Name:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2014

MATHEMATICS EXTENSION 1

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 2 hours.
- Write in black or blue pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 70

Section I: **10 marks**

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: **60 Marks**

- Attempt Question 11 - 14
- Answer on blank paper unless otherwise instructed. Start a new page for each new question.
- Allow about 1 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice sheet for Questions 1-10

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1. The focus of the parabola $(x-3)^2 = -8y$ is

a) $(3\,,-2)$ b) $(3\,,2)$ c) $(0\,,-2)$ d) $(-2\,,3)$

2. Find the acute angle between the lines $-x+2y+4=0$ and $3x+y+1=0$. Give your answer to the nearest minute.

a) 81°52’ b) 78°41’ c) 54°28’ d) 45°

3. Find the coordinates of the point that divides the interval A (-2, 7) B (12, 0) externally in the ratio 4:3.

a) (-44 , -28) b) (44, -28) c) (54, -21) d) (54, 21)

4. $\sin 2x$ equals to

a) $\frac{1-tan^2x}{1+tan^2x}$ b) $\frac{2\tan x}{1+tan^2x}$ c) $\frac{2\tan x}{1-tan^2x}$ d) $\frac{1+tan^2x}{1-tan^2x}$

5. 8 people are to be seated around a circular table. If 2 people wish to sit next to each other, how many different ways can this be done?

a) 720 b) 10080 c) 1440 d) 5040

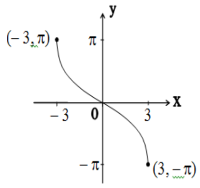
6. Use the trapezoidal rule to find an approximation for $\int_1^3 \log_e x \, dx$ using 2 subintervals.

a) 1.09 b) 1.10 c) 1.24 d) 1.25

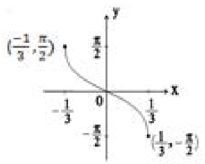
7. Find the $\lim\limits_{x \to 0} \frac{\sin x \cos x}{2x}$

a) 2 b) 1 c) $\frac{1}{2}$ d) $\frac{1}{4}$

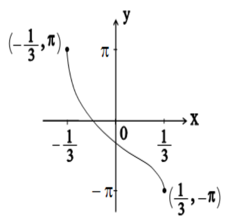
8. Which of the following is the graph of $y = -2sin^{-1}3x$?



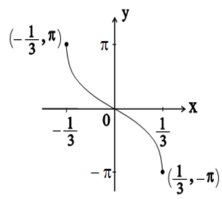
a)



b)



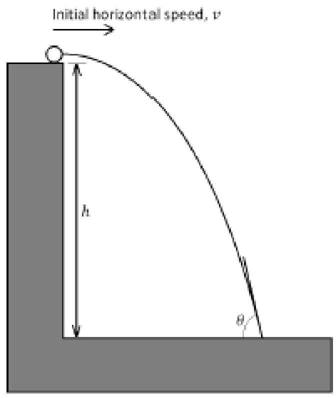
c)



d)

9. The diagram below shows the path of a projectile fired with a horizontal velocity v from a cliff of height h.

Which pair of the following values of v and h will give the greatest value of angle θ ?



a) $v = 10ms^{-1}$, $h = 30m$ b) $v = 30ms^{-1}$, $h = 50m$ c) $v = 50ms^{-1}$, $h = 10m$ d) $v = 10ms^{-1}$, $h = 50m$

10. The solution to

$$\frac{2t+1}{t-2} > 1$$

is

a) $t > -3$ b) $t > 2 \; or \; t < -3$ c) $t > -1$ d) $t > 3 \; or \; t < -2$

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE page. Extra writing papers are available.

In questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new page.

a)

Find **3**

$$\int x\sqrt{(x^2+1)^3}\ \ dx$$

using the substitution $u = x^2 + 1$

b)

The area bounded by the curve $y = cos2x$, the x-axis and lines $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ is rotated about the x-axis. Find the volume of the solid generated in exact form. **3**

c)

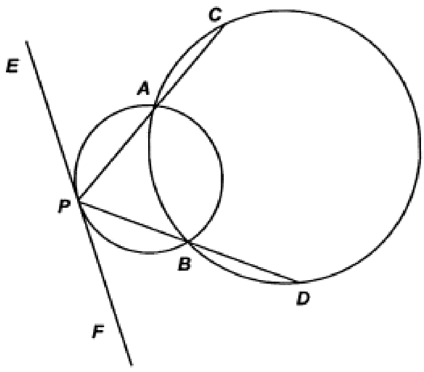
There are 4 families, and each family has exactly 4 children. Assume that the probability of giving birth to a male and giving birth to a female are even. **3**

Determine the probability that exactly 2 of the families will have exactly 2 males and 2 females as children.

Question 11 continues on page 5

d)

Copy the diagram into your answer booklet. *PAC, PBD* are straight lines. *EF* is the tangent at *P*. **3**



Prove CD || EF

e)

Prove by mathematical induction that for any positive integer $n \geq 1$. **3**

$$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\ldots\ldots\ldots\ldots+\frac{1}{(4n-3)(4n+1)}=\frac{n}{4n+1}$$

End of Question 11

Question 12 (15 marks) Start a new page.

a)

A body is in Simple Harmonic Motion and its position at a time t is given by the equation

$$x = R\cos(nt+\alpha)+1$$

The period of motion is π seconds and $0 \le \alpha \le \frac{\pi}{2}$. Initially the body is at rest 3 units to the left of the origin.

i. Find the values of R, n and α. **3**

ii. Find the velocity of the body when $t = \frac{\pi}{6}$ **1**

b)

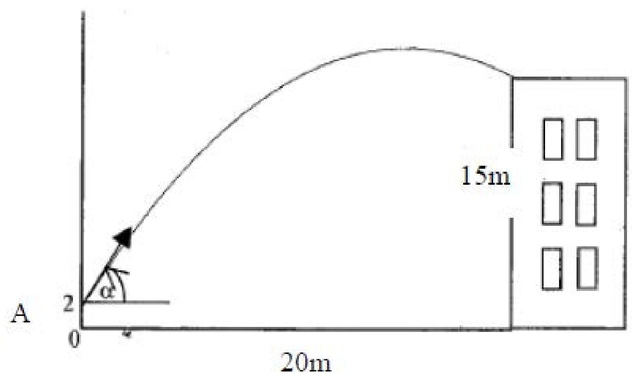
Show that the sum of the Arithmetic sequence **2**

$$\log_{10}(x-2)\,,\ \log_{10}(x-2)^2,\ \log_{10}(x-2)^3, \ldots \log_{10}(x-2)^n$$

is $\frac{n}{2}\log_{10}(x-2)^{n+1}$

c)

Andrew, whose height is 2 metres, throws a ball from area A in the direction of the Cohen building which is 15 metres high. He throws the ball with an initial velocity u at angle α, and he is 20 metres from the base of the building. (Assume $\ddot{x} = 0$ and $\ddot{y} = -10m/s^2$)



i. Show that $y = x\tan\alpha - \frac{5x^2}{u^2}(1+tan^2\alpha)+2$, at any time t. **2**

ii. Hence, find between which two angles of projection must he throw the ball to ensure that it lands on the roof of the building, or over, given that $u = 25m/s$.
(Answer to the nearest degrees). **3**

d)

i. Differentiate $x\tan^{-1}x$ **2**

ii. Hence, or otherwise, find $\int \tan^{-1}x\,dx$ **2**

End of Question 12

Question 13 (15 marks) Start a new page.

a)

At any time *t* minutes, the rate of cooling of a body with temperature *T*, when the surrounding temperature is *S*, is given by the differential equation

$$\frac{dT}{dt} = -k(T - S)$$

for some constant k.

i. Show that $T = S + Ae^{-kt}$, for some constant *A*, satisfies this differential equation. **1**

ii. A metal rod has an initial temperature of 1390°C and cools to 1060°C in 10 minutes when the surrounding temperature is 30°C. **4**

Find how much longer it will take the rod to cool to 110°C, giving your answer to the nearest minute.

b)

Find the monic cubic equation whose roots are the squares of the roots of **3**

$$x^3 + 2x + 1 = 0$$

c)

The acceleration of a particle *P* is given by the equation

$$\ddot{x} = 18x^3 + 27x^2 + 9x$$

Initially $x = -2$ and the velocity, $v = -6$.

i. Show that $v^2 = 9x^2(1 + x)^2$ **2**

ii. Hence, or otherwise, show that **2**

$$\int \frac{1}{x(1+x)}\, dx = -3t + C, \text{ for some constant C}$$

iii. Find the derivative of $\log_e(1 + \frac{1}{x})$ **1**

iv. Using your result in part iii and the initial conditions, find x as a function of t. **2**

End of Question 13

Question 14 (15 marks) Start a new page.

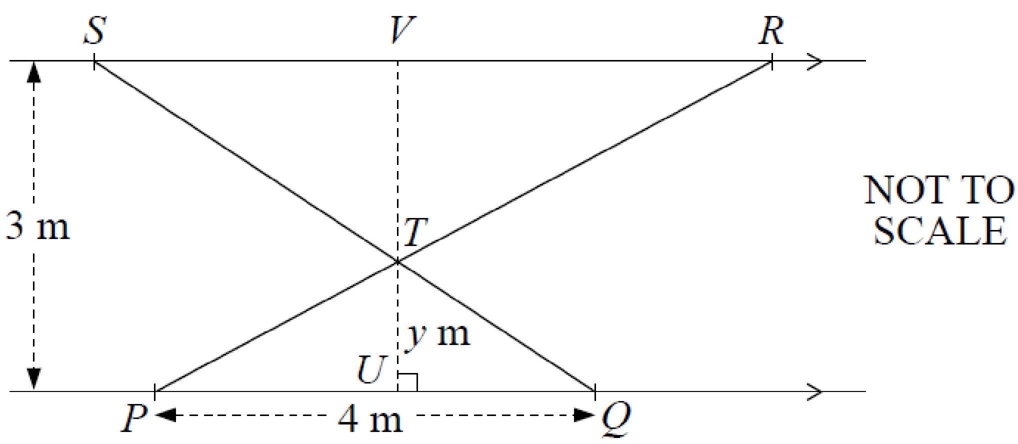
a)

Show that $x = 5sin\theta$ and $y = 5cos\theta + 1$ satisfies the equation **2**

$$y^2 + x^2 - 2y - 24 = 0$$

b)

In the diagram, PQ and SR are parallel railings which are 3 metres apart. The points P and Q are fixed 4 metres apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T Perpendicular to PQ intersects PQ at U and SR at V. The length of UT is y metres.



i. Deduce that $SR = \frac{12}{y} - 4$ **3**

ii. Hence show that the total area A of ΔPTQ and ΔRTS is **1**

$$A = 4y - 12 + \frac{18}{y}$$

iii. Find the value of y that minimises A. Justify your answer. **3**

c)

The coefficient of x^k in $(1 + x)^n$, where n is a positive integer, is denoted by c_k (so $c_k = {}^nC_k$)

i. Show that $c_0 + 2c_1 + 3c_2 + \cdots + (n + 1)c_n = (n + 2)2^{n-1}$ **3**

ii. Find the sum, **3**

$$\frac{c_0}{1 \times 2} - \frac{c_1}{2 \times 3} + \frac{c_2}{3 \times 4} - \cdots + (-1)^n \frac{c_n}{(n + 1)(n + 2)}$$

Showing all necessary working.

End of paper

Student Number 2014

Extension One Trial

Q1. A

2 A

3. C

4. B

5 C

6. ~~B~~ C

7. C

8 D

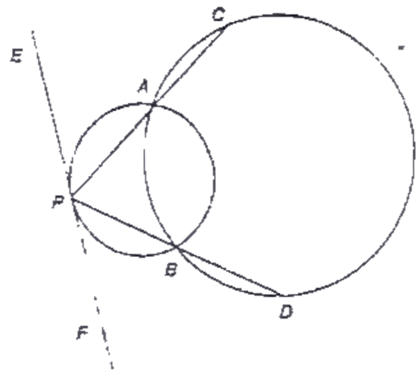
9 D

10 B

MATHEMATICS Extension 1 : Question 11

Suggested Solutions	Marks	Marker's Comments
a) Let $I = \int x\sqrt{(x^2+1)^3}\,dx$		
Let $u = x^2+1$		
$\frac{du}{dx} = 2x$		
$\therefore du = 2x\,dx$	1	
$I = \int \frac{1}{2}\sqrt{u^3}\,du$		
$= \frac{1}{2}\left[\frac{u^{5/2}}{5/2}\right]$		
$= \frac{u^{5/2}}{5}$	1	Many students failed to substitute back
$= \frac{(x^2+1)^{5/2}}{5}$	1	
b) $V = \pi\int_{\pi/4}^{3\pi/4} (\cos 2x)^2\,dx$		
$= \pi\int_{\pi/4}^{3\pi/4} \cos^2 2x\,dx$	1	~~Stu~~ some ~~ext 2~~ students made this question harder by trying to split it up.

Suggested Solutions	Marks	Marker's Comments
$= \frac{\pi}{2}\int_{\pi/4}^{3\pi/4} \cos 4x + 1\, dx$	1	Students who did not integrate $\int \cos^2 2x\, dx$ made question easier so could not gain carry forward errors.
$= \frac{\pi}{2}\left[\frac{1}{4}\sin 4x + x\right]_{\pi/4}^{3\pi/4}$		
$= \frac{\pi}{2}\left[\left[\frac{1}{4}0 + \frac{3\pi}{4}\right] - \left[0 + \frac{\pi}{4}\right]\right]$	1	
$= \frac{\pi}{2} \times \frac{\pi}{2}$		Some careless calculation errors.
$= \frac{\pi^2}{4}$ units^3		
c) Probability of exactly 2 males and 2 females $= {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$		
$= \frac{3}{8}$	1	many students stopped here.
Pr(exactly 2 families have exactly 2 males and 2 females)		
$= {}^4C_2\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^2$	1 for ${}^4C_2$	missing ${}^4C_2$ common error
$= \frac{675}{2048}$	1 for $\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^2$	

Suggested Solutions	Marks	Marker's Comments
d) 		
Construct AB and CD		
Let $E\hat{P}A = \alpha$		
$A\hat{B}P = \alpha$ (angle between tangent and chord equals angle at circumference in alternate segment)	1	students used opposite rather than alternate which is incorrect.
$A\hat{C}D = \alpha$ (exterior angle of cyclic quadrilateral ABCD equals opposite interior angle)	1	
$\therefore E\hat{P}A = A\hat{C}D = \alpha$.		
$\therefore CD \parallel EF$ (alternate angles are equal)	1	

MATHEMATICS Extension 1 : Question…11…		
Suggested Solutions	**Marks**	**Marker's Comments**
e) RTP $\frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}+\dots+\frac{1}{(4n-3)(4n+1)}=\frac{n}{4n+1}$		
Prove true for $n=1$		
LHS $\frac{1}{1\times5}=\frac{1}{5}$		Must clearly make substitution into LHS and RHS
RHS $\frac{1}{4+1}=\frac{1}{5}$	1	
$\therefore$ true for $n=1$		
Assume true for $n=k$. $k\in Z^+$		
$\frac{1}{1\times5}+\frac{1}{5\times9}+\dots+\frac{1}{(4k-3)(4k+1)}=\frac{k}{4k+1}$		
Prove true for $n=k+1$		
$\frac{1}{1\times5}+\frac{1}{5\times9}+\dots+\frac{1}{(4k-3)(4k+1)}+\frac{1}{(4k+1)(4k+5)}=\frac{k+1}{4k+5}$	1 for scaffolding	
by assumption		
LHS $=\frac{k}{4k+1}+\frac{1}{(4k+1)(4k+5)}$		
$=\frac{4k^2+5k+1}{(4k+1)(4k+5)}$		

MATHEMATICS Extension 1: Question 11

Suggested Solutions	Marks	Marker's Comments
$= \dfrac{(4k+1)(k+1)}{(4k+1)(4k+5)}$		
$= \dfrac{k+1}{4k+5}$	1	
By principle of mathematical induction true for all positive integers ≥ 1		

MATHEMATICS Extension 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
a) $x = R\cos(nt+\alpha)+1$		
period is π		
$\frac{2\pi}{n} = \pi$		
$\therefore n = 2$	1	
$\dot{x} = \cancel{R}\ -2R\sin(2t+\alpha)$		students who did not calulcte α first had trouble with R.
when $t=0$ $\dot{x}=0$		
$\therefore 0 = -2R\sin\alpha$		
Since $0 \le \alpha \le \pi/2$		
$\alpha = 0$	1	
$x = R\cos 2t + 1$		
when $x=-3$ $t=0$		
$-3 = R+1$		
$\therefore R = -4$	1	students changed to -4 stating $R>0$

C: Users Mum and dad Desktop Maths JRAHS Suggested Mk solns template_V2_no Ls.doc

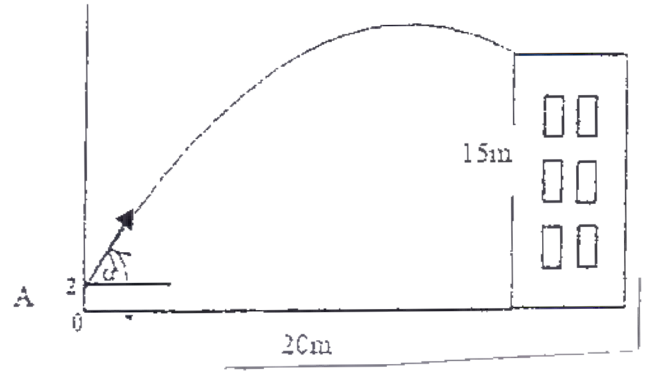
MATHEMATICS Extension 1: Question 12

Suggested Solutions	Marks	Marker's Comments
b) $a = \log_{10}(x-2)$ $d = \log_{10}(x-2)$ last term $\log_{10}(x-2)^n$ n terms $S_n = \frac{n}{2}(a+l)$ $= \frac{n}{2}\left(\log_{10}(x-2) + \log_{10}(x-2)^n\right)$ $= \frac{n}{2}\left(\log_{10}(x-2)^{n+1}\right)$ using $\frac{n}{2}(2a+(n-1)d)$ also worked (3)		

Ext 1 **MATHEMATICS: Question** 12

Suggested Solutions	Marks	Marker's Comments

c)



i) $\ddot{y} = -10$ $\qquad$ $\ddot{x} = 0$

Many students failed to include constants

$\dot{y} = -10t + c_1$ $\qquad$ $\dot{x} = c_3$

when $c_3 = u\cos\alpha$ $t = 0$

$t = 0$ $\dot{y} = u\sin\alpha$ $\qquad$ $\dot{x} = u\cos\alpha$

tes $\therefore c_1 = u\sin\alpha$

$x = ut\cos\alpha + c_4$

$\dot{y} = -10t + u\sin\alpha$ $\qquad$ when $t = 0$ $x = 0$

$\dot{y} = -5t^2 + ut\sin\alpha + c_2$ $\qquad$ $x = ut\cos\alpha$

$t = 0$ $y = 2$ $\qquad$ $t = \dfrac{x}{u\cos\alpha}$ $\qquad$ 1

$\therefore 2 = c + c_2$

$c_2 = 2$

$y = -5t^2 + ut\sin\alpha + 2$

$$y = -5\left(\frac{x}{u\cos\alpha}\right)^2 + u \times \frac{x}{\cos\alpha}\sin\alpha + 2$$

$$= \frac{-5x^2}{u^2\cos^2\alpha} + x\tan\alpha + 2 \qquad 1$$

$$= -\frac{5x^2}{u^2}\sec^2\alpha + x\tan\alpha + 2$$

Needed to have $\sec^2\alpha$ somewhere in solution

$$= x\tan\alpha - \frac{5x^2}{u^2}(1+\tan^2\alpha) + 2$$

since $\sec^2\alpha = 1 + \tan^2\alpha$

Ext 1 MATHEMATICS: Question……..

Suggested Solutions	Marks	Marker's Comments
$y \geq 15 \quad u = 25\,m/s \quad x = 20$		some students used 13 but equation had already taken +2 into account.
$15 \leq 20\tan\alpha - \frac{5\times(20)^2}{25^2} - \frac{5\times 20^2}{25^2}\tan^2\alpha + 2$	1	
$\therefore 16\tan^2\alpha - 100\tan\alpha + 81 \leq 0$		
for $16\tan^2\alpha - 100\tan\alpha + 81 = 0$	1	
$\tan\alpha = \frac{100 \pm \sqrt{100^2 - 4\times 81\times 16}}{32}$		
$= \frac{25 \pm \sqrt{301}}{8}$		
$\tan\alpha = 5.2936$ or 0.956331.		
$\therefore 44^0 \leq \alpha \leq 79^0$	1	
d(i) $\frac{dy}{dx} = \tan^{-1}x + \frac{x}{1+x^2}$	1 for each	Question well done.
(ii) $\int \tan^{-1}x\,dx = \int \frac{d}{dx}x\tan^{-1}x\,dx - \int\frac{x}{1+x^2}dx$		
$\therefore \int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + c.$		1 for $x\tan^{-1}x + c$ 1 for $-\frac{1}{2}\ln(1+x^2)$

Suggested Solutions	Marks	Marker's Comments
a) i) Substitute $T = S + Ae^{-kt}$ into eqn.		A show question needs a little more structure than one line.
$LHS = \frac{dT}{dt} = -kAe^{-kt}$		
$RHS = -k(T-S) = -k(Ae^{-kt})$ from above $= -kAe^{-kt}$		
$LHS = RHS$		
So $T = S + Ae^{-kt}$ is a solution of $\frac{dT}{dt} = -k(T-S)$	1	
ii) $S = 30$ and when $t=0$, $T = 1390$		
$\therefore 1390 = 30 + A$, $A = 1360$	1	
When $t = 10$, $T = 1060$		Most people got these 2 marks
$\therefore 1060 = 30 + 1360e^{-10k}$		
$e^{-10k} = \frac{1030}{1360}$		
$10k = \ln\left(\frac{136}{103}\right)$		
$k = \frac{1}{10}\ln\left(\frac{136}{103}\right)$ $(= 0.02779)$	1	
Find t when $T = 110$		
$110 = 30 + 1360e^{-\frac{t}{10}\ln\left(\frac{136}{103}\right)}$		
$\therefore \frac{1360}{80} = e^{\ln\left(\frac{136}{103}\right)\frac{t}{10}}$		
$\frac{136}{8} = \left(\frac{136}{103}\right)^{t/10}$		

Suggested Solutions	Marks	Marker's Comments
$\therefore t = 10\dfrac{\ln\left(\frac{136}{5}\right)}{\ln\left(\frac{136}{103}\right)}$		I sympathise with those who
$= 101.94\ldots$	1	lost the last
$= 102$ to nearest minute.		mark – remember to answer the
$\therefore$ Takes $\underline{\underline{92 \text{ more}}}$ minutes	1	question asked.

b) METHOD 1

From original equation

$$\left.\begin{aligned} \alpha+\beta+\gamma &= \text{"}-b/a\text{"} = 0 \\ \alpha\beta+\beta\gamma+\gamma\alpha &= \text{"}c/a\text{"} = 2 \\ \alpha\beta\gamma &= \text{"}-d/a\text{"} = -1 \end{aligned}\right\}$$

1

A couple of students mixed up "equation" with "polynomial"

For new equation

$$\begin{aligned} \alpha^2+\beta^2+\gamma^2 &= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha) \\ &= 0 - 2\times 2 \\ &= \underline{\underline{-4}} \end{aligned}$$

$$\begin{aligned} \alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2 &= (\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2(\alpha+\beta+\gamma)\alpha\beta\gamma \\ &= 2^2 - 2\times 0\times -1 \\ &= \underline{\underline{4}} \end{aligned}$$

$$\begin{aligned} \alpha^2\beta^2\gamma^2 &= (\alpha\beta\gamma)^2 \\ &= (-1)^2 \\ &= \underline{\underline{1}} \end{aligned}$$

1 for two of the three correct

Monic $\Rightarrow$ "a" $=1$, $\therefore$ "b" $= 4$, "c" $= 4$, "d" $= -1$

Eqn is $\underline{\underline{x^3+4x^2+4x-1=0}}$ 1

Suggested Solutions	Marks	Marker's Comments

METHOD 2

α, β, γ are x values

If y has values $\alpha^2, \beta^2, \gamma^2$ then

$y = x^2$

$x = \sqrt{y}$

Substitute into original equation

$y\sqrt{y} + 2\sqrt{y} + 1 = 0$ — 1

$\sqrt{y}(y+2) = -1$

$y(y+2)^2 = (-1)^2$ — 1

$y(y^2+4y+4) = 1$

$\underline{y^3 + 4y^2 + 4y - 1 = 0}$ — 1

c) i) $\ddot{x} = \frac{d}{dx}\left(\frac{v^2}{2}\right) = 18x^3 + 27x^2 + 9x$

$\therefore \frac{v^2}{2} = \frac{18x^4}{4} + 9x^3 + \frac{9x^2}{2} + k_1$

$v^2 = 9x^4 + 18x^3 + 9x^2 + k_2$

$= 9x^2(x^2+2x+1) + k_2$

$= 9x^2(x+1)^2 + k_2$

When $x = -2$, $v = -6$.

$\therefore 36 = 9 \times 4 \times (-1)^2 + k_2$

$\therefore k_2 = 0$ — 1

$\therefore \underline{v^2 = 9x^2(x+1)^2}$ — 1

One mark for integrating and evaluating constant.

One for tidying into given form.

Suggested Solutions	Marks	Marker's Comments
c) ii) $V^2 = 9x^2(1+x)^2$ $\therefore V = \pm 3x(1+x)$ Initially, $v=-6$ (moving to left) and $x=-2$ and acceleration $= -54$ also to left. $\therefore$ velocity always negative. $V = \frac{dx}{dt} = -3x(1+x)$ $\therefore \int \frac{dx}{x(1+x)} = -3\int dt$ $\int \frac{dx}{x(1+x)} = -3t + C$	1 1	The first mark needed hard work if you didn't start with $\pm$ sign.
iii) $\frac{d}{dx}\left(\ln\left(1+\frac{1}{x}\right)\right) = \frac{-1/x^2}{1+1/x}$ $= \frac{-1}{x^2+x} = \frac{-1}{x(x+1)}$	1	
iv) From (iii) $\int \frac{dx}{x(1+x)} = -\ln\left(1+\frac{1}{x}\right) + K$ $\therefore$ From (ii) $-3t + C = -\ln\left(1+\frac{1}{x}\right) + K$ $3t = \ln\left(1+\frac{1}{x}\right) + K_2$ When $t=0$, $x=-2$ $\therefore K_2 = \ln 2$ $\therefore e^{3t} = 2\left(1+\frac{1}{x}\right)$ $e^{3t} - 2 = \frac{2}{x}$ $\therefore x = \frac{2}{e^{3t}-2}$	1 1	People should try to be consistent with constants. Don't use same letter if it has been manipulated.

$1/4$

2014 TRIAL **MATHEMATICS Extension 1 : Question** 14

Suggested Solutions	Marks	Marker's Comments
i) $y^2+x^2-2y-24=0$		
$=(5\cos\theta+1)^2+(5\sin\theta)^2-2(5\cos\theta+1)-24$		1
$25\cos^2\theta+10\cos\theta+1+25\sin^2\theta-10\cos\theta-2-24$		
$=25(\cos^2\theta+\sin^2\theta)-25$		
$=25(1)-25$ (as $\sin^2\theta+\cos^2\theta=1$)		1
$=0$		
$=$RHS.		
b) (i) In $\triangle STR$, PTQ		
$\angle STR=\angle PTQ$ (vertically opposite angles are equal)		
$\angle TSR=\angle TQP$ (alternate angles are equal, $SR\parallel QP$)		1
$\therefore \triangle STR \mid\mid\mid \triangle PTQ$ (equiangular)		
$\frac{SR}{PQ}=\frac{RT}{TQ}$ (corresponding sides in similar triangles are in the same ratio).		1
$SR=4\times\frac{(3-y)}{y}$	1	
$=\frac{12}{y}-4$		
ii) Total area $=\triangle PTQ+\triangle STR$		
$=\frac{1}{2}\times4\times y+\frac{1}{2}\times\left(\frac{12}{y}-4\right)(3-y)$		
$=2y+\left(\frac{6}{y}-2\right)(3-y)$		1
$=2y+\frac{18}{y}-6-6+2y$		
$=4y-12+\frac{18}{y}$		
iii) $A=4y-12-\frac{18}{y}$		
$\frac{dA}{dy}=4-\frac{18}{y^2}$		
$\frac{d^2A}{dy^2}=\frac{36}{y^3}$		

MATHEMATICS Extension 1 : Question 17

Suggested Solutions	Marks	Marker's Comments
possible stat. pts when $\frac{dA}{dy}=0$		1
ie $4-\frac{18}{y^2}=0$		
$4y^2=18$		*lost 1mk you "lost" a without
$y^2=\frac{18}{4}=\frac{9}{2}$		
$y=\pm\frac{3}{\sqrt2}$		
$y>0$ as its a length; so $y=\frac{3}{\sqrt2}$ only		1
when $y=\frac{3}{\sqrt2}$ $\frac{d^2A}{dy^2}=\frac{8}{3\sqrt2}>0$		
$\therefore$ concave up		
$\therefore$ relative minimum at $y=\frac{3}{\sqrt2}$		
since there's only one stat. pt in the domain $y>0$, the relative minimum is the absolute minimum.		1
c) (i) $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+\dots\dots+{}^nC_nx^n$		poorly
let $x=1$		If you di
$\therefore (1+1)^n={}^nC_0+{}^nC_1+{}^nC_2+\dots\dots+{}^nC_n$		lost
$\therefore 2^n={}^nC_0+{}^nC_1+{}^nC_2+\dots\dots+{}^nC_n$ --- ①		1 mk
differentiate wrt x the first eqn.		
$n(1+x)^{n-1}={}^nC_1+2\,{}^nC_2x+3\,{}^nC_3x^2+\dots+n\,{}^nC_nx^{n-1}$		
sub in $x=1$,		
$n2^{n-1}={}^nC_1+2\,{}^nC_2+3\,{}^nC_3+\dots\dots+n\,{}^nC_n$ --- ②		1 mk
② + ①		
$2^n+n2^{n-1}={}^nC_0+{}^nC_1+{}^nC_1+{}^nC_2+2\,{}^nC_2+{}^nC_3+3\,{}^nC_3+\dots\dots+{}^nC_n+n\,{}^nC_n$		
$2^{n-1}(2+n)={}^nC_0+2\,{}^nC_1+3\,{}^nC_2+4\,{}^nC_3+\dots\dots+(n+1)\,{}^nC_n$		1

MATHEMATICS Extension 1 : Question 14

Suggested Solutions	Marks	Marker's Comments

(ii) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x + \dots + {}^nC_n x^n$

integrate both sides wrt x

$$\frac{(1+x)^{n+1}}{n+1} + K_1 = {}^nC_0 x + \frac{{}^nC_1 x^2}{2} + \frac{{}^nC_2 x^3}{3} + \dots + \frac{{}^nC_1 x^{n+1}}{n+1}$$

sub in $x=0$

$$\frac{1}{n+1} + K_1 = 0$$

$$K_1 = -\frac{1}{n+1}$$

$$\therefore \frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1} = {}^nC_0 x + \frac{{}^nC_1 x^2}{2} + \frac{{}^nC_2 x^3}{3} + \dots + \frac{{}^nC_n x^{n+1}}{n+1}$$ 1

Integrate both sides wrt x

$$\frac{(1+x)^{n+2}}{(n+1)(n+2)} - \frac{x}{n+1} + K_2 = \frac{{}^nC_0 x^2}{2} + \frac{{}^nC_1 x^3}{2\times 3} + \frac{{}^nC_2 x^4}{3\times 4} + \dots + \frac{{}^nC_n x^{n+2}}{(n+1)(n+2)}$$

sub in $x=0$

$$\frac{1}{(n+1)(n+2)} - 0 + K_2 = 0$$

$$K_2 = \frac{-1}{(n+1)(n+2)}$$

$$\therefore \frac{(-x)^{n+2}}{(n-)(n-2)} - \frac{x}{n+1} - \frac{1}{(n+1)(n+2)} = \frac{{}^nC_0 x^2}{1\times 2} + \frac{{}^nC_1 x^3}{2\times 3} + \dots + \frac{{}^nC_n x^{n+2}}{(n+1)(n+2)}$$ 1

sub in $x=-1$

$$0 + \frac{1}{(n+1)} - \frac{1}{(n+1)(n+2)} = \frac{{}^nC_0}{1\times 2} - \frac{{}^nC_1}{2\times 3} + \frac{{}^nC_2}{3\times 4} - \dots + \frac{(-1)^n\,{}^nC_n}{(n+1)(n+2)}$$

$$\frac{n+2}{(n+1)(n+2)} - \frac{1}{(n+1)(n+2)} = \frac{{}^nC_0}{1\times 2} - \frac{{}^nC_1}{2\times 3} + \frac{{}^nC_2}{3\times 4} - \dots + \frac{(-1)^n\,{}^nC_n}{(n+1)(n+3)}$$

$$\text{series} = \frac{n+1}{(n+1)(n+2)} = \frac{1}{n+2}$$ 1

MATHEMATICS Extension 1 : Question 14

Suggested Solutions	Marks	Marker's Comments

5)(ii) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

integrate both sides

$$\frac{(1+x)^{n+1}}{n+1} = {}^nC_0 x + \frac{{}^nC_1 x^2}{2} + \frac{{}^nC_2 x^3}{3} + \dots + \frac{{}^nC_n x^{n+1}}{n+1}$$

Integrate both sides

$$\frac{(1+x)^{n+2}}{(n+1)(n+2)} = \frac{{}^nC_0 x^2}{1\times2} + \frac{{}^nC_1 x^3}{2\times3} + \frac{{}^nC_2 x^4}{3\times4} + \dots + \frac{{}^nC_n x^{n+2}}{(n+1)(n+2)}$$

substitute $x=-1$

$$\frac{(1+-1)^{n+2}}{(n+1)(n-2)} = +\frac{{}^nC_0}{1\times2} - \frac{{}^nC_1}{2\times3} + \frac{{}^nC_2}{3\times4} - \dots + \frac{(-1)^n\,{}^nC_n}{(n+1)(n+2)}$$

$$\therefore 0 = \frac{{}^nC_0}{1\times2} - \frac{{}^nC_1}{2\times3} + \frac{{}^nC_2}{3\times4} - \dots + \frac{(-1)^n\,{}^nC_n}{(n+1)(n+2)}$$

$\therefore$ sum of series is 0.

1 for integrating twice + subbed in $x=-1$.

1 for attempting to evaluate the constant of integration

1 for fully correction solution