enemy plane. **1**

3U TRIAL

MATHEMATICS Extension 1 : Question 1 2011

Suggested Solutions	Marks	Marker's Comments
a) $\frac{d}{dx}(\tan 4x) = 4\sec^2 4x$	2	if they integrated or use inverse trig → 0 marks
b) $A(7,2)$ $B(11,6)$		
$3:-5$		
$P = \left(\frac{7\times -5 + 3\times 11}{3+-5}, \frac{2\times -5 + 3\times 6}{3+-5}\right)$	½	* If they did an internal division and got $(8\frac{1}{2}, 3\frac{1}{2})$ → 1mk
$= \left(\frac{-35+33}{-2}, \frac{-10+18}{-2}\right)$	½	
$= \left(\frac{-2}{-2}, \frac{8}{-2}\right)$	½	
$= (1, -4)$	½	
c) $\lim_{x\to 0} \frac{3\sin x\cos x}{4x}$		
$= \lim_{x\to 0} \frac{6\sin x\cos x}{8x}$	½	
$= \lim_{x\to 0} \frac{3\sin 2x}{8x}$	½	
$= \lim_{x\to 0} \frac{\sin(2x)}{(2x)} \cdot \frac{3}{4}$	½	
$= 1 \times \frac{3}{4}$		
$= \frac{3}{4}$	½	
or $\lim_{x\to 0} 3 \times \frac{\sin x}{x} \times \frac{\cos x}{1} \times \frac{1}{4}$	1	
$= 3 \times 1 \times 1 \times \frac{1}{4}$	½	
$= \frac{3}{4}$	½	
d) $\cos 2x = -\frac{1}{2}$ for $0 \le x \le 2\pi$		* If they used general solutions, then they had to use $\cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$ as $\cos^{-1}x$ is defined $0 \le x \le 2\pi$
$2x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 3\pi - \frac{\pi}{3}, 3\pi + \frac{\pi}{3}$		
$= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$		
$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$	½ mk each	

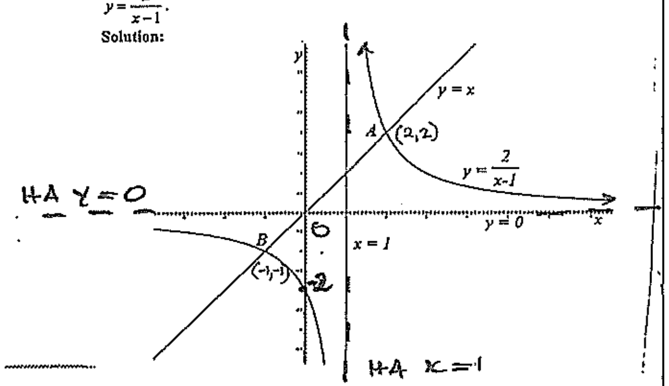
MATHEMATICS Extension 1 : Question 1 2011

Suggested Solutions	Marks	Marker's Comments
e) $x = 1 + \cos\theta$ $y = 2 - \sin\theta$		* If they left their answers in terms of inverse trig fns, they got a maximum of 1mk (as these answers are not complete).
$x - 1 = \cos\theta$ $2 - y = \sin\theta$	½	
$\cos^2\theta + \sin^2\theta = 1$	½	
$\therefore (x-1)^2 + (2-y)^2 = 1$	1	
or $2 - \sqrt{2x - x^2} = y$ or $x^2 - 2x + y^2 - 4y = 4$		
f) $\int_0^{2\sqrt{3}} \frac{dx}{4+x^2} = \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^{2\sqrt{3}}$	½	
$= \frac{1}{2}\left(\tan^{-1}\frac{2\sqrt{3}}{2} - \tan^{-1}0\right)$	½	
$= \frac{1}{2}\left(\frac{\pi}{3} - 0\right)$	½	
$= \frac{\pi}{6}$	½	

S11 TRIAL HSC **MATHEMATICS Extension 1 : Question** 2

Suggested Solutions	Marks	Marker's Comments
Q2		
a) MATHEMATICS		
No. of arrangements $= \frac{11!}{2!\times 2!\times 2!}$	1	
$= 4989600$	1	
b)(i) $T = 180 + Be^{kt}$		
when $t=0$, $T=5$		
$5 = 180 + Be^{0}$		
$B = -175$	1	
(ii) $T = 180 - 175e^{kt}$		$-\frac{1}{15}\ln\left(\frac{4}{5}\right)$ lost ½ mark
When $t=15$, $T=40$		
$40 = 180 - 175e^{15k}$		
$175e^{15k} = 140$		
$e^{15k} = \frac{140}{175}$		
$e^{15k} = \frac{4}{5}$	1	
$15k = \ln\left(\frac{4}{5}\right)$		
$k = \frac{1}{15}\ln\left(\frac{4}{5}\right)$	1	
(iii) $T = 180 - 175e^{\frac{1}{15}\ln\left(\frac{4}{5}\right)t}$		
When $t=60$		
$T = 180 - 175e^{\frac{1}{15}\ln\left(\frac{4}{5}\right)\times 60}$	1	
$= 180 - 175e^{4\ln\left(\frac{4}{5}\right)}$		
$= 108.32$	1	
$\therefore$ Temp. $= 108°C$ (nearest degree)	1	No Celsius lost ½ mark

page 2 **MATHEMATICS Extension 1 : Question** 2

Suggested Solutions	Marks	Marker's Comments
Qu 2		
(c) (i) On the same set of co-ordinate axes draw neat sketches of the graphs $y = x$ and $y = \frac{2}{x-1}$.		½ mark for $y = \frac{2}{x-1}$ with y intercept -2
Solution:		½ mark for $y = x$
	2	½ mark for H.A. $y=0$ ½ mark for V.A. $x=1$
(c)(ii) Solve $x > \frac{2}{x-1}$		1 mark for each region
At A and B $x = \frac{2}{x-1}$		
$x^2 - x - 2 = 0$		
$(x-2)(x+1) = 0$		
$x = -1$ or 2		
$\therefore$ For $x > \frac{2}{x-1}$		
$-1 < x < 1$ or $x > 2$	2	
II OR $x > \frac{2}{x-1}$		
$\frac{2}{x-1} < x$		
$\frac{2}{(x-1)} \times (x-1)^2 < x(x-1)^2$		
$x(x-1)^2 - 2(x-1) > 0$		
$(x-1)[x(x-1)-2] > 0$ 		
$(x-1)(x+1)(x-2) > 0$		
$\therefore -1 < x < 1$ or $x > 2$		
III or $x - \frac{2}{x-1} > 0 \Rightarrow \frac{x^2 - x - 2}{x-1} > 0$	$x \neq 1$ $\Rightarrow$	$(x-1)(x-2)(x+1) > 0$

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MATHEMATICS 3U 2011 JRAHS TRIALS

EXTENSION **MATHEMATICS: Question** 3

Suggested Solutions	Marks	Marker's Comments
a) (i) Number of ways	$3\times\frac{1}{2}$	for each product
$= {}^8C_4 \times {}^8C_3 \times {}^8C_2 = 109\,760$	$\frac{1}{2}$	for final answer
(ii) P(Sarah and Janice)	$\frac{1}{2}$	for ${}^7C_2$
$= \dfrac{{}^8C_4 \times {}^7C_2 \times {}^7C_1}{{}^8C_4 \times {}^8C_3 \times {}^8C_2} = \dfrac{10290}{109\,760} = \dfrac{3}{32}$	$\frac{1}{2}$	for ${}^7C_1$
	$\frac{1}{2}$	for sample space
OR $P(S \text{ and } J) = P(S)\times P(J)$	$\frac{1}{2}$	for final answer
$= \frac{3}{8}\times\frac{2}{8}$		max 1 for $\frac{1}{7}$
$= \frac{3}{32}$		max $\frac{1}{2}$ for $\frac{3}{23}$
b) $\sec^2 x + \tan x - 7 = 0$		
$\tan^2 x + \tan x - 6 = 0$	$\frac{1}{2}$	for using $\sec^2 x = \tan^2 x + 1$
$(\tan x + 3)(\tan x - 2) = 0$	$\frac{1}{2}$	for correct factorisation
$\tan x = -3$ or $\tan x = 2$	$\frac{1}{2}$	for $\tan x$ values
Reference angles : $71^\circ 34'$ and $63^\circ 26'$	$\frac{1}{2}$	for correct acute $\angle$
Hence Solution Set is :		
$\{108^\circ 26';\ 288^\circ 26';\ 63^\circ 26';\ 243^\circ 26'\}$	$\frac{1}{2}$	each for correct, corresponding pair
• General Solution : $x = n\pi + \tan^{-1}(2)$ or $x = n\pi + \tan^{-1}(-3)$		• If 1 $\angle$ omitted $-\frac{1}{2}$
For $[0, 360^\circ]$, start with $n = 0, 1, 2$ etc		• If $\tan x = 3; -2$, then max $2\frac{1}{2}$ if corresponding $\angle$'s correct
$1 + \sin x\cos x - 7\cos^2 x = 0$		• If $1 + \sin x\cos x - 7\cos^2 x = 0$ $\frac{1}{2}$
$\sin^2 x + \sin x\cos x - 6\cos^2 x = 0$		

$\because 1 \equiv \sin^2 x + \cos^2 x$.

MATHEMATICS 3U 2011 JRAHS TRIALS

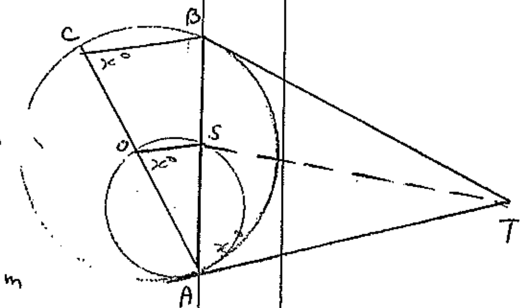
EXTENSION **MATHEMATICS: Question** 3

Suggested Solutions	Marks	Marker's Comments
c) (i) Expanding and factoring :		
$(2P - Q)\sin x + (P + 2Q)\cos x \equiv 7\sin x + 11\cos x$	$\frac{1}{2}$	for expanding and factoring
Equating coefficients of like terms :		
$2P - Q = 7 \quad \cdots (\text{i})$	$\frac{1}{2}$	for both equations
$P + 2Q = 11 \quad \cdots (\text{ii})$		
(ii)×2 : $2P + 4Q = 22 \quad \cdots (\text{iii})$		
(iii) − (i) : $5Q = 15$		
$Q = 3$	$\frac{1}{2}$	for Q value
$\Rightarrow P = 5$	$\frac{1}{2}$	for P value
		• Every mistake $-\frac{1}{2}$
(ii) From (i) :		• Careless mistakes very evident
$\displaystyle\int_0^{\pi/2} \frac{7\sin x + 11\cos x}{2\sin x + \cos x}\,dx = \int_0^{\pi/2} \frac{5(2\sin x + \cos x) + 3(2\cos x - \sin x)}{(2\sin x + \cos x)}\,dx$	$\frac{1}{2}$	• Use of substitution technique popular! Using values from (i)
$\displaystyle= \int_0^{\pi/2}\left[5 + \frac{3(2\cos x - \sin x)}{(2\sin x + \cos x)}\right]dx$	$\frac{1}{2}$	for splitting fraction and simplifying
$\displaystyle= \int_0^{\pi/2}\left[5 + \frac{3\{\frac{d}{dx}(2\sin x + \cos x)\}}{(2\sin x + \cos x)}\right]dx$	$\frac{1}{2}$	for recognising $\frac{f'(x)}{f(x)}$ form
$= \Big[5x + 3\ln\lvert 2\sin x + \cos x\rvert\Big]_0^{\pi/2}$	$\frac{1}{2}$	for each integral
$= \frac{5\pi}{2} + 3\ln\lvert 2\times 1 + 0\rvert - [5\times 0 + \ln\lvert 2\times 0 + 1\rvert]$		
$= \frac{5\pi}{2} + 3\ln 2$	$\frac{1}{2}$	for final answer

• $\frac{5\pi}{4} + 3\ln 2$ … max $2\frac{1}{2}$

• generally well answered

Suggested Solutions	Marks	Marker's Comments
$\frac{1}{2}\int_0^1 \frac{2x\,dx}{(2x+1)^2}$ $\quad u=2x+1 \quad du=2dx$; $x=0, u=1$; $x=1, u=3$		
$= +\frac{1}{2}\int_1^3 \frac{u-1}{u^2}\cdot\frac{du}{2}$	1m	Some students write $\int\frac{2x\,dx}{2x+1}$ made Q easy max 2m
$= \frac{1}{4}\int_1^3 \frac{u-1}{u^2}\,du$	1m	
$= \frac{1}{8}\int_1^3 \frac{2u\,du}{u^2} - \frac{1}{4}\int_1^3\frac{du}{u^2}$		
$= \frac{1}{8}\left[\ln u^2\right]_1^3 + \left[\frac{1}{4u}\right]_1^3$	1m	
$= \frac{1}{8}\ln 3^2 + \frac{1}{4}\left(\frac{1}{3}-1\right)$		
$= \frac{2}{8}\ln 3 - \frac{1}{6}$ #	1m	



Suggested Solutions	Marks	Marker's Comments
i) Method 1		
$\angle CBA = \angle TAB$ (angle between tangent & chord equals to angle at circumference in alternate segment)		Students can't prove $\angle CBA = \angle OSA$ correctly can't get the last mark.
Similarly $\angle TAB = \angle OSA$ ($\angle AOS$)	1m	
$\therefore \angle CBA = \angle OSA$		
$\therefore CB \parallel OS$ (2 lines are parallel if their corresponding angles are equal)	1m	
Method 2		
when 2 circles touch at a point, line through centre of one circle to point of contact will pass through centre of second circle		Students can't prove AO is diameter of small circle max 1m only

Suggested Solutions	Marks	Marker's Comments
$\therefore$ AO is diameter of small circle	1m	
$\angle OSA = 90^\circ$ (angle in semi-circle)		
Since CO is diameter of big circle		
similarly $\angle CBA = 90^\circ$		
$\therefore \angle OSA = \angle CBA$		
$\therefore CB \parallel OS$ (2 lines are parallel if corresponding angles are equal)	1m	
ii) Method 1 $CO = AO$ (radii of same circle)		
$\therefore CO/AO = 1$		
$CB \parallel OS$ (proved in i)	1m	
$\therefore \frac{AS}{BS} = \frac{CO}{AO} = 1$ (line parallel to one side of triangle divides the other 2 sides in same ratio)	1m	
$\therefore AS = BS$ #		
Method 2		
In ΔOAS, ΔCBA		
$\angle OSA = \angle CBA = 90^\circ$ (proved in i)		
$\angle OAS = \angle CAB$		
$\therefore \Delta OAS \parallel\!\mid \Delta CBA$ (equiangular)	1m	
$\frac{AO}{AC} = \frac{AS}{BS}$ (corresponding sides of similar triangles are in same ratio)		many students forgot same ratio $-\frac{1}{2}$m
$\frac{AO}{AC} = \frac{1}{2}$ (radius is half diameter in big circle)		
$\therefore \frac{AS}{BS} = \frac{1}{2}$		
w/ $\frac{AS}{AS+BS} = \frac{1}{2}$ $\quad \therefore AS = BS$	1m	

Suggested Solutions	Marks	Marker's Comments
Method 3		
$\angle OSA = 90°$ (proved in i)		
$OS \perp AB$	1m	
$\therefore BS = AS$ (line from centre of circle perpendicular to chord bisects it)	1m	
ii) $\angle CBA = 90°$ (angle in semi-circle)		MANY has proved in part i or ii
$\angle OSA = \angle CBA$ ($CB \parallel OS$, corresponding angles equal)		
$\therefore \angle OSA = 90°$	1m	
v) In $\triangle BTS$ & $\triangle ATS$		
$TA = TB$ (tangents to a circle from an external point are equal)		
TS is common	1m	
$BS = AS$ (proved in ii)		
$\therefore \triangle BTS \equiv \triangle ATS$ (SSS)	$\frac{1}{2}$m	Some students prove $\angle TAS = \angle TBS$ instead of TS $\triangle TSB \equiv \triangle TSA$ (SAS)
$\therefore \angle TSB = \angle TSA$ (corresponding angles of congruent triangles)	$\frac{1}{2}$m	
$\angle BSA = 180°$ (angle sum of straight angle)		Some students assumed OST is st. $\angle BST = \angle OSA$ (vertically opp. angles) max $\frac{1}{2}$m or $\triangle OAT$
$\therefore \angle TSB = \angle TSA = \frac{180°}{2} = 90°$		
$\angle OSA = 90°$ (proved in iii)		
$\angle OSA + \angle TSA = 90° + 90° = 180°$		
$\angle OST$ is a straight angle		
$\therefore O, S, T$ are collinear #.	1m	must state straight angle for collinear $-\frac{1}{2}$m

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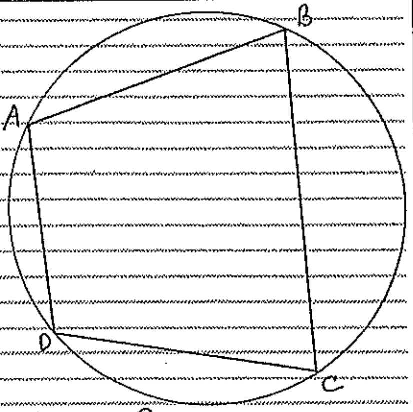
MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
		
$C = 180°$ (Opposite ∠les of a cyclic quadrilateral supplementary)	½	½ for geometry reason
$A = \cos(180° - C)$		
$A = -\cos C$		
Similarly		
$B = -\cos D$		
$\cos A + \cos B + \cos C + \cos D$		
$= -\cos C - \cos D + \cos C + \cos D \underline{\underline{= 0}}$	1½	

Suggested Solutions	Marks	Marker's Comments
To prove $11^n - 2^{2n}$ is divisible by 7 for $n \geq 1$		Although most people had the general idea, there was some very slack presentation.
Step 1 Consider $n = 1$		
$LHS = 11^1 - 2^2$		
$= 11 - 4$		
$= 7$ which is divisible by 7		
$\therefore$ True for $n = 1$	1	
Step 2 Assume true for $n = k$ where $k \in J$		
ie. $11^k - 2^{2k} = 7A$ where A integer		
RTP $11^{k+1} - 2^{2(k+1)} = 7B$ some other integer B		
Now, $11^{k+1} - 2^{2(k+1)} = 11 \cdot 11^k - 4 \cdot 2^{2k}$		
$= 11(7A + 2^{2k}) - 4 \cdot 2^{2k}$ by Assumption	½	
$= 11 \times 7A + 2^{2k}(11 - 4)$		
$= 7(11A + 2^{2k})$	2	
$= 7B$ where $B = 11A + 2^{2k}$ is an integer		

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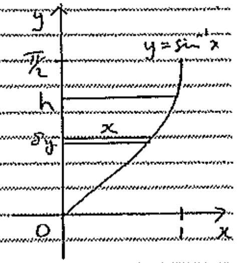
MATHEMATICS Extension 1 : Question 5 (cont)

Suggested Solutions	Marks	Marker's Comments
Thus, if true for $n = k$, also true for $n = k+1$.		
Step 3 Using steps 1 and 2, by the Principle of Mathematical Induction, thus proved $11^n - 2^{2n}$ is divisible by 7 for $n \geq 1$	½	

Suggested Solutions	Marks	Marker's Comments
c) i) $\delta V \doteq \pi x^2 \delta y$		
		
$V = \pi \int_0^h x^2 \, dy$	1	
$= \pi \int_0^h \sin^2 y \, dy$		
$= \frac{\pi}{2} \int_0^h 1 - \cos 2y \, dy$		
$= \frac{\pi}{2} \left[y - \frac{1}{2} \sin 2y \right]_0^h$	1	
$= \frac{\pi}{2} \left(h - \frac{1}{2} \sin 2h - 0 + 0 \right)$		
$= \frac{\pi}{4} (2h - \sin 2h)$	1	
ii) Find $\frac{dh}{dt}$ given that $\frac{dV}{dt} = 6$ (m³/hr)		$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$
$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{dV}{dt} \Big/ \frac{dV}{dh}$ (Chain Rule)	1	It would have been nice to see "Chain Rule" written
$\frac{dV}{dh} = \frac{\pi}{4}(2 - 2\cos 2h) = \frac{\pi}{2}(1 - \cos 2h)$		
$= \frac{\pi}{2}\left(1 - \frac{1}{2}\right)$ when $h = \pi/6$		
$= \pi/4$	1	
$\therefore \frac{dh}{dt} = 6 / (\pi/4) = \frac{24}{\pi}$		Too few people did not finish the answer in sentence form
$\therefore$ Depth increasing at rate of $\frac{24}{\pi}$ m/hr.	1	

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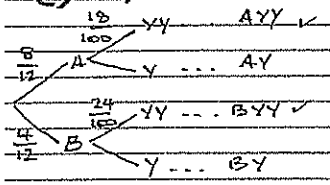
MATHEMATICS Extension 1 : Question 6

Suggested Solutions | **Marks** | **Marker's Comments**

6(a) (i) Farm A: has 8 YYs, FARM B has 4 YYs



$P(E = YY) = P(AYY \text{ or } BYY)$ — $\frac{1}{2}$

$= P(AYY) + P(BYY)$

$= \frac{8}{12} \times \frac{18}{100} + \frac{4}{12} \times \frac{24}{100}$ — $\frac{1}{2} + \frac{1}{2}$

$= \frac{2}{3} \times \frac{9}{50} + \frac{1}{3} \times \frac{6}{25}$

$= \frac{3}{25} + \frac{2}{25} = \frac{5}{25}$

$= \frac{1}{5} = 0.2$ — $\frac{1}{2}$

$\therefore$ Probability of YY is 20% q.e.d.

$\boxed{2}$

(ii) Now $P(YY) = p = 0.2$ — $\frac{1}{2}$

$\therefore P(\overline{YY}) = q = 0.8$ ✓

Using $(q+p)^{12} = (0.8 + 0.2)^{12}$ Bino Prob.

$P(X = 3\text{ YYs}) = \binom{12}{3} q^9 p^3 = \binom{12}{3}(0.8)^9(0.2)^3$ — $\frac{1}{2}$ For $\binom{12}{3}$

$= 0.2362232 01\ldots$ — $\frac{1}{2}$

$\doteqdot 24\%$ (nearest %) — $\frac{1}{2}$

Marker's comments: Many misinterpreted the Q, and got "24%"

$\boxed{2}$

(iii) $P(X \geq 3\text{ YYs}) = 1 - [P(X=0) + P(X=1) + P(X=2)]$ — $\frac{1}{2}$

at least 3 YYs

$= 1 - \left[\binom{12}{0}(0.8)^{12} + \binom{12}{1} 0.8^{11}\, 0.2^{1} + \binom{12}{2}(0.8)^{10}(0.2)^2\right]$

$= 1 - [0.068719\ldots + 0.206158\ldots + 0.283467\ldots]$ — 1

$= 1 - 0.55834574 8\ldots$

$= 0.44165425 1\ldots$

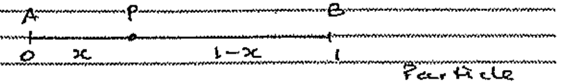
$P(X \geq 3YY) \doteqdot 44\%$ (nearest %) — $\frac{1}{2}$

$\boxed{2}$

MATHEMATICS Extension 1 : Question 6

Suggested Solutions | **Marks** | **Marker's Comments**

Q6(b)



(i) mass: $m = 1$

Resultant force towards B $= 1\ddot{x} = F_B - F_A$ — $\frac{1}{2}$

$1\ddot{x} = (1-x)^2 - 4x$

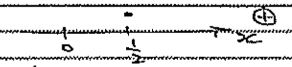
$\therefore \ddot{x} = 1 - 2x + x^2 - 4x$ — $\frac{1}{2}$

i.e. $\ddot{x} = x^2 - 6x + 1$ q.e.d.

Marker's comments: $-\frac{1}{2}$ if no mention of mass of 1 kg

$\boxed{1}$

(ii) $t = 0 \quad x = \frac{1}{2} \quad v = 0$



$\therefore \ddot{x} = \left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 1 = \frac{1}{4} - 3 + 1$

$\ddot{x} = -1\frac{3}{4} < 0$

$\therefore$ acceleration is $-1\frac{3}{4}\ \text{ms}^{-2}$

$\therefore$ Applied force is to the left ($\ddot{x} < 0$) but as starting from rest ($v = 0$)

$\therefore$ motion of particle is towards the left (towards A)

Marker's comments: $\frac{1}{2}$ For $-1\frac{3}{4}$. $\frac{1}{2}$ For Force to the L. $\frac{1}{2}$ For initially at rest. $\frac{1}{2}$ For motion to A.

$\boxed{2}$

$F_A = 2\text{ N} \quad F_B = \frac{1}{4}\text{ N}$

$F_A > F_B$

$\therefore$ resultant Force applied to the L (1)

(iii) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = x^2 - 6x + 1 \equiv (x-3)^2 - 8$

$\therefore \frac{1}{2}v^2 = \frac{1}{3}x^3 - 3x^2 + x + c \quad ; \quad \frac{1}{3}(x-3)^3 - 8x + k$ ✓ — 1

$x = \frac{1}{2} \quad v = 0 \Rightarrow c = \frac{5}{24} \quad ; \quad k = 9\frac{5}{24}$

$\therefore v^2 = \frac{2}{3}x^3 - 6x^2 + 2x + \frac{5}{12} \ ; \ v^2 = \frac{2}{3}(x-3)^3 - 16x + 18\frac{5}{12}$ ✓ — 1

* when $x = 0 \quad v^2 = \frac{5}{12}$

$v = \pm\sqrt{\frac{5}{12}}$ — $\frac{1}{2}$

$\left[v = -\sqrt{\frac{5}{12}} \text{ for } 0 \leq t \leq t_1 \ (x = 0)\right]$

$\therefore$ the speed is $\sqrt{\frac{5}{12}}\ \text{ms}^{-1}$ $(0.645\ldots)$ — $\frac{1}{2}$

$\boxed{3}$

OR $\displaystyle\int_0^v d\left(\frac{1}{2}v^2\right) = \int_{\frac{1}{2}}^{0} (x^2 - 6x + 1)\,dx$

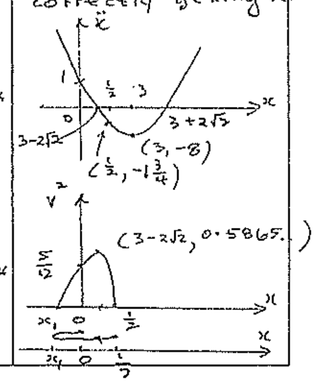
$\frac{1}{2}v^2 = \left[\frac{1}{3}x^3 - 3x^2 + x\right]_{\frac{1}{2}}^{0}$

$= 0 - \left(\frac{1}{24} - \frac{3}{4} + \frac{1}{2}\right)$

$= \frac{5}{24}$

$v^2 = \frac{5}{12}$ ETC.

Marker's comments: correctly getting to



Question (1)

[A]

(a) [3]

$$(2y - y^{-3})^{20}$$

$$T_{k+1} = \binom{20}{k}(20y)^{20-k}(-y^{-3})^{k} \quad (1)$$

$$= (-1)^k 2^{20-k}\binom{20}{k} y^{20-4k} \quad (1)$$

For constant term

$20 - 4k = 0 \Rightarrow k = 5$

$\therefore T_6$ is the constant term

$$T_6 = -2^{15}\binom{20}{5}$$
$$= -15504 \times 2^{15} \quad (1)$$
$$= -508035072$$

[2]

(b) $y = (r\sin\alpha)t - gt^2/2$ — (3)

(i) $\dot{y} = r\sin\alpha - gt$ — (1)

For max. height $\dot{y} = 0$

$\therefore t = \dfrac{r\sin\alpha}{g}$ — (2) (1)

Substitute (2) into (3)

We have.

$y_{max} = \dfrac{r^2\sin^2\alpha}{2g}$, but $y_{max} = 3h$

$\therefore r^2\sin^2\alpha = 6gh$.

$r\sin\alpha = \sqrt{6gh}$. (1)

[B]

(ii) Range $= d$ [2]

When $y = 0$

i.e. $t\left(r\sin\alpha - \dfrac{gt}{2}\right) = 0$

$\therefore T\left(\text{time of flight}\right) = \dfrac{2r\sin\alpha}{g}$ (1)

$R = (r\cos\alpha)\left(\dfrac{2r\sin\alpha}{g}\right)$

but $R(\text{range}) = d$.

$\therefore d = r\cos\alpha\left(\dfrac{2}{g}\sqrt{6gh}\right)$

$\therefore r\cos\alpha = \dfrac{gd}{2\sqrt{6gh}}$ (1)

(iii) [2]

$x = r\cos\alpha\, t$.

$y = (r\sin\alpha)t - \dfrac{gt^2}{2}$

Eliminate t

We have

$y = x\left(\dfrac{\sin\alpha}{\cos\alpha}\right) - \dfrac{gx^2}{2}\dfrac{1}{(r\cos\alpha)^2}$.

$x\left(\dfrac{r\sin\alpha}{r\cos\alpha}\right) = \dfrac{12hx}{d}$. (1)

$\therefore y = \dfrac{12hx}{d} - \dfrac{gx^2}{2} \times \dfrac{24gh}{g^2d^2}$

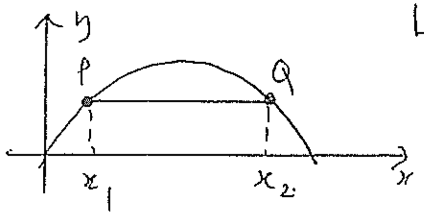
$\therefore y = \dfrac{12hx}{d}\left(1 - \dfrac{x}{d}\right)$. (1)

[2]

(iv) From (iii) When $y = h$

$$\cancel{h} = \frac{12\cancel{h}}{d}x\left(1-\frac{x}{d}\right)$$

$$\therefore \quad 12x^2 - 12dx + d^2 = 0$$ 1



Let the roots to be $x_1\, x_2$.

$$x_1 + x_2 = \frac{12d}{12} = d$$

$$x_1 x_2 = d^2/12.$$

$$\therefore \quad (x_2 - x_1)^2 = (x_2+x_1)^2 - 4x_2x_1$$

$$= d^2 - \frac{4}{12}d^2$$

$$= \frac{2d^2}{3}.$$

i.e. $PQ = (x_2 - x_1) = \frac{\sqrt{6}\,d}{3}.$ 1

or Use quad. formula

$$x = \frac{12d \pm \sqrt{144d^2 - 48d^2}}{24}.$$

Where $x_2 - x_1 = \left[\left(\frac{3+\sqrt{6}}{6}\right) - \left(\frac{3-\sqrt{6}}{6}\right)\right]d$

$$x = 100(3+\sqrt{6})t.$$

$$\therefore \quad \frac{(3+\sqrt{6})d}{6} = x_Q$$

$$\therefore \quad \frac{(3+\sqrt{6})d}{6} = 100(3+\sqrt{6})\,t \quad \Rightarrow \quad t = \frac{d}{600}.$$ 1

(v) [1]

Distance = Speed × time

$$\frac{\sqrt{6}\,d}{3} = u \times \frac{d}{600}$$ 1

$$\therefore \quad \frac{\sqrt{6}}{3} = \frac{u}{600}$$

$$u = 200\sqrt{6}\ \text{(m/s)}$$

$\therefore$ speed $\doteqdot$ 490 m/s.